The Marginal Welfare Cost of Capital Taxation:
Discounting Matters\textsuperscript{1}

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Abstract

We interpret the marginal welfare cost of capital income taxes as the present discounted value of consumption distortions. Such an asset market interpretation emphasizes the importance of the interest rate used to value future distortions, especially in the presence of uncertainty. We find that the interest rate decreases as the tax rate increases, thus increasing the welfare cost. The variations in the interest rate are caused by amplified responses of consumption to exogenous shocks as a result of capital taxation. The welfare cost may be underestimated if variations in interest rates are ignored, especially when tax rates are high.

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1 Introduction

In this paper we interpret the marginal welfare cost of capital income taxes as the present discounted value of consumption distortions. Just like prices of any other risky securities, the marginal welfare cost of taxes is determined by three factors: the stream of consumption distortions caused by inefficient allocation of resources, the covariance between consumption distortions and systematic risk, and the interest rate used to discount the stream of consumption distortions. Although the first two factors have been analyzed in the previous literature, including Lucas (1990) and Gordon and Wilson (1989), the impact of changes in capital income taxes on the interest rate has been ignored. We find that the interest rate used to discount future distortions decreases as the capital income tax rate increases, thus increasing the welfare cost of taxes. Our work brings to the forefront the importance of the interest rate used to value future consumption distortions. The welfare cost may be underestimated if variations in risky discount rates are ignored, especially when aggregate uncertainty is significant and tax rates are high.

We study the welfare cost of taxes in a general equilibrium production economy with varying degrees of uncertainty. We find that the marginal welfare cost of capital income taxes increases with the tax rate in both deterministic and stochastic environments. Moreover, the marginal welfare cost is larger in a stochastic
than in a deterministic environment, and the gap widens as the tax rate increases. In the deterministic case, the upward slope of the marginal welfare cost curve is mostly driven by increasing consumption distortions as the tax rate increases. In the stochastic case, however, variations in the interest rate and in the covariance between consumption distortions and systematic risk also play important roles. We denote consumption distortions, variations in the interest rate and variations in the covariance term as the distortion, discounting and insurance effects of capital income taxes, respectively. The discounting and insurance effects that capture the variations in the interest rate and in the covariance term are unique for a stochastic environment.

The discounting effect is the core reason for the widening gap between marginal welfare costs in the stochastic and deterministic environments. As the tax rate increases, the discounting effect increasingly dominates the insurance effect, thus reducing the interest rate used to discount future distortions and raising the welfare cost. The intuition for the increasingly dominant discounting effect is as follows. As the capital income tax rate increases, investment is further discouraged. As a result, investment constitutes an increasingly smaller share of aggregate output in the steady state. A lower investment-output ratio implies that investment needs to respond more to exogenous shocks in order to smooth consumption, which now
constitutes a larger fraction of output. Santoro and Wei (2011) show that such a mechanism can be responsible for increasingly amplified responses of consumption, and consequently of the marginal utility of consumption, to aggregate technology shocks. The amplified responses of the marginal utility of consumption typically lead to precautionary motives, which tend to reduce the interest rate used to value future consumption distortions.

In addition to the degree of aggregate uncertainty, the marginal welfare cost of capital income taxes depends on the preference and production specifications. Since we use an asset market approach to price consumption distortions, it seems important to have a production-based model that is not only able to mimic some basic asset pricing features but also tractable enough to make transparent the mechanisms introduced by capital income taxes. Jermann (1998) and Boldrin, Christiano and Fisher (2001) show that the key ingredients for such a model are habit formation in preferences and adjustment costs in production technology. Based on this consideration, we assume moderately high habit persistence and capital adjustment costs in the benchmark parameterization. We also conduct a sensitivity analysis in the same stochastic environment but with varying degrees of habit persistence and capital adjustment costs. We find that the discounting effect remains important.

\footnote{The risk-free interest rate can be overly volatile in models such as Jermann (1998) and Boldrin, Christiano and Fisher (2001). One purpose of the sensitivity analysis is to examine the robustness}
just as in the benchmark case, resulting in a higher welfare cost in the stochastic environment.

Our findings can be related to those of Chamley (1981) and Lucas (1990), which use a deterministic dynamic general equilibrium model to evaluate the welfare gain obtained by abolishing the capital income tax. Since tax reforms typically involve discrete changes in tax rates rather than abolition of a tax, we focus specifically on the welfare cost of a marginal shift in the capital income tax rate, and integrate the marginal welfare cost over the given range of tax rates to compute the total gain from discrete changes in tax rates. Our calculation yields an overall welfare gain from abolishing capital income taxes that is at the high end of Lucas’s estimate. The impact of capital income taxes on the interest rate, which is absent in a deterministic setting, contributes to the higher welfare cost of capital income taxes in the presence of aggregate uncertainty.

Judd (1987) examines the marginal efficiency cost of various factor taxes in a deterministic model. He states that “any biases of the deterministic approach relative to a more realistic model with uncertainty must arise from decreasing returns in capital intensity and third-order properties of utility functions”. We find that it is precisely the omission of the effect of capital income taxes on the interest rate that
biases the estimate of the deterministic approach, and this effect is closely related to the third-order properties of the utility function.

Our results advance the insights gained from Gordon and Wilson (1989), which examines the marginal welfare loss of capital taxation in a stochastic production economy similar to ours. They argue that past measures that ignore the negative covariance between consumption distortions and the stochastic discount factor “likely overstate the efficiency costs of a rise in the tax rate, perhaps dramatically.” The negative covariance stressed by Gordon and Wilson (1989) is also present in our framework. However, since Gordon and Wilson (1989) only examine the marginal welfare loss at a single tax rate, they do not study the declines in the interest rate accompanied by increases in the capital income tax rates. We find that the declines in the interest rate are significant enough to dominate the increasingly negative covariance as the tax rate increases.

