

1. Find the sequence $\{b_n\}_{n=0}^{\infty}$ satisfying

$$\begin{cases} b_n = 3b_{n-1} + 7, & n \geq 1, \\ b_0 = 0. \end{cases}$$

2. Find the sequence $\{a_n\}_{n=0}^{\infty}$ satisfying

$$\begin{cases} a_n = a_{n-1} + 6a_{n-2}, & n \geq 2, \\ a_0 = 2, a_1 = 1. \end{cases}$$

3. The sequence $\{f_n\}_{n=1}^{\infty}$ satisfies

$$\begin{cases} f_n = 9f_{\lfloor \frac{n}{3} \rfloor}, & n \geq 3, \\ f_1 = 1, f_2 = 4, \end{cases}$$

where $\lfloor * \rfloor$ is the floor function.

Show by strong mathematical induction that $f_n \leq n^2$ for any positive integer n .

4. Define a_n to be the number of different ways of using n blank 2×1 dominoes to cover a $2 \times n$ chessboard. Find a recurrence relation for $\{a_n\}_{n=0}^{\infty}$, and use it to give a general formula for a_n .

5. Ten different letters are intended for ten different recipients. How many different ways are there to send these letters so that four of the recipients get right letters and the other six get wrong letters? (Every recipient must get exactly one of these ten letters.)