

1. Given any set of 52 distinct integers, prove that there must be 2 of them whose sum or difference is divisible by 100.

2. Show that if 101 integers are chosen from 1 to 200 inclusive, there must be two with the property that one is divisible by the other.

3. Given a positive integer n , find the least number k such that every finite sequence x_1, x_2, \dots, x_k of integers has a consecutive subsequence x_i, x_{i+1}, \dots, x_j whose sum is divisible by n .

4. Let X be a subset of $\{2, 3, 4, 5, 6, \dots, 39, 40\}$ of 13 elements. Prove that there are two different elements of X whose greatest common divisor is greater than 1.