Instructions

- Turn off all cell phones and watch alarms!
  Put away cell phones, iPods, etc.

- There are eight (8) questions.

- Do all work in this exam booklet. You may continue work to backs of pages and the blank page at the end, but if you do so indicate where.

- Do not use any other paper except this exam booklet and the one-page “cheat sheet” that you prepared.

- Organize your work in an unambiguous order. Show all necessary steps.

- Answers given without supporting work may receive 0 credit!

- Do not write anything in the table below.

- Be prepared to show your UMass ID card when you hand in your exam booklet.

<table>
<thead>
<tr>
<th>QUESTION</th>
<th>PER CENT</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>Extra credit</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>106</td>
<td></td>
</tr>
</tbody>
</table>
1. (4 × 4% = 16%) Let \( f(x) = \sqrt{\frac{1 - 2x}{x}} \).

(a) Determine the domain of \( f \).

(b) Derive a formula for the inverse function \( f^{-1} \) of \( f \).

(c) What is \( f \left( f^{-1} \left( \frac{2}{5} \right) \right) \)? (An exact value, not a numeric approximation!)

(d) If \( g(y) = y^2 \) for all \( y \), then what is \( (g \circ f)(3x) \)?
2. \( (2 \times 8\% = 16\%) \) During a bad storm, the height of a river was measured every two hours, starting at midnight. Here is data obtained for the height \( h \), in feet above sea level, as a function of the elapsed time \( t \), in hours, since midnight:

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>6.0</td>
<td>9.9</td>
<td>11.6</td>
<td>11.1</td>
<td>8.4</td>
<td>3.5</td>
</tr>
</tbody>
</table>

(a) Using only this data, calculate the best estimate you can for the rate, in feet per hour, at which the river’s height was changing at 8 A.M. Do not round your answer; do indicate how you obtained it. (Do not try to fit a formula to this data!)

(b) Actually, the rate at time \( t = 8 \) at which \( h \) was changing with respect to \( t \) was exactly \(-1.75\) feet per hour. Write an equation for the tangent line to the graph of the function \( h \) of \( t \) at the point where \( t = 8 \). (The variables are time \( t \) and height \( h \).)
3. (4 \times 5\% = 20\%) Find each of the following limits. If the limit does not exist, say so. (Do this without using your calculator. You may want to use your calculator to confirm your answers.)

(a) \lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}

(b) \lim_{s \to \infty} \frac{7s^3 - 4s + 2}{3 + s^2 - 6s^3}

(c) \lim_{x \to 0} \frac{2x}{\sqrt{x + 4} - 2}

(d) \lim_{t \to 0} \frac{e^t - 3}{5 - \cos t}
4. $(2 \times 8\% = 16\%)$ Use the definition of the derivative in terms of limits to find:

(a) $f'(3)$ where $f(x) = x^2 - 4x + 5$.

(b) $g'(x)$ where $g(x) = \frac{1}{3 - x}$.
5. (16%) Through decay, Chemical Q loses one-third of its mass every 7 years. Twenty-five years ago, a container of Chemical Q was buried. The EPA has just dug it up and found 1,000 grams remaining. How much Chemical Q was originally buried?

(Carefully identify the variables you use!)
6. $(2 \times 8\% = 16\%)$ A magnetic field is induced by electrical current moving through a wire. The strength $B$ of the magnetic field at a distance $r$ from the center of a wire is given by:

$$B(r) = \begin{cases} 
B_0 \frac{r}{r_0} & \text{if } r \leq r_0, \\
B_0 \frac{r_0}{r} & \text{if } r > r_0 
\end{cases}$$

for certain non-zero constants $B_0$ and $r_0$.

(a) Is the function $B(r)$ continuous at $r_0$? Why or why not?

(b) Is the function $B(r)$ differentiable at $r_0$? Why or why not?
Extra credit (6%) Let
\[ f(x) = 3x - 5. \]
Illustrate the precise, $(\epsilon, \delta)$, definition that
\[ \lim_{x \to 2} f(x) = 1 \]
by finding algebraically a suitable $\delta$ for
\[ \epsilon = 0.01. \]
This page left blank for additional work.