Inflation and Stock Prices: No Illusion

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Abstract

Campbell and Vuolteenaho (2004) use VAR results to advocate inflation illusion as the explanation for the positive association between inflation and dividend yields. Using a structural approach, we find that a fully rational dynamic general equilibrium model can generate a positive correlation between dividend yields and inflation as observed in the data. The paper describes a channel by which the technology shock moves both inflation and dividend yields in the same direction, resulting in a positive correlation between the two.
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Key words: inflation illusion, dividend yield, inflation
The leading practitioner model of equity valuation, the so-called “Fed model”, implies that dividend yields (as measured by the ratio of dividends or earnings to stock prices) are highly positively correlated with inflation, a prediction borne out by the empirical evidence presented by Asness (2000, 2003).

Despite the empirical success of the Fed model, it is difficult to justify theoretically why the dividend yield, a real variable, should covary with a nominal variable such as inflation. Three hypotheses have been put forward to explain this positive correlation. The first hypothesis is that inflation (or the monetary authority’s response to inflation) damages the real economy, and in particular, lowers corporate profits. In this case, the growth rate of real dividends declines in response to inflation, driving up dividend yields. The second hypothesis associates high inflation with high real discount rates. Brandt and Wang (2003) present a model in which inflation makes investors more risk averse, driving up the required equity premium, and thus the real discount rate. Modigliani and Cohn (1979) propose a third hypothesis known as inflation illusion. According to their hypothesis, stock market investors fail to understand the effect of inflation on nominal dividend growth rates
and extrapolate historical nominal growth rates even in periods of changing inflation. From the perspective of a rational investor, this implies that stock prices are undervalued when inflation is high and overvalued when it is low.

Campbell and Vuolteenaho (2004, henceforth CV) use a decomposition approach to estimate a residual mispricing component due to inflation illusion. Their results strongly support the importance of inflation illusion in explaining the positive correlation between the dividend yield and inflation. However, as pointed out by Campbell and Ammer (1993), a shortcoming of such a VAR decomposition approach is that the results tend to overstate the importance of whichever component is treated as a residual. Wei and Joutz (2007) find evidence for structural instability in the VAR model used in their decomposition approach. They find that the post-war data do not support the inflation illusion hypothesis as the explanation for the positive correlation between inflation and dividend yields.

This paper takes an alternative approach from CV(2004). We explore an explanation for the positive association between inflation and dividend yields with no inflation illusion involved. To achieve this goal, we build a dynamic general equilibrium New-Keynesian model to study the relation between inflation and dividend yields. In contrast to a VAR framework
as in CV (2004), our structural approach achieves internal consistency of business cycle and financial variables in a general equilibrium framework. It thus enables us to study the channels through which fundamental shocks affect both inflation and dividend yields. Moreover, the VAR structure of our model solutions makes it possible to decompose the dividend yield into expected long-term dividend growth rate and discount rate components. We can then study the relative importance of each component.

This paper argues that the positive correlation between inflation and the dividend yield does not constitute evidence of inflation illusion since a correlation with a magnitude of that observed in the data is implied by this model, which does not include any irrationality. In our model, the technology shock moves both inflation and the dividend yield in the same direction, thus resulting in a positive association between the two. Moreover, the long-run expected dividend growth rate covaries with inflation, a result that is in contrast to the first hypothesis. It is the positive correlation between the long-run real discount rate and inflation which drives the positive association between dividend yields and inflation. The key element of the covariance between inflation and the real discount rate stems from fluctuations in the real risk-free rate. Our model results support the second hypothesis which advo-
cates the association between inflation and the real discount rate. However, we differ from Brandt and Wang (2003) in that there is no causal relationship from high inflation to the high real discount rate.

This paper is part of an expanding literature on the influence of the macroeconomy on the stock market. Fama (1981) advocates a proxy hypothesis to explain the negative relation between inflation and real equity returns. Marshall (1992) incorporates Fama’s hypothesis in a monetary endowment economy with transaction costs. Our model differs from this traditional literature in the following aspects. First, in our model the production sector is nontrivial. All the important variables, including consumption, dividends and inflation are endogenously determined. Second, there has been a considerable amount of work on the asset pricing implications of real business cycle models, such as Jermann (1998) and Boldrin, Christiano and Fisher (2001). However, the asset pricing implications of New-Keynesian sticky-price models, by contrast, have not been well studied. In particular, there is much room for research on the implications of sticky-price models for the relation between stocks prices and inflation. This area is particularly interesting given the possible influence of monetary policy rules on these relations.

