Errata for Real Analysis Text

12/15/2016

p. 7: Exercise 1(e): Add the condition $c \neq 0$.

p. 20: Example 1.5.4: $B(n+1)^3 - n^2$ should be $B(n+1)^2 - n^2$.

p. 34: Exercise 9: Add the condition $k > 1$.

p. 51: Theorem 3.1.11: In statement, replace the inequality $|x - y| < \delta$ by $|x - a| + |y - a| < \delta$. In proof, delete phrase “$|a_n - b_n| \to 0$ so”.

p. 64: Line 2: delete the word “below”.

p. 66: Exercise 3.4.5: Omit part (b) and delete solution on p. 526.

p. 66: Exercise 3.4.7(f): Change $2x^2 - 5$ to $2x^2 - 4.8$.

p. 66: Exercise 3.4.8: Change $\mathbb{R}$ to $(0, +\infty)$.

p. 82: Exercise 4.2.7: Add phrase “on the interval $(-\pi/2|a|, \pi/2|a|)$” at end.

p. 100: Exercise 4.5.21: Replace the interval $(0, 1)$ by $(-1, 1)$.

p. 109: Last line of proof of 5.1.7: Last phrase should read “infimum over $\mathcal{P}$ and then the supremum over $\mathcal{Q}$”.

p. 109: Line 7 from bottom: Replace $\mathcal{S}(f, P_\varepsilon) - \mathcal{S}(f, P_\varepsilon) < \varepsilon$ by $\mathcal{S}(f, P_\varepsilon) - \mathcal{S}(f, P_\varepsilon) < \varepsilon$.

p. 110: Line 5 from bottom: Replace $\Delta x_j < r$ by $\Delta x_j = r$.

p. 124: Example 5.3.4: Add “for $k > 1$” after “We show that”.

p. 125: Example 5.3.5: Last term in square brackets should be $\frac{6}{5^4}$.

p. 125: Example 5.3.5: Table caption should read “…$\int (x + 1)^3 e^{5x} \, dx$…”

p. 126: Exercise 5.3.2: Replace the phrase “Prove that $f' \in \mathcal{R}^b_a$” by “Prove that the continuous extension of $f'$ to $[a, b]$ satisfies”.

p. 133: Exercise 5.5.1: Integral coefficients $a$, $b$ on the right should be $\sqrt{a}, \sqrt{b}$.

p. 133: Exercise 5.5.7: Second integral on the right should be $f_c^b g$.

p. 146: Theorem 5.7.9: Limit is from the left: $\lim_{x \to b^-} f(x)/g(x)$.

p. 147: Theorem 5.7.11: $b$ should be $\infty$ throughout.

p. 149: Exercise 5.7.4: add the phrase “and $\int_1^\infty f$ and $\int_1^\infty g$ converge”
p. 151: Exercise 5.7.27: g should be bounded.

p. 155: Equation (5.31): \( S(f', P, \xi) \) should be \( S(|f'|, P, \xi) \).

p. 157: Line 3: Remove the paragraph starting with "Conversely," and add "Conversely, if \( f \) is continuous at 1, then for small \( \|P\| \) the quantities \( f(\xi_n) \) approximate \( f(1) \), so \( f \in \mathcal{R}_0(w) \)."

p. 175: Exercise 6.2.12: Start the problem with the sentence "Let \( 0 < a_n < r \), where \( r \) is sufficiently small."

p. 187: Problem 4(r): Replace \((1)^n\) by \((-1)^n\).

p. 193: Definition 7.1.1: Limit should be \( f = \lim_n f_n \).

p. 194: Example 7.1.6(e): Replace \( \max\{|a|, ||b||\} \) by \( \max\{|a|, |b|\} \).

p. 195: Theorem 7.1.9(d): Convergence should read \( \frac{1}{g_n} \to \frac{1}{g} \).

p. 198: Exercise 7.1.8: Delete last sentence (regarding example).

p. 199: Exercise 7.1.16: Replace \( (b)^a \) by \( (b) \).

p. 210: Exercise 12: Intervals should be closed as well.

p. 212: Theorem 7.4.2: Replace \( c_n > 0 \) by \( c_n \neq 0 \).