We organize the paper as follows. In Section 2, we describe the production economy and derive our measure of the marginal welfare cost of capital income taxes. In Section 3, we use the production economy model to examine the properties of the marginal welfare cost curves, and decompose the marginal welfare cost to analyze

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3 Bulow and Summers (1984) and Gordon (1985) also study the welfare cost of taxing risky capital income. An important limitation of their work is that they both employ a two-period framework, which alters the risk characteristics of any long-lived securities.
the distortion, discounting and insurance effects separately. Section 4 describes the sensitivity analysis. Section 5 concludes.

2 A Stochastic Production Economy

In this section, we describe a model that features a stochastic production economy, and then derive the market value of consumption distortions as a measure of the marginal welfare cost of capital income taxes.

There is a continuum of infinitely-lived identical households that own a representative firm. The government levies taxes on capital income and rebates them back in a lump sum to the households. The economy grows at a constant rate \( g \).

2.1 The Household’s Problem

The representative household maximizes an expected life-time utility function subject to a standard budget constraint.

\[
\max_{B_{t+1}, C_t} \quad E_0 \sum_{t=0}^{\infty} \beta t \frac{(C_t - bC_{t-1})^{1-\gamma}}{1 - \gamma}
\]

subject to:
\[ C_t + I_t + B_{t+1}V_t^B = (1 - \tau) R_t K_t + W_t L_t + B_t + \chi_t. \]

Here \( \beta \) is the subjective time preference and \( C_t \) is real consumption at time \( t \). The coefficient \( \gamma \) measures the curvature of the representative household’s utility function with respect to its argument \( C_t - bC_{t-1} \). When \( b > 0 \), the utility function allows for habit persistence based on the household’s own consumption in the previous period.

In the budget constraint, \( I_t \) represents investment. \( B_t \) represents private risk-free bonds held from period \( t - 1 \) to \( t \). \( V_t^B \) is the price of the risk-free bond, and \( \tau \) is the proportional capital income tax rate on capital income \( R_t K_t \). The variable \( W_t L_t \) represents labor compensation. The variable \( \chi_t \) is a lump-sum transfer of all the tax revenues from the government.

### 2.2 The Firm’s Problem

We assume that there exists a representative firm owned by households. The representative firm chooses optimal capital and labor inputs given the wage and rental rate at each period.

The firm produces output, \( Y_t \), using the Cobb-Douglas production technology:

\[ Y_t = Z_t K_t^\alpha [(1 + g)^t L_t]^{(1-\alpha)}, \]
where \( L_t \) represents labor, \( K_t \) is the capital stock, and the logarithm of the stochastic productivity level, \( Z_t \), follows

\[
z_t = \rho z_{t-1} + \sigma \xi_t, \quad \xi_t \sim N(0, 1). \tag{2}
\]

Here \( \rho \) is the persistence parameter and \( \sigma \) indexes the degree of aggregate uncertainty. The capital accumulation process is given by:

\[
K_{t+1} = (1 - \delta) K_t + \Psi(K_t, I_t), \tag{3}
\]

where \( I_t \) represents investment. We specify the functional form of \( \Psi(K_t, I_t) \) as

\[
\Psi(K_t, I_t) = \left[ \frac{(g + \delta)^\eta}{1 - \eta} \left( \frac{I_t}{K_t} \right)^{1-\eta} + \frac{\eta (g + \delta)}{\eta - 1} \right] K_t, \tag{4}
\]

where the capital supply is inelastic when \( \eta \) approaches infinity.\textsuperscript{4} The concavity of this function captures convex costs of adjustment.

\textsuperscript{4}Capital adjustment costs have been studied by Eisner and Strotz (1963), Lucas (1967) and others. The particular specification we adopt is similar to that of Jermann (1998).
2.3 Equilibrium

In equilibrium, output is equal to the sum of consumption and investment:

\[ Y_t = C_t + I_t. \]  

(5)

All the asset markets clear. Bonds are in zero net supply. Given that leisure does not enter the utility function, households will allocate their entire time endowment to productive work. Labor supply is constant and normalized to 1. It is important to note that since labor supply is inelastic in our stylized model, capital income taxes are the only source of distortion. In equilibrium, the real wage and the gross rental rate are given, respectively, by

\[ W_t = (1 - \alpha) \frac{Y_t}{L_t}. \]
\[ R_t = \alpha \frac{Y_t}{K_t}. \]

The first-order condition for investment is:

\[ \frac{1}{\Psi_I(K_t, I_t)} = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[ \alpha (1 - \tau) \frac{Y_{t+1}}{K_{t+1}} + \frac{(1 - \delta) + \Psi_K(K_{t+1}, I_{t+1})}{\Psi_I(K_{t+1}, I_{t+1})} \right] \right\}, \]  

(6)
where $\beta^{\Lambda_{t+1}}_t$ represents the stochastic discount factor and $\Lambda_t$ is given by

$$\Lambda_t = u'(C_t - bC_{t-1}) - b\beta E_t u'(C_{t+1} - bC_t).$$  \hspace{1cm} (7)

Here $\Psi_I(\bullet)$ and $\Psi_K(\bullet)$ are the derivatives of capital adjustment costs with respect to investment and capital stock. Capital income taxes distort investment decisions by reducing the marginal benefit of investment.

It is straightforward to show that both the equilibrium capital stock, which is given by 

$$h^{*} = \left[ \frac{\alpha\beta^*(1-\tau)}{1-\beta^*(1-\delta)} \right]^{1-\alpha},$$

and the equilibrium investment-output ratio, $(g + \delta)(K_s^*)^{1-\alpha}$, is monotonically decreasing in $\tau$. The inverse relationship between the investment-output ratio and the tax rate have implications for dynamic responses of consumption to exogenous shocks, as shown in Santoro and Wei (2011).

