

Assessing the Fairness of a Firm's Allocation of Shares in Initial Public Offerings: Adapting a Measure from Biostatistics

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Introduction and Outline

In 1980, Professor Gastwirth was a visiting professor at MIT and gave a seminar on statistics in law. Two Harvard law students attended the course. One of them went on to become a partner in a premier New York law firm specializing in securities law. Realizing that the case he was working on involved a significant statistical component, he contacted his former Professor to be his statistical expert.

During the high-tech boom of the late 90's, individuals or firms who received shares at the initial offering price were often making substantial profit by selling their shares shortly after they bought them. Some institutions, e.g. hedge funds, tried to increase the number of shares they were allocated by sharing their profits with their brokers or investment firms. This practice is prohibited by law as the Wall Street firms should serve members of the public fairly. Indeed several major firms, e.g. Credit Suisse First Boston, paid substantial penalties to the Securities and Exchange Commission (SEC).

In this case, the client was a relatively small investment firm that was accused of "sharing profits" with some customers, primarily because these customers paid so-called "inflated" or "excessive" commissions on other trades made through the firm on or near the days they received IPO (Initial Public Offering) shares.

The case involved both legal and statistical issues. Prior to the complaint, a commission was deemed to be "excessive" if it exceeded five percent. Although the published rule does state that commissions less than 5% might also be judged excessive, no alternative guideline is provided. Since almost all the so called "inflated" or "excessive" commissions were less than 5%, indeed, less than 3%, the propriety of the regulators changing the criteria for commissions to be deemed "excessive" *without* conducting the standard rule-making procedures was a major legal issue.

A key statistical issue concerned the fairness of the IPO allocation process used at the firm. If there were a set of "profit sharing" customers, one would expect that they would be favored in the allocation process. Thus, one would expect that their IPO share allocations would be greater than their "expected" or "fair" share relative to those of other customers which would usually be less. This problem has analogies in

epidemiology and equal employment. In the former one studies whether the members of a population that were exposed to a particular health risk differs significantly from the unexposed population. In discrimination cases, the fraction of jobs or promotions a minority group received is compared to their percentage of qualified applicants. In both contexts, the Cochran-Mantel-Haenszel (CMH) test is a widely used measure of disparity between two classes of subjects. In our application, the two groups were the alleged “profit sharers” and other similar customers who requested IPO shares. The CMH measure was adapted in order to provide an interpretation of the difference between the two groups in terms of a ratio that could be easily conveyed to the hearing panel.

A major issue we faced was the definition of “fairness” in the allocation of IPO shares. This was needed, as our CMH-based measure requires a determination of the expected share of customers under fair allocation practices. Arthur Levitt, the Chairman of the SEC from 1993 to 2001, has stated that it is reasonable for a firm to allocate shares to its “best” customers, especially those who had given it substantial business prior to the market and IPO “boom.” A measure of the fairness of the allocations of a particular IPO needs to incorporate the previous business of the customers who requested shares in that IPO in order to determine which ones received “less” than their appropriate share and which customers were “favored.” The measure of the fairness of the allocations of IPO shares we proposed compares the actual number of shares the alleged “profit sharers” received to the number they would have been expected to receive based on their fraction of the business all requesting customers gave the firm during the previous twelve months.

The issue of a group of subjects being favored over another is central in discrimination law cases. We expressed our measure in terms of the “selection ratio” used to evaluate an employment practice for possible disparate impact. It will be seen that the group of alleged “profit sharers” were not favored. Indeed, the selection ratio measure indicates that the “profit sharers” were actually disadvantaged in the allocation process.

Background: Adapting the Cochran-Mantel-Haenszel (CMH) Test Statistic

No specific SEC rules govern the process of allocating shares in a securities offering. NASD (National Association of Securities Dealers) Conduct Rule 2110 requires member firms to observe just and equitable principles of trade. SEC or NASD rules prior to this case precluded brokerage firms from charging commissions in excess of 5%. Unlike many brokerage houses, this firm did not have a schedule for commissions for trades of various sizes. From its inception the firm allowed its customers to place their own commissions on their trades. As a consequence of the deregulation of the securities industry, this practice was legal and was not unique to the firm.

