

MATH 127H Practise Exam 1

October, 2006

You are allowed calculators and one page of notes. Please show all of your work and clearly mark your answer. Unless otherwise specified, all answers must be in the precise values, not numerical approximations.

1. Consider the function $f(x) = \frac{3x-1}{x+2}$.

- (a) What is the domain of $f(x)$?
- (b) Find a formula for f^{-1} .
- (c) What is the domain and range of f^{-1} ?

2. Find the value of

(a)

$$\lim_{x \rightarrow \infty} \frac{x^3 - 3x + 5}{x^3 + 20x^2 + 80x + 100}.$$

(b)

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - 1}{5x^2}$$

(c)

$$\lim_{x \rightarrow 0} \frac{\sin(5x^2)}{3x^2}$$

3. The sequence $\{a_n\}$ satisfies

$$\begin{cases} a_n = \frac{1}{3}a_{n-1} + 6, & n \geq 2, \\ a_1 = 2. \end{cases}$$

Assume that $\lim_{n \rightarrow \infty} a_n$ exists. Find its value.

4. The function $f(x)$ is defined by

$$f(x) = \begin{cases} x^2 + c^2, & \text{if } x < 1, \\ 2cx, & \text{if } x \geq 1. \end{cases}$$

For what value of c is $f(x)$ a continuous function?

5. $f(x) = x^2 - x$ and $g(x) = \frac{1}{1+x}$. Compute the following derivatives by definition.

- (a) $f'(3)$.
- (b) $g'(0)$.

6. Consider the sequence $\{N_k\}$ given by the recursive relation

$$\begin{cases} N_{k+1} = \frac{2\lambda N_k}{\lambda + N_k}, & n \geq 1, \\ N_1 = 10. \end{cases},$$

where λ is a constant. Assume that

$$\lim_{k \rightarrow \infty} N_k = 50.$$

Find the value of λ .

7. The radioactive compound UMass-127 decays with time, losing half its mass every 567 years. Suppose you have 3 grams of UMass-127 now.

(a) Find a formula for the mass M , in grams, of UMass-127 as a function of time t , in years from now.

(b) When will there be only one gram remaining? Give your answer to the nearest year.