DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MATH 455.2 April 14, 2005
EXAM 2

Your Name: ______________________________

Your student ID: ____________________

This exam paper consists of 7 questions. It has 6 pages. Show your work: all your answers must be justified. When proving using induction make sure you include words to describe what you are assuming and what you are trying to show.
No calculators, books or notes are allowed!

1. (20)	__________
2. (20)	__________
3. (15)	__________
4. (15)	__________
5. (10)	__________
6. (15)	__________
7. (15)	__________
TOTAL (110)	

1. *(20 pts)* Consider the recurrence relation $4a_n = 4a_{n-1} - a_{n-2}$, $n \geq 2$.

   (a) *(10 pts)* Find the general form of the solution.

   (b) *(10 pts)* Give a formula for a_n valid for $n \geq 0$ that satisfies the initial conditions $a_0 = 3$, $a_1 = 2$.

2. *(20 pts)* Suppose $c_1, c_2, \ldots$ is a sequence with $c_0 = 1$ and, for $n \geq 1$, satisfying the recurrence relation $c_n = c_{n-1} + 3 \cdot 2^{n-1}$.

   (a) *(5 pts)* Compute c_1, c_2, c_3, and c_4.

   (b) *(15 pts)* Guess a formula for c_n valid for $n \geq 0$ and prove that your guess is correct using mathematical induction. Use words to explain your logic and process.

3. *(15 pts)* How many one-to-one functions are there from the set of 5 elements to the set of 7 elements? Justify your answer, but do not attempt to write them all down.

4. *(15 pts)* Give an example of a function $f : \mathbf{Z} \to \mathbf{Z}$ which is onto, but not one-to-one.

5. *(10 pts)* Prove that $2 + 4 + 6 + \ldots + 2n$ is $\Theta(n^2)$. You may use limits.

6. *(15 pts)* Show that in a group of 3 people whose sum of ages is 67 years there will always be two people whose sum of ages is at least 45.

7. *(15 pts)* Prove that a countable union of countable sets is countable, i.e. if $A_1, A_2, A_3, \ldots$ are countable sets then so is $A = \cup_{i=1}^{\infty} A_i$. (Hint: Let a_{ij} be the j-th element in A_i. Recall the proof of the countability of $\mathbf{N} \times \mathbf{N}$.)