The paper is organized as follows: Section 1 describes a standard New-
Keynesian sticky-price model. Section 2 studies the theoretical implications of the model. Section 3 conducts the quantitative analysis in which dividend yields are decomposed into the long-run dividend growth rate and discount rate components. Section 4 concludes.

1 The Model

In this section, we describe a standard New-Keynesian model.

1.1 Preferences

Representative households maximize expected lifetime utility of consumption, subject to a sequential budget constraint:

$$\max E_t \sum_{n=0}^{\infty} \beta^n \left[ \frac{(C_{t+n} - bC_{t+n-1})^{1-\sigma} - 1}{1 - \sigma} - \frac{I_{t+n} \theta}{\theta} + g \left( \frac{M_{t+n}}{P_{t+n}} \right) \right]$$  \hspace{1cm} (1)

such that

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t R_t} + a_t V_t^a = W_t L_t + a_{t-1} (V_t^a + D_t^a) + \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + T_t.$$  \hspace{1cm} (2)

The coefficient $\beta$ is the subjective discount factor and $C_t$ is real consump-
tion at time $t$. The coefficient $\sigma$ measures the curvature of the representative agent’s utility function with respect to its argument $C_t - bC_{t-1}$, where $b$ measures the habit persistence based on aggregate consumption in the previous period. The parameter $\tau$ reflects the degree of disutility from working, $L_t$ is the labor supply at time $t$, $\theta$ indexes the degree of labor supply elasticity, and $g \left( \frac{M_t}{P_t} \right)$ represents the utility obtained from holding real money balances.

In the budget constraint, $B_t$ represents the household’s holding of nominal bonds from period $t$ to $t + 1$. The vector $a_t$ represents the vector of other financial assets held at period $t + 1$ and chosen at $t$, including real bonds, shares of the representative firm and possibly other assets. The vectors $V_t^a$ and $D_t^a$ are corresponding vectors of asset prices and current period payouts in real terms; $P_t$ and $R_t$ represent the aggregate price level and the gross nominal interest rate respectively. $W_t$ represents the real wage, and $T_t$ is a lump sum money injection in period $t$.

The first-order conditions with respect to the labor supply, the nominal
bond, and the real equity are given by\(^1\):

\[
W_t - \tau \frac{L_t^{\theta-1}}{\Lambda_t} = 0, \quad (3)
\]

\[
E_t \left[ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{R_t}{P_{t+1}/P_t} \right] = 1, \quad (4)
\]

\[
E_t \left[ \beta \frac{\Lambda_{t+1}}{\Lambda_t} (V_{t+1} + D_{t+1}) \right] = V_t. \quad (5)
\]

Here \(\Lambda_t\), defined as \(\Lambda_t = (C_t - bC_{t-1})^{-\sigma}\), is the marginal utility of consumption and the Lagrange multiplier for the household’s budget constraint. \(V_t\) denotes the real value of the firm, and \(D_t\) represents dividends in real terms. Denote \(\lambda_t\) as the log-linearized deviation of the marginal utility of consumption from its steady state value:

\[
\lambda_t = -\frac{\sigma}{1 - b} c_t + \frac{\sigma b}{1 - b} c_{t-1}. \quad (6)
\]

For the remainder of the paper, all lower case letters represent log-linearized deviations of corresponding variables from their steady state values.
1.2 Production Technology and Price-Setting

We follow Bernanke, Gertler and Gilchrist (1998) in assuming a wholesale sector for production and a retail sector for pricing.

1.2.1 Wholesale Sector

Competitive firms produce the wholesale good, make decisions on how much output to produce and how much to invest.

