MATH 300 Final Practice Exam

1. H_1 and H_2 are subgroups of $\mathbb{Z}$. Prove that $H_1 + H_2$ is also a subgroup of $\mathbb{Z}$.

2. Prove Cantor's Theorem. That is, for every set A, $|A| < |\mathcal{P}(A)|$.

3. (M, d) is a metric space.
(a) State the definition of open sets in (M, d).
(b) Prove the following statements.
(1) M and $\emptyset$ are open.
(2) If $U_1, \cdots, U_n$ are open sets in M, then $\cap_{k=1}^{n} U_k$ is also open.
(3) If $\mathcal{F}$ is a family of open sets in M, then $\cup_{U\in\mathcal{F}} U$ is also open.

4. (M, τ) is a topological space. $A \subset M$.
(a) State the definition of interior points of A and the definition of $\text{Int}(A)$.
(b) Prove that $\text{Int}(A)$ is a subset of A and an open subset of M.
(c) Prove that, if X is a subset of A and an open subset of M, then $X \subset \text{Int}(A)$.

5. (a) State the definition of continuous functions from a metric space to a metric space.
(b) State the definition of continuous functions from a topological space to a topological space.
(c) (M, d) is a metric space, and τ_d is the topology of M induced by d. Prove that $f : (M, d) \to (M, d)$ is continuous if and only if $f : (M, \tau_d) \to (M, \tau_d)$ is continuous.