On automorphisms of structures in logic and orderability of groups in topology

by Ataollah Togha

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Abstract

We investigate properties of non-standard models of set theory, in particular, the *countable recursively saturated* models while having the automorphisms of such models in mind. The set of automorphisms of a model forms a group that in certain circumstances can give information about the model and even recover the model. We develop results on conditions for the existence of automorphisms that fix a given initial segment of a countable recursively saturated model of ZF.

In certain cases an additional axiom **V** = **OD** will be needed in order to establish analogues of some results for models of Peano Arithmetic. This axiom will provide us with a definable global well-ordering of the model. Models of set theory do not automatically possess such a well-ordering, but a definable well-ordering is already in place for models of Peano Arithmetic, that is, the natural order of the model.

We investigate some finitely presentable groups that arise from topology. These groups are the *fundamental groups* of certain manifolds and their orderability properties have implications for the manifolds they come from. A group $(G, \circ)$ is called left-orderable if there is a total order relation $<$ on G that preserves the group operation $\circ$ from the left.

Finitely presentable groups constitute an important class of finitely generated groups and we establish criteria for a finitely presented group to be non-left-orderable. We also investigate the orderability properties for Fibonacci groups and their generalizations.