Production Technology of Wholesale Sector  Wholesale goods are produced using the following technology:

\[ Y_{w,t} = Z_t K_t^{1-\alpha} L_t^\alpha, \]  

(7)

where the logarithm of the technology level, \( Z_t \), follows an AR1 process:

\[ z_t = \rho z_{t-1} + \varepsilon_{z,t}, \text{where } \varepsilon_{z,t} \sim N(0, \sigma^2_z), 0 < \rho < 1. \]  

(8)

The wholesale firms choose the labor input optimally to maximize their profits. The maximization condition shows that the relative price of the wholesale good is equal to the real wage over the marginal product of labor.
Define $\mu_t$ such that

$$
\exp(\mu_t) = \frac{P_{w,t}}{P_t} = \frac{W_t}{MPL_t}.
$$

(9)

Log-linearizing the above equation and substituting in equations (3) and (7) yields:

$$
\mu_t = \frac{\theta - \alpha}{\alpha} y_t - \frac{\theta}{\alpha} z_t - \frac{\theta}{\alpha} (1 - \alpha) k_t - \lambda_t.
$$

(10)

**Investment Decisions** Wholesale firms make investment decisions given the above production technology. We assume convex capital adjustment costs in the investment technology so that,

$$
K_{t+1} = (1 - \delta) K_t + \phi \left( \frac{I_t}{K_t} \right) K_t,
$$

(11)

where

$$
\phi \left( \frac{I_t}{K_t} \right) = \frac{\delta^\eta}{1 - \eta} \left( \frac{I_t}{K_t} \right)^{1-\eta} + \frac{\eta \delta}{\eta - 1}.
$$

(12)

The parameter $\eta$ measures the adjustment cost. As $\eta$ approaches 0, adjustment cost is zero. The capital adjustment costs increase as $\eta$ becomes larger.
The maximization problem facing the wholesale firms is:

\[
\max_{I_t} \sum_{n=0}^{\infty} E_t \left\{ \beta^n \frac{\Lambda_{t+n}}{\Lambda_t} \left[ \frac{P_{w,t+n}}{P_{t+n}} Y_{w,t+n} - W_{t+n} L_{t+n} - I_{t+n} \right] \right\}.
\]  \tag{13}

The first-order condition for investment is:

\[
\frac{1}{\phi' \left( \frac{I_t}{K_t} \right)} = E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left[ \frac{P_{w,t+1}}{P_{t+1}} (1 - \alpha) \frac{Y_{w,t+1}}{K_{t+1}} \right] \right. \\
+ \left. \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left[ 1 - \delta - \phi' \left( \frac{I_{t+1}}{K_{t+1}} \right) \left( \frac{I_{t+1}}{K_{t+1}} \right) + \phi \left( \frac{I_{t+1}}{K_{t+1}} \right) \right] \frac{1}{\phi' \left( \frac{I_{t+1}}{K_{t+1}} \right)} \right\}
\]  \tag{14}

where the left-hand side is the value of a marginal unit of installed capital in terms of the consumption good today, and the right hand side is the present value of the marginal benefits.

### 1.2.2 Pricing Decisions of Retail Sector

We assume that there exists a continuum of retailers of measure one. Retailers buy the wholesale good in a competitive market and differentiate it at no resource cost. Households and wholesale firms then purchase the final good, which is a CES aggregate of these retail goods, for consumption and investment respectively.
Let $Y_{j,t}$ be the quantity of output sold by retailer $j$, measured in units of wholesale goods, and let $P_{j,t}$ be the nominal price. Total final goods, $Y_t$, are the following composite of individual retail goods:

$$Y_t = \left[ \int_0^1 Y_{j,t}^\gamma d\gamma \right]^{\frac{1}{\gamma}}, \quad 0 < \gamma < 1$$

(15)

where $\gamma$ indexes the elasticity of substitution between retail goods.

Given equation (15), the demand facing each retailer is

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{\frac{1}{\gamma}} Y_t. \quad (16)$$

Competition drives the profits of the output aggregator to zero in equilibrium, determining $P_t$ as

$$P_t = \left[ \int_0^1 P_{j,t}^{\gamma} d\gamma \right]^{\frac{1}{\gamma-1}}. \quad (17)$$