### 2.4 An Asset Price Measure of Welfare Cost

Given the equilibrium allocations in the production economy, we define the marginal cost of capital income taxes and relate it to the normalized market value of consumption distortions.

For any arbitrarily given $\tau$, we consider two scenarios starting from the initial state $s_0$ characterized by $\{K_s^*, C_s^*, Z_s\}$, which represent, respectively, the steady-
state values of capital stock and consumption at a given $\tau$ and the steady-state level of the aggregate productivity. In the first scenario, the marginal tax rate remains unchanged at $\tau$. We use $\{C^\tau_t\}_{t=0}^\infty$ to represent the consumption stream for the representative household in the economy with the tax rate $\tau$ at each period $t$. The first superscript, $\tau$, stresses the dependence of consumption on the given tax rate, and the second superscript, 0, indicates that the tax rate remains unchanged at $\tau$.

Under the alternative scenario, there is a permanent marginal shift in the capital income tax rate from $\tau$ to $\tau + \varepsilon$ starting in the initial period. After that, the economy moves on a dynamic path from the initial state $(s_0)$ consistent with a tax rate $\tau$ to a long-run state compatible with the higher marginal tax rate, $\tau + \varepsilon$. We use $\{C^{\tau,\varepsilon}_t\}_{t=0}^\infty$ to represent the alternative consumption stream along this transition path resulting from the marginal tax rate increase. Here the first superscript, $\tau$, stresses that the initial state of the economy is consistent with the tax rate $\tau$, and the second superscript, $\varepsilon$, indicates that the tax rate shifts up from $\tau$ by $\varepsilon$.

We assume the same aggregate technology process, $\{Z_t\}_{t=0}^\infty$, for the two scenarios. In the deterministic environment, $Z_t$ is equal to 1 for all $t$.

We measure the welfare cost of this marginal tax change in terms of the compensation that renders the representative household indifferent between the con-
sumption streams with or without the permanent marginal increase in the tax rate. This compensation is measured as a fraction of consumption for each period in the economy with the marginally higher tax rate.

Let $\lambda$ be the fraction that will serve as a compensating consumption supplement, and define the indirect utility function $\mathcal{U}$ for the representative household by:

$$
\mathcal{U}(\lambda|\tau) = \mathcal{U}_0 \left( \left\{ (1 + \lambda) C_t^{\tau,\varepsilon} \right\}_{t=0}^{\infty} \right),
$$

where $\mathcal{U}(\bullet)$ represents lifetime utility from any consumption stream.

Here $\mathcal{U}(\theta, \varepsilon|\tau)$ is interpreted as the utility the consumer would enjoy from the alternative consumption stream $\{C_t^{\tau,\varepsilon}\}_{t=0}^{\infty}$ and a consumption supplement $\theta C_t^{\tau,\varepsilon}$ for each period.

We define the unique positive value of $\theta$ that satisfies the condition:

$$
V(\theta, \varepsilon|\tau) = V(0, 0|\tau),
$$

as the welfare cost of raising the capital income tax rate from $\tau$ to $\tau + \varepsilon$. This measure reflects the welfare cost of the marginal increase in the tax rate in the long run, as well as during the transition period. Here $\theta$ is a function of both $\varepsilon$ and $\tau$.

Using the implicit function theorem, we can derive the derivative of $\theta(\varepsilon, \tau)$ with
respect to $\varepsilon$:

$$
\theta_{\varepsilon}(\varepsilon, \tau) \bigg|_{\varepsilon=0} = -\frac{V_{\varepsilon}(0,0|\tau)}{V_{\theta}(0,0|\tau)}.
$$

(9)

Since $\theta(0, \tau)$ is equal to zero according to our definition, the welfare cost of a permanent discrete change from $\tau$ to $\tau + \varepsilon$ can be approximated locally as

$$
\theta(\varepsilon, \tau) \simeq \varepsilon \theta_{\varepsilon}(\varepsilon, \tau) \bigg|_{\varepsilon=0}.
$$

(10)

Here $\theta_{\varepsilon}(\varepsilon, \tau)$ is the marginal welfare cost of a marginal tax rate increase of $\varepsilon$ in an economy characterized by an initial capital income tax rate of $\tau$.

Given the utility function in our economy, we have the following representation of the marginal welfare cost of taxes:

$$
\theta_{\varepsilon}(\varepsilon, \tau) = E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{\Lambda_{\tau,0}}{\Lambda_0} \frac{\partial}{\partial \varepsilon} \theta(-C_t^{\varepsilon}) \right\} \frac{1}{W_\tau},
$$

(11)

where $\Lambda_{\tau,0}$ is the marginal utility of consumption in the first scenario, when the tax rate stays at $\tau$, and $\beta^t \frac{\Lambda_{\tau,0}}{\Lambda_0}$ is the stochastic discount factor. The denominator $W_\tau$ is given by

$$
W_\tau = E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{\Lambda_{\tau,0}}{\Lambda_0} C_t^{\tau,0} \right\}.
$$
The marginal welfare cost of capital income taxes is now the ratio of the values of two securities. The numerator represents the value of a security that is a claim to the differences in the consumption streams (consumption distortions) due to the marginal increase in the capital income tax rate. The denominator, instead, represents the value of a security that is a claim to the consumption stream in an economy without the marginal increase in the current tax rate \( \tau \).

The marginal welfare cost of capital income taxes is now effectively the normalized market value of consumption distortions, with the denominator serving as the normalizing factor.

### 3 Decomposition of the Marginal Welfare Cost

In this section, we apply equation (11) to compute the marginal welfare cost for all tax rates. We describe the main features of the marginal welfare cost curve and discuss its decomposition. The details of the computation are contained in Appendix A.

