PRACTICE PROBLEMS FOR MIDTERM #1
MATH 455.2, SPRING 2005

Note: On the exam, you do not need a numerical answer unless you are asked for it. It is *better* to not simplify, things like $\binom{a}{b}$, $P(n,k)$ and $n!$ are encouraged in your answers. **No calculators will be allowed!**

(1) Give a numerical value for $\binom{10}{7}$.

(2) Give a numerical value for $P(6,4)$.

(3) How many ways are there to place 5 identical balls into 3 distinct boxes?

(4) How many ways can the 25-member cooking club choose its President, Vice President, and Secretary?

(5) How many ways can 22 players be divided into two teams of 11 for a soccer game?

(6) How many distinct arrangements of the letters in HIPPOPOTAMUS are there?

(7) How many numbers less than 800,000 can be formed by rearranging the digits in 219,338?

(8) How many ways can you arrange the letters of the alphabet so that there are exactly 5 letters between the a and the b?

(9) Count the number of 5-card poker hands with 4 of the same denomination.

(10) What is the probability that after a pair of dice is rolled the minimum of the numbers showing is 4?

(11) How many strings of 4 letters begin or end with one of the five vowels?

(12) How many integers from 1 to 300 are divisible by 4 or by 14?

(13) A bag has 3 red, 5 orange, 4 green, and 7 white balls. How many distinguishable collections of 3 balls can be drawn from the bag?

(14) Prove by induction that
$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2} \right)^2.$$

(15) Prove by induction that $(1+x)^n \geq 1 + nx$ for any positive integer n and any $x \geq -1$.

(16) When a group of n businessmen arrives at a meeting each person shakes hands with all the other people present. Guess the number of handshakes occur. Prove your guess by induction.