# DEPARTMENT OF MATHEMATICS AND STATISTICS UNIVERSITY OF MASSACHUSETTS <br> MATH 455 March 3, 2005 <br> EXAM 1 

Your Name: $\qquad$

Your Section: $\qquad$

This exam paper consists of 9 questions. It has 6 pages. You do not need a numerical answer unless you are asked for it. It is okay to use $\binom{a}{b}, P(n, r), n$ ! or $a^{b}$ in your answers. Each answer must be justified. No calculators, books or notes are allowed!

1. (10) $\qquad$
2. (10) $\qquad$
3. (10) $\qquad$
4. (10) $\qquad$
5. (10) $\qquad$
6. (10) $\qquad$
7. (10) $\qquad$
8. (20) $\qquad$
9 a. (10) $\qquad$
9 b. (10) $\qquad$ (bonus)

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1. Give a numerical value for

$$
\frac{\binom{11}{4}}{P(11,2)}
$$

2. How many solutions to $x_{1}+x_{2}+x_{3}+x_{4}+x_{5}=50$ are there in positive integers (i.e. each $x_{i} \geq 1, i=1,2,3,4,5$ )?
3. How many distinct arrangements of the letters in REVERSE start with an E and end with an R ?
4. How many 5 -card hands from the standard 52 card deck have "three of a kind", i.e. three cards of the same denomination and two others of different denomination? (Caution: Do not include full-house hands!)
5. Count the number of ways 12 players can be split into 3 teams of four for a bridge tournament. (Assuming there is no order of the teams.)
6. What is the number of 4 -letter words that either start or end with a vowel? (Note: "word" means a string letters, a vowel is one of the five letters $\{a, e, i, o, u\}$.)
7. What is the probability that a number between 1 and 100 (inclusive) chosen at random will be divisible by either 2,3 or 5 ?

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8. Prove by induction that for any $n \geq 1$

$$
\frac{1}{1 \cdot 3}+\frac{1}{3 \cdot 5}+\frac{1}{5 \cdot 7}+\cdots+\frac{1}{(2 n-1)(2 n+1)}=\frac{n}{2 n+1} .
$$

9. A rook is a chess piece that may move any number of unoccupied squares either vertically or horizontally.
a) How many ways to place 8 indistinguishable rooks on the chess board so that no rook is attacked by another?
b) (Bonus.) How many ways to place 8 indistinguishable rooks on the chess board so that each unoccupied square is attacked by at least one rook?