We describe the benchmark parameterization in Table 1. Since our focus is on valuing stochastic consumption distortions, it is important to have a model that can reproduce key asset market statistics. Jermann (1998) shows that a combination
of high habit persistence and capital adjustment costs help to generate reasonable
asset return statistics in a model like ours. Based on this consideration, we introduce
moderately high habit persistence and capital adjustment costs by setting \( \beta \) equal to
0.6 and \( \eta \) equal to 4.24. The other parameter values are similar to those in Jermann

3.1 Features of the Marginal Welfare Cost Curve

Figure 1 plots the marginal welfare cost, \( \theta (\varepsilon, \tau) \), for a range of initial tax rates \( \tau \)
in both a deterministic (\( \sigma = 0 \)) and a stochastic (\( \sigma = 0.016 \)) environment. These
two environments are otherwise the same except for the difference in \( \sigma \). Here we
approximate \( \varepsilon \) with a value of 0.01. McGrattan and Prescott (2005) estimate the
corporate tax rate to be 0.43 for periods of high tax rates. For illustration, we pick
this as one of the extreme values that the capital income tax rate can take.

Figure 1 displays three salient features about the marginal welfare cost curves:

1. The marginal welfare cost curves are convex and upward sloping in the tax

\[\text{Footnotes:}\]
rate in both the deterministic and stochastic cases.

2. The marginal welfare cost curve is higher and steeper in the stochastic case than in the deterministic case.

3. The gap between the marginal welfare cost curves in the stochastic and deterministic cases widens as the tax rate increases.

Since the total welfare cost of a discrete change in the tax rate is the integral of the marginal welfare cost for the given range of tax rates, steeper slopes imply higher total welfare costs. The gap between the two marginal welfare cost curves can thus be translated into the difference in the total welfare gain of eliminating capital income taxes by integration.

According to a recent CBO study, the effective tax rate on capital income is 36.1 percent for equity-financed corporate business in the United States. This is also the tax rate Lucas (1967) uses in computing the welfare gain of abolishing the capital income taxes. Since the representative firm in our model is equity-financed, we compute the marginal and total welfare gain at the tax rate of 0.36 to compare with the previous study, and provides a benchmark assessment of the welfare gain from similar tax changes.

We find that the total welfare gain of eliminating capital income taxes with
\[ \tau = 0.36 \] is equivalent to 3.43 percent of lifetime consumption in the deterministic case, but is equal to 4.92 percent in the stochastic case. The part of total welfare gain attributed to uncertainty amounts to around 40 percent of the total welfare gain in the deterministic case. Ignoring uncertainty may lead to an underestimation of the welfare cost of capital income taxes.

In a deterministic environment, the upward slope of the marginal welfare cost curve is mostly driven by increasing consumption distortions as the tax rate increases. In a stochastic environment, however, variations in both the stochastic discount factor and the covariances between the stochastic discount factor and consumption distortions also play important roles.

### 3.2 Decomposition

Here we decompose the numerator of the marginal welfare cost, as defined in equation (11), into two elements. The decomposition is as follows:

\[ \theta_{\varepsilon} (\varepsilon, \tau) = \Phi_{\varepsilon} + \Omega_{\tau}, \]  

(12)
where

$$
\Phi_\tau = \frac{\sum_{t=0}^{\infty} \left\{ E_0 \left( \frac{\beta_t^{t_0^{0,0}}}{\Lambda_0^{t,0}} \right) E_0 \left[ \frac{\partial (-C_t^{r,e})}{\partial \varepsilon} \right] \right\}}{W_\tau}, \quad \Omega_\tau = \frac{\sum_{t=0}^{\infty} \text{cov}_0 \left[ \frac{\beta_t^{t_0^{0,0}}}{\Lambda_0^{t,0}}, \frac{\partial (-C_t^{r,e})}{\partial \varepsilon} \right]}{W_\tau}. \tag{13}
$$

The decomposition reveals multiple forces determining the marginal welfare cost in the presence of uncertainty. In $\Phi_\tau$, $E_0 \left[ \frac{\partial (-C_t^{r,e})}{\partial \varepsilon} \right]$ represents the expected consumption distortion that varies with the tax rate. We label this the distortion effect of capital income taxes. A corresponding term is also present in a deterministic environment. Also in $\Phi_\tau$, $E_0 \left( \frac{\beta_t^{t_0^{0,0}}}{\Lambda_0^{t,0}} \right)$ is the price of a $t-th$ period discount bond, which determines the term structure used to value future consumption distortions.

It is important to note that in a stochastic environment the interest rate not only varies with time, but also depends upon the capital income tax rate, $\tau$. We call that the discounting effect of capital income taxes. In $\Phi_\tau$, the product of $E_0 \left[ \frac{\partial (-C_t^{r,e})}{\partial \varepsilon} \right]$ and $E_0 \left( \frac{\beta_t^{t_0^{0,0}}}{\Lambda_0^{t,0}} \right)$ represents expected consumption distortions at period $t$ valued at the price of the $t$-th period discount bond. The second element, $\Omega_\tau$, captures the variations in the covariance between the stochastic discount factor and consumption distortions as the tax rate changes. We call this effect the insurance effect of capital income taxes.