In the case, the regulators examined about two and half years of data and identified several groups of accounts as “profit sharers” at different times during the investigation. The final set of accounts was selected by a new criterion for “excessive” commissions and the alleged violations were claimed to have occurred in a six month period. A stock transaction was categorized as an “inflated” rate commission if the trade

involved 10,000 or more shares with a commission of 20 or more cents per share. Also, for the accounts with such trades, trades of 1,000 or more shares with commission of 75 or more cents per share were also considered “inflated.” These criteria were met by about 30 institutional accounts of the firm. The regulator’s expert decided to drop several of these because their “trading pattern” was not similar to that of the other members of this group. This final set of allegedly favored customers will be referred to as the APS (Alleged Profit Sharers) group in this paper.

The Cochran-Mantel-Haenszel (CMH) test compares two groups on a binary response, stratifying the data into K subgroups or strata with similar levels of other relevant covariates. Agresti provides a full discussion of the test and its applications. The data are given by a series of K 2x2 contingency tables. Traditionally, in each table the rows correspond to the "Treatment group" values (e.g "Placebo", "Drug A") and the columns to the "Response" values (e.g "No change," "Improvement"). The value of the test statistic is given by

$$CMH = \frac{\sum_{i=1}^K (observed_i - expected_i)}{\sqrt{\sum_{i=1}^K V_i}},$$

where V_i is the variance of the usual test of the equality of proportions in the i th stratum. The CMH test statistic is used to test the null hypothesis that the response is conditionally independent of the treatment.

If we consider the APS group as the treatment group and a comparable set of other customers as the control, then for every IPO a 2x2 table containing the number of requests each group made and their success rate can be formed. The data for each IPO define a separate stratum. This analysis was carried out after we noticed that the aggregate success rates of retail (non-institutional) customers exceeded those of the APS customers. This was quite surprising as institutional customers typically give investment firms much more business than retail customers. The CMH test for the 2x2 tables across all IPO allocations obtained that their success rates were not statistically significantly different.

The success rate analysis does not account for volume differences. That is, the fact that a customer was allocated IPO shares does not contain information as to the size of the allocation with respect to the total number of IPO shares the firm had available and the number of shares allocated to other customers. Thus, in addition to the CMH formal test, it is useful to have a measure of the magnitude of the difference $\sum (observed_i - expected_i)$ relative to the total expected value of the response. We use

$$\frac{\sum_{i=1}^K (\text{observed}_i - \text{expected}_i)}{\sum_{i=1}^K \text{expected}_i}$$

to measure the disparity between the actual and expected allocations as a percent of the expected number of shares. The numerator of this measure is the numerator of the CMH statistic while the denominator reflects their expected total shares under a “fair” system. Positive values of the ratio indicate an allocation exceeding their expected number whereas negative values indicate allocations below expected. This measure is similar to the attributable risk type measures used in epidemiology. To implement this measure one needs to determine the expected number of shares a customer would receive under a “fair” allocation system.

Expectancy Analysis: Was the allocation fair?

As already stated, there were no specific criteria or rules governing the allocation process set by the SEC, NASD or the firm. In our search for understanding allocation practices in the street, we came across statements of two SEC commissioners that it is reasonable for a firm to consider its business relationship with a customer in allocating IPO shares. The problem is to translate the factors a firm could appropriately take into account in the calculation of a fair expected number of shares for each customer who requests shares in a particular IPO. Some measurable factors that could be used in evaluating a customer’s business relationship with the firm are: (1) how long the customer has been with the firm, (2) the amount of business an account generated for the firm over a period of time, and (3) how active the account has been. While the potential of a customer to generate future business is also a reasonable factor, it is quite subjective and brokers are usually not asked to record such predictions at the time they allocate shares.

Data on the length of time a customer had been with the firm was not as accurate as we first thought. Some customers came with a newly hired broker so their association with the broker was longer than would be indicated by the firm’s records. Also, several major customers had substantial other business relationship with the firm. The only highly reliable data available to us were the trading data of all customers over a period of two years from which we could easily extract the commission business each customer gave the firm. Since an investment firm would naturally allocate more shares to customers who had given it more business than to other customers, the relationship between the allocations of shares in all IPO deals and the previous year’s commission business customers gave the firm was studied. We focused on one year’s prior business since it would account for seasonal variations in the trading patterns of different customers. The firm also thought that its brokers would have a relatively accurate picture of the business their customers gave them over the last year.

