

The George Washington University  
Department of Mathematics  
**Math 6710: Mathematical Logic** (CRN 16723)  
Fall 2016  
MW 3:45–5:00pm  
1957 E Street, Room 313

**Professor**

Valentina Harizanov

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Office Hours: M 2:10–3:35pm

W 2:10–3:35pm

At other times by appointment.

**Mathematics department**

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**Description**

This course will present a selection of topics in *model theory* and *computability theory*. Model theory emerged as a distinct field in the 1940's through the works of Gödel, Löwenheim, Malcev, Skolem, Tarski, and others. It provides a rigorous mathematical framework for the notions of language, provability, model, and truth. One of the fundamental results of model theory is *Gödel's completeness theorem*, which establishes a correspondence between syntactic provability and semantic truth in first-order logic. In the 1930's, Church, Gödel, Kleene, Post, Turing, and others developed computability theory, a precise mathematical theory of algorithms. Their results paved the way for the invention of computers. One of the first results of computability theory was Turing's theorem that not all problems can be solved algorithmically; those that can are called decidable, computable, or recursive. Interaction of computability theory with model theory, has resulted in computable model theory and, more generally, in algorithmic mathematics. *Gödel's incompleteness theorem* (refereeing to a different meaning of completeness) is a striking early result in computable model theory, which strongly influenced all further thought about the foundation of mathematics. It showed that Hilbert's dream of establishing consistency of mathematics is impossible. Another important negative result in computable mathematics is the undecidability of the Hilbert's Tenth Problem of 1900, which was finally established in 1970.

**Required background**

Math 2971 or an equivalent. Advanced undergraduate students may also take this course for credit.

**Reading Material**

Course material from various sources will be provided in class. Book sources include:

*Mathematical Logic: Applications and Theory* by J.E. Rubin, Sounders College Publishing, 1990.

*Model Theory: An Introduction* by D. Marker, Springer, 2002.

*Model Theory* by C.C. Chang and H.J. Keisler, Dover, 2012.

*Computability* by N.J. Cutland, Cambridge University Press, 1980.

*Computability Theory* by S.B. Cooper, Chapman & Hall, 2004.

*Classic Set Theory* by D. Goldrei, Chapman and Hall, 1996.

*Turing Computability* by Robert Soare, Springer, summer 2016.

## **Learning Outcomes**

As a result of completing this course students should be able to:

1. Know Gödel's Completeness Theorem and various constructions of models for consistent theories;
2. Know computable and partial computable functions, and be able to classify sets as decidable, computably enumerable and non-computably enumerable;
3. Apply Compactness Theorem to various problems in logic and mathematics, such as to show the existence of nonstandard models of number theory;
4. Analyze the proof and ramifications of Gödel's Incompleteness Theorem.

## **Grading**

Based on class participation (10%), take-home assignments (50%), midterm project and its in-class presentation (20%), and take-home final exam (20%).

## **Academic integrity code**

Academic dishonesty is defined as cheating of any kind, including misrepresenting one's own work, taking credit for the work of others without crediting them and without appropriate authorization, and the fabrication of information. For the remainder of the code, see:

[studentconduct.gwu.edu/code-academic-integrity](http://studentconduct.gwu.edu/code-academic-integrity)

## **Support for students outside the classroom**

*Academic support services:* <http://advising.columbian.gwu.edu/academic-support>

*Mental health services:* 202-994-5300

The University's Mental Health Services offers 24/7 assistance and referral to address students' personal, social, career, and study skills problems. Services for students include: crisis and emergency mental health consultations confidential assessment, counseling services (individual and small group), and referrals. [counselingcenter.gwu.edu/](http://counselingcenter.gwu.edu/)

**Emergency preparedness:** [www.gwu.edu/~gwalert](http://www.gwu.edu/~gwalert)