Abstract

We present some structural theorems on the lattice $\mathcal{L}(V_\infty)$ of computably enumerable vector spaces and its factorlattices. The reader who is not familiar with these concepts can look at the introduction where the basic definitions are given together with some of the known results in the field. Some easy facts, that may or may not be new, or not available in the literature are also stated there.

In Chapter 2 we prove the existence of a *major* subspace V of a given c.e. space W such that no basis of V is extendable.

In the first section of Chapter 3, we prove that the strongly supermaximal spaces together with the co-finite dimensional spaces form a filter in $\mathcal{L}(V_\infty)$. We also prove that this is not true for the class of all supermaximal and co-finite dimensional spaces. We do prove, however, that if V is a fixed supermaximal space, then the class of all $k-thin$ spaces V_1 such that $V_1 =^* V$, is the least filter that contain the space V and the co-finite dimensional spaces.

In the second section of Chapter 3, we give a necessary and sufficient condition for the principal filter B_n of the equivalence class of a quasimaximal subset I of a computable basis to lift to an isomorphic principal filter in $\mathcal{L}^*(V_\infty)$. As a

corollary of this characterization we obtain a solution to an open problem posed by Downey and Remmel.

In Chapter 4 we construct an embedding of a special type of finite distributive lattices into principal filters of the lattice $\mathcal{L}^*(V_\infty)$. The images of the meet irreducible elements are given specific Turing dependence degrees. The structure of these Turing degrees, partially ordered by Turing reducibility, is antiisomorphic with the structure of the meet irreducible elements with partial order inherited from the lattice.

Finally, in Chapter 5 we construct a computably enumerable space V with any increasing sequence of Turing degrees as an initial segment of its dependence degree spectrum, and such that the lattice of c.e. subspaces of V_∞/V has no nontrivial computable automorphisms.