MATH 300 Practise Exam 2

March, 2007

The actual exam consists of two parts. In the first part, you will be asked to slove a combinatorial problem. In the second part of the exam, you will be asked to prove the following 3 theorems.

1. Prove by mathematical induction that, for any integer $n \geq 1$,

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{(2n-1) \cdot (2n+1)} = \frac{n}{2n+1}.$$

2. Prove by mathematical induction that, for any integers $n \geq r \geq 0$,

$$\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}.$$

3. Prove that, for any integer $n \geq 2$,

$$\sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} (2k+1) \binom{n}{2k+1} = n \cdot 2^{n-2},$$

where $\lfloor \frac{n-1}{2} \rfloor$ is the greatest integer not greater than $\frac{n-1}{2}$.