MATH 127H Final Practise Exam

December, 2006

You are allowed calculators and one page of notes. Please show all of your work and clearly mark your answer. Unless otherwise specified, all answers must be in the precise values, not numerical approximations.

1. Compute the following limits. (5 points each)

(a)

$$\lim_{x\to 0} \frac{x+\sin x}{4x^3 - x}.$$

(b)

$$\lim_{x\to +\infty} \frac{(\ln x)^2}{x}.$$

(c)

$$\lim_{x\to 0^+} \left(\frac{1}{\sin x} - \frac{1}{x}\right).$$

(d)

$$\lim_{x\to 0} (1+3x)^{\frac{2}{x}}.$$

2. Compute the general anti-derivatives of the following functions. (5 points each)

(a)

$$x^{127} + \frac{1}{x^{127}}.$$

(b)

$$\frac{e^{3x} + \sin 5x}{7}.$$

3. (10 points) Solve the initial value problem

$$\begin{cases} \frac{dy}{dx} = \frac{x}{x+1}, & x \geq 0 \\ y(0) = 5. \end{cases}$$

4. (15 points) Perfect Paper Cup Company (PPCC) wants to make a cylindrical cup (with open top, of course, so people can drink out of it). The cup is to be made from exactly 22 in^2 of paper-not counting any paper wasted in cutting out the circular bottom, in attaching the bottom to the cylindrical piece, or in fastening together the ends of the rectangle thats rolled up to make the cylindrical piece.

PPCC asks you to solve its problem: What should the cups dimensions be so that it will hold the maximum amount of liquid when it is filled to the brim?

5. (15 points) Travelling at 90 feet per second along a narrow road in your Hummer one dark night, you suddenly see a deer 500 feet ahead on the road. The deer, stunned by the glare of your headlights, freezes in place. You slam on the brakes, decelerating the car at a constant rate, until the car comes to a complete stop 10 seconds later. Are you able to stop your car before it hits the deer?

Use methods of calculus! Clearly indicate the meaning of the variables used.

Suggestion: Find the cars velocity function. Then find where the car stops.

6. $f(x) = e^{-x^2+4x-6},\ x \in \mathbb{R}$.

(a) (10 points) Find intervals where $f(x)$ is increasing and decreasing. Find all local extrema of $f(x)$. Does $f(x)$ have global extrema?

(b) (10 points) Find the absolute maximum and minimum of $f(x)$ on $[0, 5]$.

(c) (10 points) Find intervals where $f(x)$ is concave up and concave down. Find all inflection points of $f(x)$.