

MATH 300 Practise Exam 3

April, 2007

The actual exam consists of two parts. In the first part, you will be asked to solve a recursive relation. In the second part of the exam, you will be asked to prove the following 3 theorems.

1. $X = \{1, 2, 3, 4, \dots, 2006, 2007\}$. Find the least integer n so that every subset of X with n or more elements contains 3 consecutive integers, and justify your answer. (*Hint.* you need show: (i) There is a subset of X with $n - 1$ elements that does not contain 3 consecutive integers; (ii) Every subset of X with n or more elements contains 3 consecutive integers.)

2. Prove that, for any $z, w \in \mathbb{C}$,

$$|z + w|^2 + |z - w|^2 = 2(|z|^2 + |w|^2).$$

3. Let $\{F_n\}$ be the Fibonacci sequence. Prove that $F_{k+1}^2 - F_k^2 = F_{k-1}F_{k+2}$ for all integers $k \geq 1$.