GWU Department of Mathematics

Topics in Model Theory: Classical and Computable

(CRN 46947 Math 6720: Topics in Logic)

Spring 2015

TuTh 3:45–5:00p.m.

Duquès Hall, Room 362

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- **Description**
  We will cover a variety of important topics in model theory, and their counterparts in computable model theory. Model theory, emerged as a distinct field in the 1940's through the works of Gödel, Löwenheim, Malcev, Skolem, Tarski, and others. Early model theory provided a rigorous framework for the notions of language, meaning, and truth. A model, a concept used in all of sciences, describes a portion of reality by using a formal language to express properties under study. Model theory investigates structural nature of mathematical objects and looks at the ways in which algebraic structures or classes of algebraic structures can be described or classified. Computable model theory, which has received increasing attention in the last few decades, studies computability-theoretic phenomena in countable structures and investigates algorithmic nature of model-theoretic constructions. While some constructions are algorithmic, or can be replaced by algorithmic ones yielding the same results, others are intrinsically non-algorithmic. Examples of negative results in computable model theory include the undecidability of the Hilbert's tenth problem, and the undecidability of the word problem on groups.

The course will be, in some sense, self-contained. We will start by reviewing the fundamental concepts of classical model theory. We will then move to more advanced topics such as amalgamation of structures and interpreting one structure in another. We will pay attention to algorithmic content in theories and structures. Selection of particular examples will take into consideration the backgrounds and interests of the students.

- **Required Background**
  Mathematical maturity and familiarity with basic abstract algebra. Math 6720 can be taken for credit repeatedly. Advanced undergraduate students may also take this course for credit.

- **Grading**
  Based on class participation and in-class presentation (30%), take-home assignments (50%), and an independent project (20%).

- **Textbook and other course material**

  (1) *A Shorter Model Theory* by Wilfred Hodges, Cambridge University Press, 1997.

  Other material, including papers on computable model theory, will be provided in class.

  (2) E. Fokina, V. Harizanov, and A. Melnikov, "Computable model theory," in the volume *Turing's Legacy: Developments from Turing Ideas in Logic*, Cambridge University Press, 2014, pp. 124–194 (survey chapter without proofs).

  (3) V. Harizanov, "Pure computable model theory," in the volume: *Handbook of Recursive Mathematics*, vol. 1, North-Holland, Amsterdam, 1998, pp. 3–114 (survey chapter with sample proofs).