First, we investigated whether aggregate historical commission business would be a good predictor of IPO allocations especially in comparison to the new cents per share criterion used by the regulator. We compared the fraction of IPO shares allocated to customers requesting those shares to the twelve month average of the cents per share they put on trades and the corresponding average of their total commission business. As the total commission the customers gave over the year prior to the IPO was not normally distributed the Spearman correlation coefficient was used to assess the strength of the relationship between the number of shares allocated and total commission. The average Spearman correlation between IPO shares allocated and total commissions in all offerings was .58 and was highly significant (p -value $< .0001$). In contrast, the Spearman correlation coefficient between IPO shares allocated and cents per share was only .04, a non-significant result. This calculation was made because the regulator defined a commission to be “inflated” in terms of cents per share. It showed that a customer’s allocation was not related to their average commission in that metric.

We considered an allocation of IPO shares to a set of requesting customers to be fair if the fraction of the total number of shares a customer received was proportional to their share of prior business. In detail, for each offering the expected number of shares a customer who expressed interest would receive was determined by its fraction of the total prior business given the firm amongst the customers who were interested. Once this expected fraction, F , was determined, the expected number of shares the customer should receive is just F times the number of shares the firm had to allocate. For example, if the firm had 20,000 shares to allocate and a customer’s fraction F of prior business given by all requesting customers over the last year was .25, the customer would be expected to receive 5,000 shares.

To assess whether the APS customers received more than their fair share, for each offering we summed the expected fractions (F s) of the APS customers who requested shares to determine their total, say FT . The expected number of shares the APS customers should receive, assuming an allocation proportional to prior business, is FT times the number of shares the firm had to allocate. To continue the example, if FT were .60, e.g., three APS customers with F ’s of .25, .20 and .15 asked for shares, they would be expected to receive $.6 \times 20,000 = 12,000$ shares of the offering. By comparing the actual number of shares to the expected number of shares we can assess whether the APS customers received more or less than their fair share. This is accomplished by calculating the difference between the actual and expected numbers of shares for each offering and summing over all IPO deals. The results can be summarized by comparing the actual fraction of shares received by the APS customers in the offerings of interest to their fraction expected. The few new customers, i.e. the customers who had not executed any trades in the year prior to the month of the offering, were excluded from the computations, as they would be expected to receive zero shares since they had no business in the prior twelve months. The effect of this was quite small as about 96% of all IPO shares went to customers with prior business. Of course, some of these customers might have had an account with the firm but had not been active in the year preceding the month of an IPO deal.

To further illustrate how the calculations are carried out, we present the following hypothetical example. Suppose the firm had a total of 1,000 shares of an IPO to distribute to the 10 customers who requested shares. If we assume that two of these customers were new to the firm and were allocated a total of 100 shares, there remain 900 shares to allocate to the remaining 8 customers. Also assume that among the 8 customers, the alleged “profit sharers” fraction of the prior twelve-month business is .60 and they were allocated 500 shares. Table 1 below summarizes the calculations.

Table 1: Hypothetical Example to Illustrate the Calculation of Expected Number of Shares Allocated to APS Customers and Comparison with Actual Number

	Number of Received IPO Shares	Fraction of Aggregate Prior Year Business	Expected Number of IPO Shares	Difference (Received –Expected)
APS	500	0.6	$900 \times 0.6 = 540$	-40
All Other Accounts	400	0.4	$900 \times 0.4 = 360$	+40
Total	900	1.0	900	0

Table 2 reports the number of shares *in all IPO deals* the APS customers received and the number of shares they would have been expected to receive on the basis of their fraction of the commissions received by the firm in the previous 12 months.

