PRACTICE PROBLEMS FOR MIDTERM #2
MATH 455.2 SPRING 2005

Note: Apart from the problems below you are responsible for the previous homework problems, material covered in lectures and in sections: 8.1, 8.2, 8.3, 7.1, 7.2, 7.3, 7.4, 7.5, 9.1, and 9.2. When proving using induction make sure you *include words to describe what you are assuming and what you are trying to show.* As before, no calculators, notes, or books will be allowed on the test.

(1) Show that the sequence $0, 1, 3, 7, \ldots, 2^n - 1, \ldots$ satisfies the recurrence relation $c_n = 2c_{n-1} + 1$ for all $n \geq 1$.

(2) Prove that $F_n = 3F_{n-3} + 2F_{n-4}$ for all $n \geq 4$, where F_n denotes the n-th Fibonacci number.

(3) With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking a combination of steps that are either one- or two-stair increments. Let c_n be the number of different ways to climb a staircase with n stairs ($n \geq 1$). Find a recurrence relation for c_n. Find the general formula for c_n.

(4) Use iteration method to guess an explicit formula for the sequence given recursively by $a_k = \frac{a_{k-1}}{2a_{k-1} - 1}$ for all $k \geq 1$ and $a_0 = 2$. Prove that your guess is correct using induction.

(5) Solve the SOLHRRCC: $r_k = 8r_{k-1} - 16r_{k-2}$ for $k \geq 2$ and $r_0 = 1$, $r_1 = 4$.

(6) If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are both 1-1, is $f + g : \mathbb{R} \to \mathbb{R}$ also 1-1? If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are both onto, is $f + g : \mathbb{R} \to \mathbb{R}$ also onto? If $f : \mathbb{R} \to \mathbb{R}$ is 1-1 and $c > 0$, is $c \cdot f : \mathbb{R} \to \mathbb{R}$ also 1-1?

(7) How many integers must you pick to be sure that at least two of them have the same remainder when divided by 7?

(8) There are 50 students in your class. All of them are known to be between 18 and 30 years old. You want to make a bet that the class contains at least k students of the same age. How large can you make k and yet be sure to win your bet?

(9) On a square tray with side 1 meter you place 51 dimes. Prove that no matter how you place them there will be three dimes that can be covered with a plate of radius 15 cm.

(10) Let $f : X \to Y$ and $g : Y \to Z$. If $g \circ f$ is 1-1 must f be 1-1? What about g? Prove or give a counterexample.

(11) Let $f : X \to Y$ and $g : Y \to Z$. If $g \circ f$ is onto must f be onto? What about g? Prove or give a counterexample.

(12) Prove that any infinite set contains a countable infinite subset.

(13) Show that the set of all finite sequences of 0's and 1's is countable.