1 Abstract of Dissertation

We first investigate Turing degrees of countable groups in some well-known classes of nonabelian groups. The *Turing degree spectrum* of a countable structure A is the set of Turing degrees of all isomorphic copies of A. The *Turing degree of the isomorphism type* of A, if it exists, is the least degree in the degree spectrum of A. So far degrees of the isomorphism types have been studied for abelian and 2-step nilpotent groups. We show that there are countable centerless (and hence non-nilpotent) groups whose isomorphism types have arbitrary Turing degrees. It then follows that their Turing degree spectra coincide with the upper cones of degrees above arbitrary Turing degrees. We also show that there are various countable centerless groups whose isomorphism types do not have a degree.

We next analyze computational and topological properties of spaces of left orders on computable groups. A group is *computable* if its domain is computable and its group-theoretic operation is computable. A group is *left-orderable* if there is a linear order on its domain, which is left-invariant under the group operation. The space of orders on a group G consists of all possible orders on G. We define the *Turing degree spectrum of left orders* on a computable group G to be the set of all Turing degrees of possible left orders on G. We formulate a general sufficient condition for the degree spectrum of left orders to include all Turing degrees. A relativized version of this result gives conditions which assure that the degree spectrum which is closed upwards includes all Turing degrees above an arbitrary Turing degree. On the other hand, modified assumptions of our general result provide the conditions for the space of left orders on a countable group to be homeomorphic to the Cantor set. We then use our theorems to examine the complexity of left-invariant orders for various classes of groups. A group is *fully left-orderable* if every partial left order on a group extends to a left order. We show that the degree spectrum of left orders on a fully left-orderable group includes all Turing degrees above an arbitrary degree. Using the properties of the lower central series, we prove that the Turing degree spectrum of left orders on a finitely generated free group contains all Turing degrees. We also show that the space of left orders on $\mathbb{Z}^{\omega}$ is homeomorphic to the Cantor set. We further prove that the space of standard orders for the fundamental group of a closed, connected and orientable surface of genus greater than 1 is homeomorphic to the Cantor set.