

DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MATH 455 March 3, 2005
EXAM 1

Your Name: _____

Your Section: _____

This exam paper consists of 9 questions. It has 6 pages. You do not need a numerical answer unless you are asked for it. It is okay to use $\binom{a}{b}$, $P(n, r)$, $n!$ or a^b in your answers. Each answer must be justified. No calculators, books or notes are allowed!

- 1. (10) _____
- 2. (10) _____
- 3. (10) _____
- 4. (10) _____
- 5. (10) _____
- 6. (10) _____
- 7. (10) _____
- 8. (20) _____
- 9 a. (10) _____
- 9 b. (10) _____ (*bonus*)

TOTAL (110)

1. Give a numerical value for

$$\frac{\binom{11}{4}}{P(11, 2)}.$$

2. How many solutions to $x_1 + x_2 + x_3 + x_4 + x_5 = 50$ are there in *positive* integers (i.e. each $x_i \geq 1$, $i = 1, 2, 3, 4, 5$)?

3. How many distinct arrangements of the letters in REVERSE start with an E and end with an R?

6. What is the number of 4-letter words that either start or end with a vowel?
(Note: “word” means a string letters, a vowel is one of the five letters $\{a, e, i, o, u\}$.)
7. What is the probability that a number between 1 and 100 (inclusive) chosen at random will be divisible by either 2, 3 or 5?

8. Prove by induction that for any $n \geq 1$

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}.$$

9. A rook is a chess piece that may move any number of unoccupied squares either vertically or horizontally.

a) How many ways to place 8 indistinguishable rooks on the chess board so that no rook is attacked by another?

b) (*Bonus.*) How many ways to place 8 indistinguishable rooks on the chess board so that each unoccupied square is attacked by at least one rook?