

1. How many distinct arrangements of the letters in MASSACHUSETTS start with MA?
  
2. A straight is a poker hand consisting of 5 cards whose ranks form a sequence. The highest possible straight is AKQJ10 (Broadway) and the lowest straight is 54321 (the wheel). How many 5-card hands from the standard 52 card deck are straights (including straight flushes, royal flushes and ect.)?
  
3. (a) (10 points) How many solutions to  $x_1 + x_2 + x_3 = 17$  are there in non-negative integers (i.e.  $x_i \geq 0$   $i = 1, 2, 3$ )?  
(b) (10 points) How many solutions to  $x_1 + x_2 + x_3 = 17$  in integers satisfy  $0 \leq x_i \leq 7$ ,  $i = 1, 2, 3$ .
  
4. A bag has 3 red, 5 orange, 4 green, and 7 white balls.  
(a) How many distinguishable collections of 3 balls can be drawn from the bag?  
(b) How many distinguishable collections of 5 balls can be drawn from the bag?

5. Prove that, for  $n \geq 1$ ,

$$\sum_{k=1}^n k \binom{n}{k}^2 = n \binom{2n-1}{n-1}.$$

6. Prove that, for  $n \geq 0$ ,

$$\sum_{k=0}^n k^2 \binom{n}{k} = n(n+1)2^{n-2}.$$