In the next sections, we examine each of the three effects in detail.
3.2.1 Distortion

To understand the distortion effect, it is useful to examine the path of consumption distortions for a particular tax rate. In Figure 2, we plot the percentage differences in the two consumption paths \( \{C_t^{\tau,0}\}_{t=0}^{\infty} \) and \( \{C_t^{\tau,\varepsilon}\}_{t=0}^{\infty} \) as a fraction of \( \{C_t^{\tau,0}\}_{t=0}^{\infty} \) for \( \tau = 0.36 \). As mentioned above, \( \{C_t^{\tau,0}\}_{t=0}^{\infty} \) represents the consumption path under the first scenario, where the marginal tax rate remains unchanged at \( \tau \), while \( \{C_t^{\tau,\varepsilon}\}_{t=0}^{\infty} \) represents the corresponding path under the second scenario, where the tax rate shifts permanently to \( \tau + \varepsilon \) at period 0.

Although those two scenarios start at the same initial steady state \( \{K_{ss}^{0.36}, C_{ss}^{0.36}, Z_{ss}\} \) and are subject to the same aggregate technology process, their consumption paths diverge as soon as the marginal tax change is implemented. The resulting path of consumption distortions demonstrates the short-run and long-run trade-offs following a marginal shift in the tax rate, a point also made by Bernheim (1981) and Lucas (1990).

Under the second scenario, consumption temporarily increases above what it would be under the first scenario, resulting in negative consumption distortions. Increases in consumption in the initial periods accommodate the convergence of the capital stock to a lower level consistent with a higher tax rate in the long run. However, as time passes, distorted investment decisions also lead to both lower
capital stock and lower output. Eventually consumption declines and converges to a lower long-run equilibrium level relative to that under the first scenario, where the tax rate remains unchanged at \( \tau \). Consequently, the amount of consumption distortions fluctuates around a positive value in the long run.

In Figure 2, we also plot a corresponding relative measure of consumption distortions in a deterministic economy. The same short-run and long-run trade-off applies, and the amount of consumption distortions eventually converges to a positive value.

### 3.2.2 Discounting

The rate that is used to discount consumption distortions is a function of the stochastic discount factor, \( \beta \frac{\Lambda^0}{\Lambda_0^{\rho}} \), which in turn depends upon the given capital income tax rate \( \tau \). The interest rate plays a crucial role in determining the welfare cost of taxes. In this section, we examine how the interest rate varies across different capital income tax rates and determines the welfare cost of taxes together with other forces.

In order to isolate the effect of the stochastic discount factor on the marginal welfare cost, we compare two methods of valuing consumption distortions by discounting the same path of consumption distortions differently. In the first method, we use \( \beta \), the discount factor in the deterministic case, while in the second method,
we use the stochastic discount factor \( \frac{\beta^{\Lambda_t,0}}{\Lambda_0^0} \), which reflects consumption fluctuations across different states contingent upon the tax rate \( \tau \).

The upper left panel of Figure 3 plots the present discounted value of strips of deterministic consumption distortions, \( \frac{\partial(-C_t^{\tau,\epsilon})}{\partial \epsilon} |_{\sigma=0} \), at \( \tau = 0.36 \) using the above two methods. The horizontal axis denotes the time period for the strip, and the vertical axis denotes the market value of the corresponding strip at period 0. The market value of the \( n \)-th strip computed with the first method, \( \beta^t \frac{\partial(-C_t^{\tau,\epsilon})}{\partial \epsilon} |_{\sigma=0} \), stays below the value computed with the second method, \( E_0 \left( \beta^t \frac{\Lambda_t,0}{\Lambda_0^0} \frac{\partial(-C_t^{\tau,\epsilon})}{\partial \epsilon} |_{\sigma=0} \right) \) for almost all the time periods. This indicates that the values of discount bonds in the stochastic environment are higher than those in the deterministic case. We plot the differences between \( \beta^t \frac{\partial(-C_t^{\tau,\epsilon})}{\partial \epsilon} |_{\sigma=0} \) and \( E_0 \left[ \beta^t \frac{\Lambda_t,0}{\Lambda_0^0} \frac{\partial(-C_t^{\tau,\epsilon})}{\partial \epsilon} |_{\sigma=0} \right] \) in the upper right panel of Figure 3. This difference measures the strength of the discounting effect resulting from the presence of uncertainty. In the left panel of Figure 4, we plot the differences between those two terms for three different tax rates: 0.001, 0.13, 0.36. As the tax rate increases, the discounting effect becomes stronger.\(^7\)

The intuition for the increasing discounting effect is as follows. As the capital income tax rate increases, investment is further discouraged. As a result, investment constitutes an increasingly smaller share of aggregate output in the steady state.

\(^7\)The humped shape of the curves is explained by both the rising and then stabilizing of consumption distortions and the decaying value of the discount factor as \( t \) increases.
lower investment-output ratio implies that investment needs to respond more to exogenous shocks in order to smooth consumption, which now constitutes a larger fraction of output. Santoro and Wei (2011) show that such a mechanism can be responsible for amplifying the responses of consumption, and consequently of the marginal utility of consumption, $\Lambda$, to aggregate technology shocks in an economy with higher tax rates. The amplified responses of the marginal utility of consumption typically lead to precautionary motives, which tend to reduce the interest rate used to value future consumption distortions. (paraphrase this sentence to be different from the introduction).

### 3.2.3 Insurance

The term, $\text{cov}_{0} \left[ \beta^{t} \frac{\Lambda^{x,0}}{N_{0}} \sigma, \frac{\partial (-C^{x,e})}{\partial \epsilon} \right]$, in $\Omega_{\tau}$ denotes a covariance between the stochastic discount factor and consumption distortions conditional on the information at time 0. A negative covariance between the two implies that consumption distortions are large in states when such distortions are valued less. Such desirable coincidence reduces the welfare cost of taxes and acts as insurance in the presence of uncertainty (see, for example, Gordon and Wilson, 1989). That is the reason we call this effect the *insurance effect* of capital income taxes.