Because retail goods are heterogeneous, retail firms set prices taking the demand curves and the price of the wholesale good as given. Profits from retail activity are rebated lump-sum to households. We incorporate sticky prices into the model as Calvo (1983). We assume that at each period $\varphi$ frac-
tion of randomly chosen retail firms are free to set prices, while the rest have to set their prices according to $P_{j,t} = \frac{P_{j,t-1}}{P_{j,t-2}} P_{j,t-1}$. Defining $\pi_t \equiv \ln \left( \frac{P_t}{P_{t-1}} \right)$, the pricing decision facing the price setters is:

$$\max_{P_t^*} \sum_{n=0}^{\infty} \left\{ (1 - \varphi)^n E_t \left[ \beta^n \frac{\Lambda t+n}{\Lambda t} \left( \frac{P_t^* \Pi_{m=0}^{n-1} \pi_{t+m} - P_{w,t+n}}{P_{t+n}} \right) Y_{j,t+n}^* \right] \right\}, \quad (18)$$

where $Y_{j,t+n}^*$ is the demand for good $j$ given optimally chosen $P_t^*$.

After log-linearizing equation (17) around the nonstochastic steady state and substituting in the optimal price $P_t^*$ and equation (10), we obtain the Phillips curve:

$$\pi_t = \frac{1}{1 + \beta} \pi_{t-1} + \frac{\beta}{1 + \beta} E_t \pi_{t+1} + \varphi \left[ 1 - \beta (1 - \varphi) \right] \mu_t, \quad (19)$$

where $\mu_t$, as defined in equation (10), is the log-linearized deviation of retailers’ real marginal cost from its steady state.

In our model, a positive technology shock causes the wholesale sector to expand production. Since a fraction of retail firms cannot adjust their retail prices to absorb the increase in supply, the relative price of the wholesale good $\mu$, which is also the real marginal cost of the final good, has to decline.
in response to a positive technology shock. The proportional relation between the real marginal cost and the output gap no longer holds in a model with production. The impact of technology shocks on the real marginal cost is transmitted to inflation through the price-setting behavior of retailers as described in the Phillips curve.

1.3 Monetary Policy Rule

We assume that the monetary authority responds to the deviations of inflation and output from their steady state values\(^5\). In particular, monetary policy is described by the following interest rate reaction function:

\[
r_t = \rho_r r_{t-1} + (1 - \rho_r) \left[ (1 + \rho_\pi) \pi_t + \rho_y y_t \right] + \varepsilon_{r,t}, \quad \varepsilon_{r,t} \sim N \left( 0, \sigma_r^2 \right),
\]

(20)

where \(0 < \rho_r < 1\), and \(\rho_\pi, \rho_y > 0\). Here \(\rho_r\) indexes the degree of interest rate smoothing. A positive \(\rho_\pi\) guarantees that the central bank adjusts the short-term nominal interest rate so that the targeted ex post real interest rate rises when inflation exceeds its target value, which is assumed to be the steady state rate of inflation. A positive \(\rho_y\) indicates a countercyclical monetary policy.
The government creates fiat money and makes lump-sum transfers, $T_t$, to households. Its budget constraint takes the following form:

$$M_t = M_{t-1} + T_t.$$ 

The government adjusts the mix of financing between money creation and lump-sum transfers to support the interest rate rule given by equation (20).

1.4 Equilibrium

In equilibrium, all produced goods are either consumed or invested:

$$Y_t = C_t + I_t. \quad (21)$$

Financial markets equilibrium requires that $a_t$ equals 1 for all $t$ and that the private bonds, $B_t$, are in zero net supply.

2 Inflation and Stock Prices

In this section, we use the log-linearization methods to solve the model. The VAR structure of the model solutions makes it possible to decompose the
log of the dividend yield into components related to future dividend growth rates and discount rates.

The model solution can be represented by a loglinear state space system, with the vector of state variables, $s_t$, following a first order autoregressive process with multivariate normal i.i.d impulses:

$$s_t = As_{t-1} + B\xi_t,$$  \hspace{1cm} (22)

where the square matrix $A$ governs the dynamics of the system, and $B$ summarizes the covariance structure of the impulses. In the model economy considered here, $s_t$ contains the nominal interest rate $r_t$, the inflation rate $\pi_t$, consumption $c_t$, the aggregate technology level $z_t$, and the capital stock, $k_{t+1}$, which is determined based on information available at time $t$. The vector $\xi_t$ contains normalized impulses, namely, $\begin{bmatrix} \frac{\varepsilon_{r,t}}{\sigma_r} & \frac{\varepsilon_{z,t}}{\sigma_z} \end{bmatrix}$.