Table 2: Comparison of Actual and Expected Number of Shares Allocated to the APS Customers

	Actual Number	Expected Number	Difference
57 IPOs	1,279,775	1,806,585.992	-526,811

As the total number of IPO shares allocated to all customers with prior business was 3,118,025, the actual and expected percentages of these offerings allocated to these accounts are given in a bar-plot in Figure 1. The APS customers received 41.04% of all IPO shares but on the basis of their prior commissions they would have been expected to receive 57.94% so they only were allocated $41.04/57.94 = .7083$ or 70.83% of their expected number. Thus, in relative terms, the APS received 70.83% of the IPO shares they would have been expected to receive had shares been allocated in proportion to a customer’s commissions during the previous twelve months.

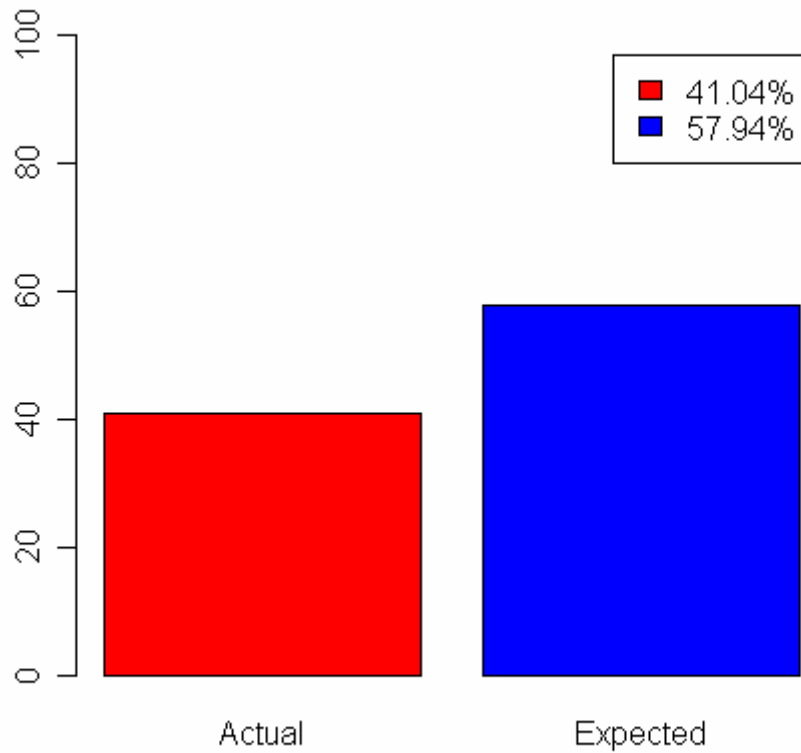


Figure 1. Comparison of Actual and Expected Percent of Shares Allocated to APS Customers

To confirm that these results reflecting the allocation process of IPO shares were not unique to the six-month period specific to the law case, a similar expectancy analysis was conducted for the subsequent six months. In that period the APS received about 66.15% of their expected number of shares or equivalently, 33.85% fewer shares than would be expected on the basis of their business during the previous twelve months.

A Sensitivity Analysis: Demonstrating the Robustness of the Results

When presenting the results of a statistical analysis in court it is useful to provide a sensitivity analysis showing that the main inference remains unaffected by the inevitable deviations of the data from the theoretical ideal. Several such analyses are described in our related paper listed in the references. Here we summarize an analysis answering an interesting non-statistical question posed by the lawyers.

For purposes of the regulators’ analysis, an inflated rate transaction was a stock trade involving: (i) commissions of \$.20 per share or more on 10,000 shares or more, or (ii) commissions of \$.75 or more on 1,000 shares or more. What effect would these two criteria have on the number of shares the APS customers would be expected to receive had these criteria been in existence at the time and enforced by the firm? To answer this question we adjusted the prior business for all customers who had executed such trades in the twelve-month period prior to the review period as well as during the review period as follows. All trades satisfying criterion (i) were assigned commissions calculated by setting the cents per share commission rate to \$.19. All trades satisfying criterion (ii) were assigned commissions by setting cents per share to \$.74. All other commissions were left intact. Then the expected number of shares for each IPO was calculated from these adjusted data, which removed the alleged “excessive” component of commissions given by the APS customers from the analysis. Note that the prior business of the APS customers is reduced and, consequently, their expected fraction of IPO shares becomes smaller.

