Prove the following propositions.

1. For any $z, w \in \mathbb{C}$,
$$||z| - |w|| \leq |z - w|.$$

2. For any $z_1, \cdots, z_n \in \mathbb{C}$,
$$|\sum_{k=1}^{n} z_k| \leq \sum_{k=1}^{n} |z_k|.$$
(*Hint.* Use mathematical induction on n.)

3. For any $z, w \in \mathbb{C}$,
$$|z + w|^2 + |z - w|^2 = 2(|z|^2 + |w|^2).$$

4. If $n = p_1^2 + p_2^2$ and $m = q_1^2 + q_2^2$, where $p_1, p_2, q_1, q_2 \in \mathbb{Z}$, then there exist $r_1, r_2 \in \mathbb{Z}$ such that $mn = r_1^2 + r_2^2$.
(*Hint.* Consider the squares of absolute values of certain complex numbers.)

5. (a) Points z_1 and z_2 are mirror images of each other across the straightline $B\overline{z} + \overline{B}z + c = 0$, where $B \in \mathbb{C}$, $c \in \mathbb{R}$, if and only if $B\overline{z}_1 + \overline{B}z_2 + c = 0$.
(b) Find the mirror image of the origin across the line $(3 + 5i)\overline{z} + (3 - 5i)z + 7 = 0$.