The lower left panel of Figure 3 plots $E_{0} \left[ \beta^{t} \frac{\Lambda^{x,0}}{N_{0}} \sigma \right] E_{0} \left[ \frac{\partial (-C^{x,e})}{\partial \epsilon} \right]$ and $E_{0} \left( \beta^{t} \frac{\Lambda^{x,0}}{N_{0}} \frac{\partial (-C^{x,e})}{\partial \epsilon} \right)$.
for all periods when $\tau$ is equal to 0.36. As shown in equation (12), the difference between the two variables represents $\text{cov}_0 \left( \frac{\beta^t \Lambda^t,0}{\Lambda_0}, \frac{\partial(-C_{t,0}^{t,0})}{\partial z} \right)$, the covariance between the stochastic discount factor and consumption distortions for all time periods. The lower right panel of Figure 3 plots this difference. The difference remains largely negative. Such a negative covariance implies that consumption distortions are higher in states when they are valued less, reflecting the insurance effect of capital income taxes. This desirable property would reduce the welfare cost of taxes in the presence of uncertainty.

In the right panel of Figure 4, we plot $\text{cov}_0 \left( \frac{\beta^t \Lambda^t,0}{\Lambda_0}, \frac{\partial(-C_{t,0}^{t,0})}{\partial z} \right)$ for all time periods with a tax rate of 0.001, 0.13 or 0.36. As the tax rate increases, the covariance becomes more negative, reflecting a stronger insurance effect. The increasingly negative covariance can be explained by both amplified responses of the marginal utility of consumption and higher consumption distortions as the tax rate increases.

### 3.3 Distortion, Discounting and Insurance Across Tax Rates

In all, we identify three effects of capital income taxes on the marginal welfare cost: the distortion, discounting and insurance effects. The distortion effect is present in both stochastic and deterministic environments, while the second two effects, which are closely related to second-order moments, are present only in stochastic
Since the latter two effects appear only in stochastic environments, the difference between the marginal welfare cost curves in the stochastic and deterministic environments reflects the strength of the discounting and insurance effects. We use the decomposition analysis above to examine the gap between the marginal welfare cost curves in stochastic and deterministic environments shown in Figure 1. We follow the decomposition equation (12) and plot the two decomposed elements, $\Phi$ and $\Omega$. The sum of the two elements represents the marginal welfare cost for each tax rate.

The left panel of Figure 5 plots $\Phi$ for both the deterministic ($\sigma = 0$) and stochastic ($\sigma = 0.016$) cases for all tax rates. The curve $\Phi$ corresponding to the stochastic case stays above that of the deterministic case. The fact that $\Phi$ is increasing even without uncertainty shows that the distortion from capital income taxes increases with the tax rate. Since the distortion effect on consumption is present in both the deterministic and stochastic cases, and the discounting effect is present only in the presence of uncertainty, the gap between the curves across different degrees of uncertainty mainly reflects the importance of the discounting effect.\(^8\) The widening gap between the two curves reflects stronger discounting effects as the tax rate.

\(^{8}\text{Our simulation shows that the difference between the paths of the deterministic consumption distortions and the expected path of the stochastic consumption distortions is fairly small.}\)
increases.

The right panel of Figure 5 plots $\Omega_r$ for both the deterministic ($\sigma = 0$) and stochastic ($\sigma = 0.016$) cases for all tax rates. The curve $\Omega_r$ corresponding to the stochastic case is increasingly negative as the tax rate increases, whereas it is equal to zero in the deterministic case.

As the capital income tax rate increases, the decline in the interest rate used to value consumption distortions raises the marginal welfare cost of taxes, while the increasingly negative covariance between consumption distortions and the stochastic discount factor reduces it. Two opposite forces are at work. We find that the discounting effect from declines in the interest rate increasingly dominates the insurance effect as the tax rate increases. This dominance explains the widening gap between the marginal welfare cost curves in the stochastic and deterministic environments, as displayed in Figure 1.

4 Sensitivity Analysis

In this section, we conduct a sensitivity analysis to examine the impact that preference and production specifications have on the marginal welfare cost. We use the same economic environment but assume varying degrees of habit persistence,
capital adjustment costs and risk aversion. Specifically we compare across the following five cases: Case I \( (b = 0, \eta = 0, \gamma = 5) \), Case II \( (b = 0.6, \eta = 0, \gamma = 5) \), Case III \( (b = 0, \eta = 4.24, \gamma = 5) \), Case IV \( (b = 0.6, \eta = 4.24, \gamma = 3) \), and Case V \( (b = 0.6, \eta = 4.24, \gamma = 5) \). Here Case V is our benchmark model, Case I to III share the same value of risk aversion parameter \( \gamma \), but differ in terms of either the degree of habit persistence or capital adjustment costs or both. Case IV share the same habit persistence and capital adjustment costs as the benchmark model, but differs in the degree of risk aversion.

Figure 6 plots the marginal welfare cost curves for both the stochastic and deterministic environments from Case I to Case V. In all the cases, the marginal welfare cost curve in the stochastic environment stays above that in the deterministic environment particularly for medium-high and high tax rates and has a steeper slope throughout. The results show that the increasing dominance of the discounting effect is present in all specifications, although the dominance is the strongest in the benchmark case, when there are moderately high habit persistence, high capital adjustment costs and high risk aversion. A reduction in the degree of habit persistence, capital adjustment costs or risk aversion lessens the dominance of the discounting effect, but nevertheless preserves its qualitative significance. As shown in Santoro and Wei (2011), as long as investment plays a major role in smoothing
consumption and the investment-output ratio declines as tax rate increase, there will be discounting and insurance effects at work.\(^9\) The strong discounting effect in the benchmark framework is due to the presence of both habit formation and capital adjustment costs.