This system provides us with the log of investment, $i_t$, the log of dividends, $d_t$, and the log of the marginal utility of consumption, $\lambda_t$, as linear combinations of $s_{t-1}$ and $\xi_t$. We derive expressions for the log of dividends and the dividend yield below to examine the relationship between inflation and stock prices.
We assume that the firm finances its capital stock solely through retained earnings. The dividends to shareholders are then equal to

\[ D_t = Y_t - W_tL_t - I_t. \]  

(23)

Substituting in the aggregate resource constraint (21), the level of dividends, \( D_t \), can also be written as

\[ D_t = C_t - W_tL_t. \]  

(24)

To derive an expression for the log of dividends, we first recognize that given the Cobb-Douglas structure of the production function, equation (9) implies the following:

\[ \mu_t = w_t + l_t - y_t. \]  

(25)

Substituting the above equation, the Phillips curve and the log-linearized equation for the resource constraint (21) into the log-linearized equation for
dividends (24), we can write the log-linearized dividend as\(^7\):

\[
d_t = \frac{\hat{c} \left( 1 - \hat{c} + \hat{d} \right)}{\hat{d}} c_t - \frac{(1 - \hat{c}) (\hat{c} - \hat{d})}{\hat{d}} i_t \\
- \frac{(\hat{c} - \hat{d})}{\hat{d}} \frac{(1 + \beta)(1 - \varphi)}{\varphi [1 - \beta (1 - \varphi)]} \left( \pi_t - \frac{1}{1 + \beta} \pi_{t-1} - \frac{\beta}{1 + \beta} E_t \pi_{t+1} \right)
\]

(26)

where \(\hat{c}\) and \(\hat{d}\) are respectively the steady state ratio of consumption and dividends to output. Substituting the model solutions for consumption, investment and inflation into this equation, the log of dividends can be rewritten as

\[
d_t = d_A s_{t-1} + d_B \xi_t.
\]

(27)

Accordingly, based on equation (6), the log of the marginal utility can be written as

\[
\lambda_t = \lambda_A s_{t-1} + \lambda_B \xi_t.
\]

(28)

We focus on the value of a conglomerate consisting of retail and wholesale firms. This makes sense since both sectors are owned by households. We compute the value of the firm using the lognormal pricing formulae. By assuming that \(\Lambda\) and \(D\) are conditionally lognormal with joint distribution given by (27) and (28), we are able to obtain closed form solutions for the
value of a claim to a single random future payout. The firm value is then obtained by aggregating the value of claims to each of the infinite sequence of dividends. Such a nonlinear method allows for time-varying risk premia and expected returns as shown in Jermann (1998).

The log-linear dynamic valuation framework of Campbell and Shiller (1988) shows that the log of the dividend yield can be decomposed into two components, the long-run discount rate and the long-run expected dividend growth rate.

\[
d_t - v_t = E_t \sum_{j=0}^{\infty} \beta^j r_{t+1+j}^a - E_t \sum_{j=0}^{\infty} \beta^j \Delta d_{t+1+j},
\]

(29)

where \( r_t^a \) denotes the real stock return, and \( \Delta d \) denotes the dividend growth rate. The decomposition says that changes in the dividend yield must be associated with changes in expectations of future dividend growth or real returns.

We can simulate the dividend yield after computing the firm value. The regression coefficient of the log dividend yield on simulated inflation is given by:

\[
\beta_\pi = \frac{cov(d_t - v_t, \pi_t)}{var(\pi_t)},
\]

(30)
which can be further decomposed into the regression coefficients of the two components of the dividend yield on inflation:

$$
\beta_\pi = \frac{\text{cov}\left(E_t \sum_{j=0}^{\infty} \beta^j r^{s}_{t+1+j}, \pi_t\right)}{\text{var}(\pi_t)} - \frac{\text{cov}\left(E_t \sum_{j=0}^{\infty} \beta^j \Delta d_{t+1+j}, \pi_t\right)}{\text{var}(\pi_t)}. \quad (31)
$$

The VAR structure of the model solutions makes it convenient to derive an expression for the long-run expected dividend growth, $E_t \sum_{j=0}^{\infty} \beta^j \Delta d_{t+1+j}$. It is then possible to obtain an analytical expression for the second term of equation (31). The regression coefficient of the long-run discount rate on inflation can then be inferred. Such a decomposition allows us to study the relative importance of each component for the positive correlation between inflation and the dividend yield.