p. 244: Example 8.3.7: The inequality defining \( A \) should be \( 0 < x \leq 2/\pi \).

p. 264: Inequality (8.6) should read \( \rho(f_n(x_n), f_n(a)) \geq \varepsilon \).

p. 270: Corollary 8.7.6: Last line of proof: Change \( A \) to \( B \).

p. 271: Theorem 8.7.8: Change \( \alpha \) to \( \phi \) in proof.

p. 296: Third line from top: Change \( \leq 1 \) to \( \leq \|T\| \).

p. 305: Exercise 9: \( \nabla(f \circ \alpha) = \alpha' \) should read \( (\nabla f) \circ \alpha = \alpha' \).

p. 329: Exercise 7: \( P(x \pm 1) \) should read \( P(x \pm 1, y \pm 1) \).

p. 350: 2nd line: Inequality should read \( |J_k| < |I_k| + \varepsilon/2^k \).

p. 351: Exercise 3: Rephrase as follows:

Let \( \mathcal{U} \) be the collection of all bounded open subsets of \( \mathbb{R} \) and \( \mathcal{K} \) the collection of all compact sets. Prove that \( \lambda^*(A) \) equals each of the following:

(a) \( \inf \left\{ \sum_j \lambda^*(U_j) : U_j \in \mathcal{U} \text{ and } \bigcup_j U_j \supseteq A \right\} \).

(b) \( \inf \left\{ \sum_j \lambda^*(K_j) : K_j \in \mathcal{K} \text{ and } \bigcup_j K_j \supseteq A \right\} \).
p. 352: Replace line 16 from top by: Since \((C \cap E) \cup (C \cap E^c \cap F) = C \cap (E \cup F)\), by (10.4),

p. 353: Replace the dyadic expansions .00220... and .22202... in Figure 10.3 by .0220... and .2202..., respectively.

p. 354: Replace the dyadic expansions .00220... and .22202... below Figure 10.4 by .0220... and .2202..., respectively.

p. 354: Line 12 from bottom: Replace “(Figure 11.2)” by “(Figure 10.3)”.

p. 354: Replace the dyadic expansions .00220... and .22202... below Figure 10.4 by .0220... and .2202..., respectively.

p. 354: Line 12 from bottom: Replace “(Figure 11.2)” by “(Figure 10.3)”.

p. 364: Middle: Replace \(A := \bigcup_{n=1}^{\infty} A_k\) by \(A := \bigcup_{k=1}^{\infty} A_k\).

p. 364: First line of Proposition 10.5.12: replace “be” by “is”.

p. 378: Ex. 11: Replace \(\leq\) by \(\geq\).

p. 517: Sec. 1.2.1(f): Replace \(bc(bd^{-1})\) by \(bc(bd)^{-1}\).

p. 518: Ex. 10: Replace \(x = a - b\) by \(x = (a - b)/2\).

p. 520: Sec. 2.1, Ex. 3: Replace min by max.

p. 521: Sec. 2.1, Ex. 17: Replace \(a^n\) by \(a_n\).

p. 521: Sec. 2.1, Ex. 22: Replace \(d j x\) by \(d + j x\).

p. 531: Sec. 4.5, Ex. 1(s): Limit = \(+\infty\) in all cases.

p. 541: The solution to 6(a) should read: Diverges for all \(p\).

p. 552: Sec. 8.3, Ex. 3(b): Add the point \((1, 0, 1)\).

p. 561: Ex. 9.7.1(b) : Last term should be \(\frac{\partial^3 f}{\partial y^3} (dy)^3\).

p. 566: Ex. 10.3.4 : Replace n’s by k’s.

Solution Manual

p. 4: Section 1.2, Indices \(j\) and \(k\) in problem 7 should start at 0.

p. 20: Section 2.3, answer to 1(b) should be \(-1, 0, 2\).

p. 26: Replace the solution to 3.1.10 with the following:

