

Prove the following propositions.

1. If  $H$  is a subgroup of  $\mathbb{Z}$  and  $n \in H$ , then  $n\mathbb{Z} \subset H$ .
2. If  $H_1$  and  $H_2$  are subgroups of  $\mathbb{Z}$ , then  $H_1 + H_2$  and  $H_1 \cap H_2$  are also subgroups of  $\mathbb{Z}$ .
3. For  $p, q \in \mathbb{Z}$ ,  $\text{g.c.d.}(p, q) = 1$  if and only if there exists  $x, y \in \mathbb{Z}$  s.t.  
$$xp + yq = 1.$$
4. For  $p, q \in \mathbb{Z}$ , if  $d \mid p, q$  and  $d = xp + yq$  for some  $x, y \in \mathbb{Z}$ , then  $d = \text{g.c.d.}(p, q)$ .
5.  $\sqrt{2}$  is an irrational number.