### 3 Quantitative Predictions on Inflation and Stock Prices

The objective of the quantitative evaluation is to examine the model’s ability to explain the positive association between the dividend yield and inflation, while maintaining reasonable business cycle implications. The role of deep
structural parameters, and in particular, monetary policy parameters, can also be studied in such a quantitative framework. We first start with the calibration of the model.

### 3.1 Calibration

The model parameters can be categorized into the following three groups:

#### 3.1.1 Monetary Policy Rule Parameters

The monetary policy rule is characterized by the set of four parameters, \( \{\rho_r, \rho_\pi, \rho_y, \sigma_r\} \). They are respectively set to \( \{0.8, 0.23, 0.5, 0.002\} \). These parameter values are conventional in New-Keynesian models. Sensitivity analysis will be carried out to examine the significance of different monetary policy rules on the relation between inflation and stock prices.

#### 3.1.2 Preference

The preference-related parameters consist of \( \{\beta, \sigma, b\} \). The subjective time discount rate, \( \beta \), is set to \( 1.03^{-0.25} \), which implies a steady state annualized real interest rate of 3 percent. Each model period is considered as one quarter. We set \( \sigma \) to 3. The parameter \( b \), which indexes the degree of habit persistence,
is set to 0.9, similar to the value used in Jermann (1998).

3.1.3 Production and Investment

There are nine production-related parameters: \{δ, α, ρ, σ_z, η, γ, θ, ϕ, τ\}. The capital depreciation rate δ is 0.025. The constant labor share in a Cobb-Douglas production function is \frac{2}{3}. The persistence parameter of the technology process is 0.95, and σ_z is set to 0.007, as is standard in real business cycle models. The parameter η stands for the inverse of the elasticity of the investment-capital ratio with respect to Tobin’s Q. We set η to \frac{1}{0.23}, which is the value used in Jermann (1998) and Boldrin, Christiano and Fisher (1999). The parameter γ represents the degree of monopolistic competition in the economy. As γ approaches 1, the economy is close to perfect competition. We set γ to 0.3 in the benchmark case and conduct some sensitivity analysis. The parameter θ describes the elasticity of labor supply, which determines the responsiveness of real wage, and consequently real marginal cost, to outside shocks. Excessive responsiveness of real marginal cost to outside shocks may imply low inflation persistence. We set θ to 2 as standard in the New-Keynesian literature to capture inflation persistence in our model. The parameter ϕ is the fraction of firms that cannot set prices for a given period.
and is set equal to 0.5. The parameter $\tau$ is set so that the fraction of labor used for production is 0.3.

### 3.2 Model Results

The benchmark model presented have generated reasonable business cycle and asset return statistics, and most importantly, provided a rational explanation for the positive association between inflation and the dividend yield. Table 1 shows the model’s predictions on the standard deviations and first order autocorrelations of the nominal interest rate, inflation and the dividend yield. The aggregate statistics implied by the model are reasonably close to the data, except for the standard deviation of the log dividend yield. Because the stock prices in our model are entirely determined by fundamentals, it is understandable that the dividend yield is less volatile than in the data\(^9\). The model also generates historical consumption and investment volatility relative to that of output.

Table 2 reports the regression coefficients of the dividend yield and its components on inflation as implied by the theoretical model. The theoretical model predicts a regression coefficient of inflation of similar magnitude as their empirical counterpart, which is reported to be 4.01 in CV (2004). The
decomposition of regression coefficients shows a strong positive correlation between the long-term real discount rate and current inflation. We further regress the simulated expected long-term real risk-free interest rate against current inflation. The results show that the key element of covariance between inflation and real discount rate pertains mostly to fluctuations in the real risk-free rate, rather than fluctuations in the required excess return on equities.\textsuperscript{10}

In our model, inflation increases in response to a negative technology shock, as a result of the increase in marginal cost. To the extent that high inflation coincides with negative technology shocks, dividends will be low when inflation is high, and in a stationary model the long-run dividend growth rate covaries with inflation due to the low initial dividend level. However, a negative technology shock reduces the value of the firm and therefore stock prices by more than the reduction in dividends. As a result, dividend yields (as defined as the ratio of dividends to stock prices) increase. Thus we see the positive correlation between inflation and dividend yields.

