DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MATH 455 May 17, 2004
FINAL EXAM, DURATION 2 HOURS

Your Name: _____

Your Section: _____

This exam paper consists of 7 questions. It has 6 pages. Show your work: all your answers must be justified. No calculators, books or notes are allowed!

1. (15) _____

2. (15) _____

3. (15) _____

4. (15) _____

5. (15) _____

6. (20) _____

7. (15) _____

TOTAL (110)

1. *(15 pts)* Count the number of ways 33 players can be split into three soccer teams: Spartak, Dinamo, and Torpedo. (Note: Each soccer team has 11 players.)

2. *(15 pts)* Let $f : \mathbf{N} \times \mathbf{N} \to \mathbf{N}$ be defined by the formula $f(m, n) = 2^m 5^n$.

   (a) *(10 pts)* Is f one-to-one?

   (b) *(5 pts)* Is f onto?

3. *(15 pts)* Let B_4 denote the set of all length 4 binary strings. Consider the Hamming distance function $Hd : B_4 \times B_4 \to \mathbf{Z}$.

   (a) *(5 pts)* What is the number of preimages of 0 under Hd?

   (b) *(5 pts)* What is the number of preimages of 1 under Hd?

   (c) *(5 pts)* What is the number of preimages of 5 under Hd?

4. *(15 pts)* Draw a finite state automaton for the input alphabet $\{0, 1\}$ that accepts the set of all strings that end in 01. Clearly indicate the initial and accepting states.

5. *(15 pts)* Consider the error correcting code $C = \{00000, 01011, 10101, 11110\}$. This is a $(5, 4, d)$ error correcting code.

   (a) *(5 pts)* Is C linear?

   (b) *(5 pts)* Find d. How many errors can C correct? Explain.

   (c) *(5 pts)* Is C perfect?

6. *(20 pts)* Let G be a simple graph with n vertices.

   (a) *(10 pts)* Prove that the number of edges of G is at most $n(n-1)/2$.

   (b) *(10 pts)* Is it possible that the degrees of the vertices of G are all different? Justify.

7. *(15 pts)* Recall that a simple graph is *bipartite* if it is possible to color its vertices into two different colors in such a way that no two vertices of the same color are connected with an edge. Prove that a graph is bipartite if and only if it does not contain circuits with an odd number of edges.