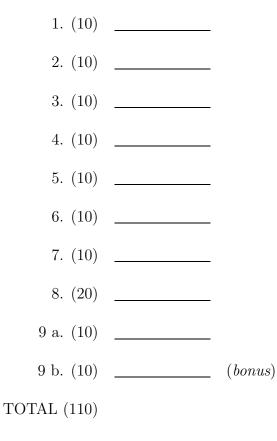
DEPARTMENT OF MATHEMATICS AND STATISTICS UNIVERSITY OF MASSACHUSETTS MATH 455 March 3, 2005 EXAM 1

Your Name: _____

Your Section: _____

This exam paper consists of 9 questions. It has 6 pages. You do not need a numerical answer unless you are asked for it. It is okay to use $\binom{a}{b}$, P(n,r), n! or a^b in your answers. Each answer must be justified. No calculators, books or notes are allowed!



1. Give a numerical value for

$$\frac{\binom{11}{4}}{P(11,2)}.$$

2. How many solutions to $x_1 + x_2 + x_3 + x_4 + x_5 = 50$ are there in *positive* integers (i.e. each $x_i \ge 1$, i = 1, 2, 3, 4, 5)?

3. How many distinct arrangements of the letters in REVERSE start with an E and end with an R?

4. How many 5-card hands from the standard 52 card deck have "three of a kind", i.e. three cards of the same denomination and two others of different denomination? (Caution: Do not include full-house hands!)

5. Count the number of ways 12 players can be split into 3 teams of four for a bridge tournament. (Assuming there is no order of the teams.)

6. What is the number of 4-letter words that either start or end with a vowel? (Note: "word" means a string letters, a vowel is one of the five letters $\{a, e, i, o, u\}$.)

7. What is the probability that a number between 1 and 100 (inclusive) chosen at random will be divisible by either 2, 3 or 5?

8. Prove by induction that for any $n\geq 1$

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}.$$

- 9. A rook is a chess piece that may move any number of unoccupied squares either vertically or horizontally.
 - a) How many ways to place 8 indistinguishable rooks on the chess board so that no rook is attacked by another?

b) (*Bonus.*) How many ways to place 8 indistinguishable rooks on the chess board so that each unoccupied square is attacked by at least one rook?