Table 3 reports the number of IPO shares the APS accounts received and the number of shares they would have been expected to receive on the basis of their fraction of the adjusted commissions they would have had given the firm in the previous twelve months. For each offering the expected number of shares for the APS customers is determined from their fraction of the adjusted total commission business during the previous twelve months of all customers requesting that specific offering as described in the main expectancy analysis.

Table 3: Comparison of Actual and Adjusted Expected Number of Shares Allocated to the APS Customers

	Actual Number	Expected Number	Difference
57 IPOs	1,279,775	1,542,538.836	-262,763.836

Figure 2 displays a bar-plot with the percentages of actual and expected number of shares for the APS group. The actual remains the same as in Figure 1. The new expected value, after the adjustment, is now 49.47%. In relative terms, even after subtracting from their prior business the allegedly “excessive” component of their commission, the APS customers received 17.04% fewer IPO shares they would have been expected to receive had shares been allocated in proportion to a customer’s *adjusted* commissions during the previous twelve months.

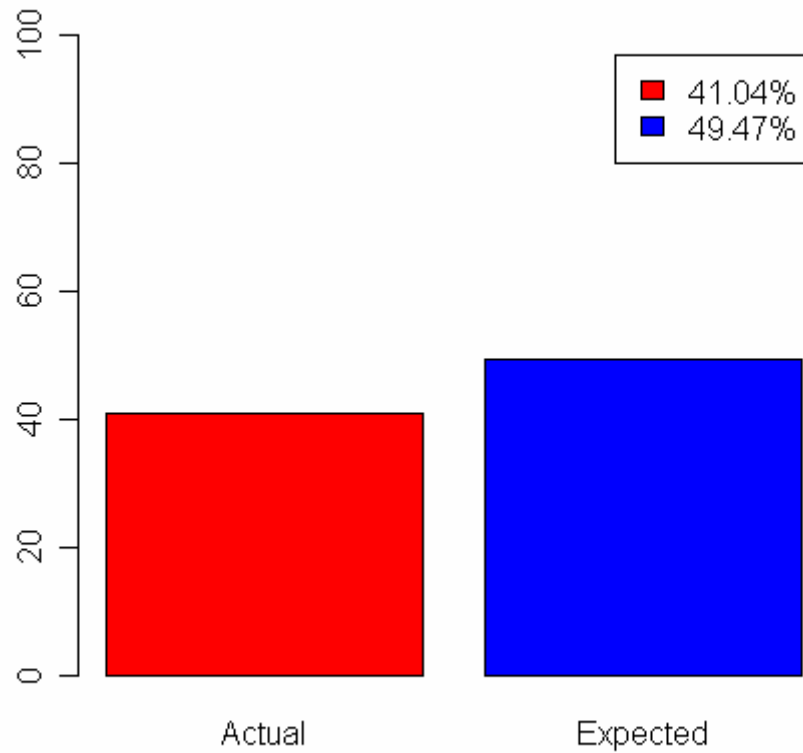


Figure 2. Comparison of Actual and Adjusted Expected Percent of Shares Allocated to the APS Customers

Drawing on an Analog in Equal Employment: Selection Ratio Analysis

To further appreciate the magnitude and importance of the shortfall in the number of IPO shares received by the APS customers identified as “profit sharers” by the regulator, this shortfall can be interpreted as a “*selection ratio*” used in equal employment cases. It is the ratio of the pass rate of a minority group of job applicants to the pass rate of job applicants from the majority group and is used to assess whether a job requirement (*e.g.*, attaining a certain score on a test) has a “disparate impact” on minority applicants. In that application, the government guideline is that selection ratios less than .80 or four-fifths (*e.g.*, minority applicants receive passing scores on a test at a rate less than 80%, or four-fifths, the rate of Caucasians) indicate a disparate impact so that the specified job requirement needs to be shown to be job-related.

Although the selection ratio is defined for the comparison of two success rates, Kairys *et al.* and Gastwirth and Greenhouse have translated it to our situation. Suppose the fraction π of job applicants is minority and they receive the fraction p ($<\pi$) of the jobs.

