

**LOGIC SEMINAR  
SPRING 2008**

Previous seminars at: [http://home.gwu.edu/~harizanv/index.html#GW\\_Logic\\_Seminar\\_](http://home.gwu.edu/~harizanv/index.html#GW_Logic_Seminar_)

**Thursday, April 24, 2008**

4:00–5:00 p.m.

Speaker: Nate Ackerman, University of Pennsylvania

<http://www.math.upenn.edu/~nate/>

Place: Monroe Hall (2115 G Street), Room 267

Title: *Vaught's Conjecture and Vaught Trees: Part II*

**Friday, April 18, 2008**

2:00–3:00 p.m.

Speaker: Nate Ackerman, University of Pennsylvania

<http://www.math.upenn.edu/~nate/>

Place: Monroe Hall (2115 G Street), Room 267

Title: *Vaught's Conjecture and Vaught Trees: Part I*

Abstract: In 1961 Robert Vaught published a ground breaking paper on countable model theory entitled *Denumerable Models of Complete Theories*. In this paper he set out many of the fundamental results concerning countable model theory. At the end of this paper he made a conjecture that every countable first order theory in every model of set theory had either countably or continuum many countable models. This conjecture is what is now called *Vaught's Conjecture* and is one of the oldest open problems in model theory.

One of the most significant steps towards resolving this conjecture is a result of Morley's which says: For every countable first order theory one of three possibilities occurs. (1)  $T$  has a perfect kernel of countable models. (2)  $T$  has countably many models. (3)  $T$  has  $\omega_1$  many models.

In this talk we will prove this by showing that if  $T$  does not have a perfect kernel of countable models then we can arrange these models in a hierarchy of complexity called the Vaught tree. Further we will show that each level of the Vaught tree has either a perfect kernel of models or is countable.

If we have time, we will also show that the statement  $T$  satisfies *Vaught's Conjecture* is absolute between models of set theory.

**Thursday, March 13, 2008**

4:00–5:00 p.m.

Speaker: Joseph S. Miller, University of Connecticut

<http://www.math.uconn.edu/~josephmiller/>

Place: Monroe Hall (2115 G Street), Room 267

Title: *Extracting information is hard*

Abstract: Can randomness – or more technically, “information” – be effectively extracted from a semi-random source? A special case of this question was answered by von Neumann in 1951. He described a simple method for extracting an unbiased random sequence from the flips of a biased coin. A more general form of the question was asked by Reimann and Terwijn in the context of algorithmic randomness, so we will start with an introduction to Kolmogorov complexity, effective Hausdorff dimension, and Martin-Löf randomness. Kolmogorov complexity measures the information content of a finite binary string. Informally, the complexity of a string is defined to be the length of the shortest program that generates it. A closely related notion, effective (Hausdorff) dimension, measures the information density of infinite binary sequences. We can now formulate the question in terms of effective dimension: is there a sequence that has effective Hausdorff dimension  $1/2$  – in other words, a half-random sequence – that does not compute a sequence of higher effective dimension? As it turns out, such sequences exist. We will discuss this and related results.

**Thursday, March 6, 2008**

4:00–5:00 p.m.

Speaker: Andrei Morozov, Sobolev Institute of Mathematics and Novosibirsk State University, Russia

<http://www.math.nsc.ru/~asm256/>

Place: Monroe Hall (2115 G Street), Room 267

Title: *Countable structures  $\Sigma$ -definable over classical continuous number fields*

Abstract: We give a complete characterization of countable structures that are  $\Sigma$ -definable over classical number systems, such as reals, complex numbers, and quaternions. Some of the work that will be presented is with M. Korovina.

**Logic/Topology/Algebra talk**

**Thursday, February 21, 2008**

4:00–5:00 p.m.

Speaker: Jennifer Chubb, GWU

<http://home.gwu.edu/%7Ejchubb/>

Place: Monroe Hall (2115 G Street), Room 267

Title: *Computability and Buttsworth's group, Part II*

Abstract: We will continue to analyze Buttsworth's construction of a family of groups, each admitting a countable infinity of orderings. We will also examine the computability theoretic properties of the groups and the orderings.

**Logic/Topology/Algebra talk**

**Thursday, February 7, 2008**

4:00–5:00 p.m.

Speaker: Jennifer Chubb, GWU

<http://home.gwu.edu/%7Ejchubb/>

Place: Monroe Hall (2115 G Street), Room 267

Title: *Computability and Buttsworth's group*

Abstract: A group is orderable if there is an ordering of its elements that is left and right invariant. The collection of all possible orderings of a group forms a natural topological space, and this space is compact. Neumann asked for a group admitting infinitely many distinct orderings, whether the number of orderings is necessarily a power of 2. Buttsworth constructed a family of groups, each admitting a countable infinity of orderings, and so negatively answered Neumann's question. We review this construction and examine the computability theoretic properties of the groups and the orderings.

**Friday, January 25, 2008**

4:00–5:00 p.m.

Place: Monroe Hall (2115 G Street), Room 267

Speaker: Russell Miller, Queens College and Graduate Center, CUNY

<http://qcpages.qc.cuny.edu/~rmiller>

Title: *Locally computable structures*

Abstract: Computable model theory has always restricted itself to the study of countable structures. The natural numbers form the standard domain and range for partial computable functions, due to the finiteness of the programs and computations in the Turing model. In this talk we continue to use this standard model of computation, but in such a way as to allow consideration of uncountable structures. We no longer aspire to describe a structure  $S$  globally. Instead, we content ourselves with a local description: a list of the finitely-generated substructures of  $S$ , up to isomorphism. Initially we require that these substructures should be computable, and that the list of them be given effectively. If this can be done, then  $S$  is said to be locally computable. (Such an  $S$  must have only countably many finitely-generated substructures, up to isomorphism.) Then we consider more sophisticated effective descriptions of  $S$ , by naming embeddings among the substructures which reflect the inclusions in  $S$ . We will prove several theorems about locally computable structures, but since the notion of local computability is extremely new, much of this talk will be devoted to definitions and examples.

## ***OTHER LOGIC TALKS***

**Graduate Student Seminar**

**Thursday, April 17, 2008**

4:00–5:00 p.m.

Monroe Hall (2115 G Street), Room 267

Speaker: Karen Lange, University of Chicago

Title: *The relative strength of the Atomic and Homogeneous Model Theorems*

Abstract: Reverse mathematics and computable model theory both attempt to measure the strength of classical mathematical principles, the former from a proof-theoretic perspective and the latter from a computable one. I will present computability results on

the Homogeneous Model Theorem (HMT), an existence theorem for general model-theoretic structures found throughout mathematics. I will discuss how these results provide insight into the reverse mathematical strength of HMT. In particular, I will discuss how HMT compares with the Atomic Model Theorem studied by Hirschfeldt, Shore, and Slaman.

**Graduate Student Seminar**

**Friday, March 7, 2008**

1:00–2:00 p.m.

Monroe Hall (2115 G Street), Room 267

Speaker: Sara Quinn, University of Notre Dame

Title: *A tour of computable structure theory*

Abstract: Computable structure theory is a branch of mathematical logic in which the algorithmic complexity of algebraic structures is examined. In other words, we determine how complicated it is to answer certain questions about algebraic structures, such as: “Are these two structures isomorphic?” There is a hierarchy we use to rank the complexity of these questions. In this talk I will give all of the necessary definitions, and then describe the kinds of questions that we ask in computable structure theory.