Assume \(\lim_{x \to c} f(x)\) does not exist. Suppose first that \(\lim_{x \to c} f(x)\) does not exist. Then, by the Cauchy criterion for functions, there exist sequences of points \(x_n, y_n < c\) with \(x_n, y_n \to c\), and \(\varepsilon > 0\) such that \(|f(x_n) - f(y_n)| \geq \varepsilon\) for all \(n\). Construct a subsequence \(\{x'_n\}\) of \(\{x_n\}\) such that \(x'_n \uparrow c\). Let \(\{y'_n\}\) denote the corresponding subsequence of \(\{y_n\}\), and choose a subsequence \(\{y''_n\}\) such that \(y''_n \uparrow c\). Let \(z_n\) be any sequence with \(z_n \downarrow c\). Then not both \(f(x''_n) - f(z_n) \to 0\) and \(f(y''_n) - f(z_n) \to 0\).
This proves the assertion for the case that \( \lim_{x \to c^-} f(x) \) does not exist. The proof for \( \lim_{x \to c^+} f(x) \) is similar. Finally, suppose \( \lim_{x \to c^-} f(x) \) and \( \lim_{x \to c^+} f(x) \) both exist but are unequal. Then choose any \( a_n \uparrow c \) and \( b_n \downarrow c \).

p. 31: Delete (b). Replace the solution of (a) with the following:
For example on an interval \([[(k + 1)^{-1}, k^{-1}]\), let \( a_n \) strictly decrease to \((k + 1)^{-1}\), where \( a_1 = 1/k \). Consider the polygonal graph obtained by connecting \((a_1, 0)\) to \((a_2, 1)\) to \((a_3, 0)\), etc.

p. 45: Replace solution to Exercise 4.5.3(c) by the following:
\[
a_n = \frac{(1 + 1/n)^n - e}{1/n}
\]
is of the form \( 0/0 \) hence has the same limit as
\[
-n^2 \frac{d}{dn} (1 + 1/n)^n = (1 + 1/n)^n \left[ n/(1 + 1/n) - n^2 \ln(1 + 1/n) \right].
\]
The first factor tends to \( e \). The second factor may be written
\[
\frac{(n + 1)^{-1} - \ln(1 + 1/n)}{n^{-2}},
\]
which has the same limit as
\[
\frac{-(n + 1)^{-2} + n^2/(1 + n^{-1})}{-2n^{-3}} \to -1/2.
\]
Therefore, \( a_n \to -e/2 \).

p. 55: Replace solution to Exercise 8 by the following:
(a) Let \( x_0 \) be a continuity point of \( f \) with \( f(x_0) > 0 \). Choose \( \delta > 0 \) such that \( f(x) > f(x_0)/2 \) for \( x_0 - 2\delta < x < x_0 + 2\delta \). Let \( g \) be a continuous function that has value 1 on \([x_0 - \delta, x_0 + \delta]\), 0 on the complement of \([x_0 - 2\delta, x_0 + 2\delta]\) and linear in between. Then \( \int_a^b fg > 0 \).
(b) Similar to (a).

p. 57: In 5.3.2, delete the phrase "The desired result follows by summing on \( j \)."

p. 72: The solution to 6(a) should read: Diverges for all \( p \).

p. 88: The numerator in (†) should be \((1 - x^2/n^2)\).

p. 157: Replace solution in 4(b) by the following:
This follows by taking \( K = C_r(x_0) \) in (a) and noting that by continuity \( f(x_K) \to f(x_0) \) as \( r \to 0 \).

p. 110: Replace solution of 14(b) by the following:
Let \( x \in X \) and \( r > 0 \). Suppose that \( B_r(x) \setminus \{x\} \) contains only finitely many \( a_n \), say \( a_{n_1}, \ldots, a_{n_k} \). Set \( s = \min\{\|x - a_{n_1}\|, \ldots, \|x - a_{n_k}\|\} \).
Then $s > 0$ so and $B_s(x) \setminus \{x\}$ is a nonempty open set disjoint from \{a_1, a_2, ...\} contradicting the hypothesis. Therefore, $B_r(x)$ contains infinitely many $a_n$.

p. 127,8: Solution to 4(b) should read
\[ d_{g_x}(h) = \frac{\|x\|^2h - (2x \cdot h)x}{\|x\|^4} \] and \[ d_{g_x}(x) = -\frac{x}{\|x\|^2}. \]

p. 130: Second line of solution to 11:
Switch the matrices $[z_u, z_u]$ and $[z_x, z_y]$ in the first equality.

p. 151: Ex. 10.3.4: Replace $n$’s by $k$’s.