To further demonstrate the importance of accounting for the discounting effect in a stochastic environment, in Table 2 we report the percentage differences in the marginal and total welfare costs between a stochastic and deterministic environment when the tax rate is at 0.36. The tax rate is similar to the capital income tax rate on equity-financed corporate business in the United States. The total welfare costs can also be considered as total welfare gain from abolishing capital income taxes. In our benchmark case (Case V), the part of the total welfare cost attributed to a combination of discounting and insurance effects amount to 43.37 percent of the total welfare cost in the deterministic case. In the standard business cycle framework (Case I), the part of the total welfare cost attributed to discounting and insurance effects amounts to 3.98 percent of the total welfare cost in the deterministic case. Discounting effect also accounts for varying degrees of percentage differences

\(^9\)We also investigate the discounting and insurance effects in the case of elastic labor supply. If labor elasticity is not very high in response to technology shocks (as indicated by the empirical literature), the self-insurance through adjustment of labor supply would be limited, and investment would remain the main vehicle to smooth consumption. As a result, the discounting effect still dominates, especially when tax rates are high.
in marginal and total welfare cost in Case II to IV. The benchmark case, which features moderately high habit persistence, high capital adjustment costs and risk aversion, demonstrates the strongest impact of discounting effect. This is consistent with Santoro and Wei (2011), which finds that the impact of capital income taxes on asset prices is the strongest for this combination of preference and production specifications.\textsuperscript{10}

Since Case I is a well-known real business cycle setup, we conduct a decomposition of the marginal welfare curves into $\Phi_\tau$ and $\Omega_\tau$, in Figure 7 to further illustrate the discounting and insurance effects. The figure indicates that both the discounting and the insurance effects are present in this case, albeit at a smaller magnitude. The discounting effect is weaker relative to the insurance effect for relatively low tax rates. However, as the tax rate increases, the discounting effect dominates the insurance effect, just as in the benchmark case.

\textsuperscript{10}We also conduct a sensitivity analysis using different values of $\sigma$. We find that the discounting effect strengthens as the degree of aggregate uncertainty increases. We also perform sensitivity analyses using different risk aversion coefficients and different specifications of the adjustment costs. Detailed results are available from the authors upon request.
5 Conclusion

In this paper, we employ an asset market approach to analyze the determinants of the marginal welfare cost of capital income taxes. We show that the marginal welfare cost can be thought of as the normalized market value of consumption distortions. Such a perspective brings to the forefront the importance of the stochastic discount factor and its covariance with consumption distortions in valuing future consumption distortions.

We find that the marginal welfare cost curve is convex in and upward sloping with the tax rate in both stochastic and deterministic environments. Everything held equal, the marginal welfare cost curve in the stochastic environment lies above and is steeper than that in the deterministic case. A decomposition of the marginal welfare cost shows that such features can be explained by three effects of capital income taxes: the distortion, discounting and insurance effects. The upward slope of the marginal welfare cost curve results from the increasingly inefficient allocation of resources as the tax rate increases, which represents the distortion effect common to deterministic and stochastic environments. The widening gap between the marginal welfare cost curves between the stochastic and deterministic cases is attributed to the dominance of the discounting effect over the insurance effect. The magnitudes of these two effects are determined, respectively, by the decrease in the interest rate
and the increasingly negative covariance between the stochastic discount factor and consumption distortions as the tax rate increases. We find that the welfare cost of capital income taxes can be under-estimated if variations in risky discount rates are ignored, especially when tax rates are high.
References


Appendix

A. Computing the Marginal Welfare Cost at \( \tau \)

We assume that the economy starts from the deterministic steady state consistent with a given tax rate \( \tau \). Based on our stylized model, the initial state can be characterized by \( \{K_{ss}, C^*_{ss}, Z_{ss}\} \), which represent, respectively, the steady-state values of capital stock and consumption given \( \tau \) and the steady-state level of aggregate productivity.

Now we consider two scenarios starting from the initial state. In the first scenario, the marginal tax rate remains unchanged at \( \tau \). We denote the optimal decisions under such a scenario as

\[
K_{t+1} = g^{\tau,0}(K_t, C_{t-1}, Z_t), C_t = h^{\tau,0}(K_t, C_{t-1}, Z_t)
\]

where \( g^{\tau,0} (\bullet) \) and \( h^{\tau,0} (\bullet) \) represent respectively the optimal decision rules for next period capital stock and current consumption. The first superscript, \( \tau \), stresses the dependence of those decision rules on the given tax rate, and the second superscript, 0, indicates that the tax rate remains unchanged at \( \tau \). A dynamic path under this scenario is characterized by the initial state, \( \{K^*_{ss}, C^*_{ss}, Z_{ss}\} \), and the optimal
decision rules described above.

Now consider the alternative scenario, under which there is a marginal shift in the capital income tax rate from $\tau$ to $\tau + \varepsilon$ in the initial period. Under this scenario, the transition path to the new long-run equilibrium is characterized by the initial state $\{K_{ss}^\tau, C_{ss}^\tau, Z_{ss}\}$ and the optimal decision rules dictated by $g^{\tau,\varepsilon} (K_t, C_{t-1}, Z_t)$ and $h^{\tau,\varepsilon} (K_t, C_{t-1}, Z_t)$ for all $t$. Here the first superscript $\tau$ stresses that the initial state of the economy is consistent with the tax rate $\tau$, and the second subscript stresses that the optimal decision rules are derived under the marginally higher tax rate, $\tau + \varepsilon$.

We assume the same aggregate technology process $\{Z_t\}_{t=0}^\infty$ with or without the marginal shift in the tax rate. In computing the marginal welfare cost, we approximate $\varepsilon$ with a small positive value of $0.01$. Consequently, the consumption distortion at period $t$, $\frac{\partial (-C^\tau,\varepsilon)}{\partial \varepsilon}$, is approximated by $\frac{C^\tau,0 - C^\tau,0.01}{0.01}$ for all $\tau$.