Assume there is a total of N applicants of which n are hired. Then the minority success rate is $np/\pi N$ and the majority success rate is $n(1-p)/(1-\pi)N$. The selection ratio then is given by

$$\frac{\frac{np}{\pi N}}{\frac{n(1-p)}{(1-\pi)N}} = \frac{\frac{p}{1-p}}{\frac{\pi}{1-\pi}}$$

Notice that the selection ratio is the ratio of the odds a hire is minority to the odds an applicant is minority. In our application the fraction of shares the APS received is p and their fraction of prior business is π .

The selection ratio for the APS customers calculated from the data in Figure 1 is .5053 well below the value of the government guideline, .80. This indicates that on the basis of their prior commission business the APS customers were disadvantaged relative to other customers in the IPO allocation process used at the firm. The selection ratio for the data in Figure 2, where the alleged “excessive” or “inflated” commissions were adjusted downward to comply with the “new” criterion, is .711, still below the .80 threshold. Thus interpreting the results of the expectancy analysis in terms of the guidelines used in equal employment cases shows that the selection ratio of the APS group indicates they were disfavored rather than favored by the firm.

A Final Note

While the IPO frenzy during the late 1990’s was unusual, there remains a substantial IPO market as new companies grow and decide to go public. The methodology described in this paper should be useful both to regulators and individual firms who desire to check the fairness of their IPO allocations. In particular, a firm can readily monitor its individual brokers to ensure that none of them is favoring a few customers by giving them noticeably more shares than their prior business merits. If a customer has an unusually high success rate or is often allocated more shares than expected, the firm could then examine the pattern of commission business the customer gives to see whether it is concentrated near the time the customer received shares in an IPO and is related to the profit potential of the IPO.

The “pattern” that the regulatory body in the case motivating this research used really does not capture “profit sharing” activity around the days of an IPO. Recall that only trades of 10,000 shares or more with commissions of 20 cents or more per share or trades of at least 1,000 shares with commissions of 75 cents per share or more were considered “inflated” or “excessive”. Thus, a customer who traded 5,000 shares and placed a commission of 50 cents per share would go undetected even though he/she gave a commission of \$2,500, exceeding a \$2,000 commission on a trade of 10,000 at 20 cents per share. It would seem more appropriate to consider the total commission business near the receipt of an IPO and examine the ratio of these commissions to their projected profit from the IPO shares they were allocated. If there had been a formal or informal *quid pro*

quo arrangement or understanding these ratios would be expected to concentrate around the understood “share.”

As far as the authors know, this is the first use of concepts originally developed in biostatistics in a securities law case. The factors used to determine the “expected” number of shares a customer would receive under a “fair” system rely on practices considered appropriate in the industry. In another case, other factors might be incorporated provided reliable data on them are available.

References and Further Reading

Agresti, A. (1990). *Categorical Data Analysis*. New York: Wiley.

Gel, Y. Miao, W. and Gastwirth, J. L. (2005). The Importance of Checking the Assumptions Underlying Statistical Analysis: Graphical Methods for Assessing Normality. *Jurimetrics Journal*, 46, 3-26.

Gastwirth, J. L. (1988). *Statistical Reasoning in Law and Public Policy. Volume 1 Statistical Concepts and Issues of Fairness*. San Diego: Academic Press.

Gastwirth, J. L., Bura, E. and Modarres, R. (2005). Statistical Methods for Assessing the Fairness of the Allocation of Shares in Initial Public Offerings. *Law, Probability and Risk*. 4, 143-158.

Gastwirth, J. L., Modarres, R. and Bura, E. (2005). The Use of the Lorenz Curve, Gini Index and Related Measures of Relative Inequality and Uniformity in Securities Law. *Metron*. In press.

Gastwirth, J. L. and Greenhouse, S. W. (1987). Estimating a common relative risk: Application in equal employment. *Journal of the American Statistical Association*, 82, 38-45.

Kairys, D., Kadane, J. B., and Lehoczky, J. P. (1997). Jury Representativeness: A Mandate for Multiple Source Pools. *California Law Review*, 65, 776-827.