We use the perturbation method (Dynare 4.3) to compute the second-order approximation to optimal decision rules. In order to examine the accuracy of our computed decision rules, we calculate the relative Euler equation errors of the key first-order condition, equation (6), for 850,000 periods across all tax rates. The maximum absolute value of the relative Euler equation errors range from $6.6e - 13$ to $9.8e - 7$, showing a reasonably high degree of computational accuracy. Given
the optimal decision rules, we simulate the optimal paths of endogenous variables and use Monte-Carlo methods to compute the marginal welfare cost using equation (11).
Table 1: Benchmark Parameterization

<table>
<thead>
<tr>
<th>Category</th>
<th>Parameter Definition</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>trend growth rate</td>
<td>$g$</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>depreciation rate</td>
<td>$\delta$</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>share of capital income</td>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>Preferences</td>
<td>subjective time preference</td>
<td>$\beta (1 + g)^{1-\gamma}$</td>
<td>0.992</td>
</tr>
<tr>
<td></td>
<td>curvature of utility function</td>
<td>$\gamma$</td>
<td>5</td>
</tr>
<tr>
<td>Technology Process</td>
<td>persistence</td>
<td>$\rho$</td>
<td>0.995</td>
</tr>
<tr>
<td></td>
<td>standard deviation</td>
<td>$\sigma$</td>
<td>0.016</td>
</tr>
</tbody>
</table>
Table 2: The Importance of Discounting Effect in a Stochastic Environment

<table>
<thead>
<tr>
<th>Percentage Differences</th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
<th>Case IV</th>
<th>Case V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal Welfare Cost</td>
<td>5.76</td>
<td>3.35</td>
<td>15.09</td>
<td>7.98</td>
<td>33.82</td>
</tr>
<tr>
<td>Total Welfare Cost</td>
<td>3.98</td>
<td>6.91</td>
<td>16.94</td>
<td>10.66</td>
<td>43.37</td>
</tr>
</tbody>
</table>

The entries are the percentage differences in the marginal welfare cost and the total welfare cost between the stochastic and deterministic environments at the tax rate $\tau = 0.36$ for the following different cases: Case I ($b = 0, \eta = 0, \gamma = 5$), Case II ($b = 0.6, \eta = 0, \gamma = 5$), Case III ($b = 0, \eta = 4.24, \gamma = 5$), Case IV ($b = 0.6, \eta = 4.24, \gamma = 3$) and Case V ($b = 0.6, \eta = 4.24, \gamma = 5$).
Figure 1: The Marginal Welfare Cost Curves

The figure plots the marginal welfare cost curves, \( \theta_\epsilon (\varepsilon, \tau) \), for a range of initial tax rates \( \tau \) in both a deterministic \((\sigma = 0, \text{dashed})\) and a stochastic \((\sigma = 0.016, \text{solid})\) environment. The values are computed as averages of 1000 simulations each 2000 periods long.
Figure 2: Paths of Consumption Distortions ($\tau = 0.36$)

The figure plots the percentage differences in the two consumption paths $\{C_t^{\tau,0}\}_t=0^\infty$ and $\{C_t^{\tau,\varepsilon}\}_t=0^\infty$ as a fraction of $\{C_t^{\tau,0}\}_t=0^\infty$ for $\tau = 0.36$ in both a deterministic ($\sigma = 0$) and stochastic ($\sigma = 0.016$) environment.
The upper left panel of Figure 3 plots the present discounted value of strips of deterministic consumption distortions, \( \frac{\partial (-C_t^{\tau,c})}{\partial \varepsilon} \bigg|_{\sigma=0} \), at \( \tau = 0.36 \) using the deterministic and then stochastic discount factor. The horizontal axis denotes the time period for the strip, and the vertical axis denotes the market value of the corresponding strip at period 0. The upper right panel of Figure 3 plots the differences between the two curves on the left. The lower left panel of Figure 3 plots
$$E_0 \left[ \beta_t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}} \right] E_0 \left[ \frac{\partial (-C_t^{\tau,\pi})}{\partial \pi} \right] \text{ and } E_0 \left( \beta_t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}} \frac{\partial (-C_t^{\tau,\pi})}{\partial \pi} \right) \text{ for all periods when } \tau \text{ is equal to } 0.36. \text{ The lower right panel of Figure 3 plots this difference. The values are computed as averages of 1000 simulations}$$
Figure 4: Discounting and Insurance Effects for Different Tax Rates

The figure re-produces the upper right and lower right panels of Figure 3 for three different tax rates. The values are computed as averages of 1000 simulations.
The figure plots the decomposition of the marginal welfare cost of taxes into $\Phi(\tau)$ and $\Omega(\tau)$ as a function of tax rates under the benchmark parameterization. The values are computed as averages of 1000 simulations each 2000 periods long.
The figure plots the marginal welfare cost curves, $\theta_\varepsilon(\varepsilon, \tau)$, for a range of initial tax rates $\tau$ in both a deterministic ($\sigma = 0$, dashed) and a stochastic ($\sigma = 0.016$, solid) environment when there are varying degrees of habit persistence ($b$), capital adjustment costs ($\eta$) and risk aversion parameter ($\gamma$). In all panels the horizontal axes represent tax rates, and the vertical axes represent the marginal welfare costs. The values are computed as averages of 1000 simulations each 2000 periods long.
Figure 7: The Decomposition of the Marginal Welfare Cost Curves \((b = 0, \eta = 0)\)

The figure plots the decomposition of the marginal welfare cost of taxes into \(\Phi(\tau)\) and \(\Omega(\tau)\) as a function of tax rates when there are neither habit persistence nor capital adjustment costs. The values are computed as averages of 1000 simulations each 2000 periods long.