The intuition can be seen more clearly using the log-linearized form of
the dividend yield:

\[ d_t - v_t = -(E_t \lambda_{t+1} - \lambda_t) - (E_t d_{t+1} - d_t) + \beta E_t (d_{t+1} - v_{t+1}). \] (32)

After substituting equations (27) and (28) into the above equation, we can solve for the dividend yield as

\[ d_t - v_t = \tilde{y}_A s_{t-1} + \tilde{y}_B \xi_t, \] where

\[ \tilde{y}_A = (\lambda_A + d_A) (I - A)(I - \beta A)^{-1}, \] (34)

\[ \tilde{y}_B = (\lambda_B + d_B) - (\lambda_A + d_A - \beta \tilde{y}_A) B. \] (35)

Considering the special case when \( \beta \) approaches 1. In this case, equations (33) to (35) imply that \( d_t - v_t \to d_t + \lambda_t \), or equivalently the log-linearized firm value \( v_t \) approaches \( -\lambda_t \). The intuition is clear. The higher \( \beta \) makes the discount rate relatively more important than dividends in determining firm value.

This approximate relationship also enables us to decompose \( \beta_\pi \) as:

\[ \beta_\pi \approx \frac{cov(d_t, \pi_t)}{var(\pi_t)} + \frac{cov(\lambda_t, \pi_t)}{var(\pi_t)}. \] (36)
As shown in our benchmark case, typically technology shocks move dividends and inflation in opposite directions, resulting in a negative covariance between the two. In order to obtain a positive relationship between the dividend yield and inflation, the covariance between the marginal utility of consumption and inflation must be a large positive value. In other words, the pricing kernel should be specified such that the covariance between \( \lambda_t \) and \( \pi_t \) is high. Our regression results show a large regression coefficient of the long-run discount rate on inflation, which further demonstrates that it is the comovement of inflation and the long-run real discount rate that explains the positive correlation between inflation and dividend yields.

The impulse responses further illustrate the dynamics of the model. Figure 1 plots the impulse responses of both business cycle and financial variables in response to one standard deviation of the two fundamental shocks. In particular, we examine the impulse responses of the nominal interest rate, inflation, consumption, dividends, marginal utility of consumption, and the dividend yield.

In our model, technology shocks are persistent, while monetary policy shocks are assumed to be white noise. As a result, the impact of monetary policy shocks on inflation and the dividend yield is fairly small and short-lived
as compared to that of technology shocks.

By sequentially assigning zero variance to one of the exogenous shocks, we are able to distinguish the importance of each type of exogenous shock in the regression coefficient $\beta_\pi$. There is little positive association between the dividend yield and inflation when the variance of technology shocks is zero. This observation indicates that in our model, technology shocks are the primary forces behind the positive correlation between the dividend yield and inflation.

As is now evident, the dividend yield and inflation are positively correlated because they both decline in response to positive technology shocks. This finding is along the same line as Fama’s proxy hypothesis.

### 3.3 Sensitivity Analysis

Table 3 reports results of the sensitivity analysis. The first column indicates alternative calibrations of a given parameter while keeping others at their benchmark values. Below we discuss sensitivity analysis of selected preference, production and policy rule parameters.
3.3.1 Preference Parameters

As shown in equation (6), given the negative correlation between inflation and both the level and growth rate of consumption, the higher is $\sigma$ or $b$, the higher is the positive correlation between the marginal utility of consumption and inflation and also the higher the positive association between inflation and dividend yields. When $\sigma$ is set to 1, there is little positive association between the dividend yield and inflation. When only $b$ is set to zero, the regression coefficient of the dividend yield on inflation is only slightly smaller than the benchmark case. The results indicate that habit formation is not required for the positive correlation results. However, allowing for habit formation delivers reasonable relative volatilities of consumption and investment as compared to output.

3.3.2 Production Parameters

When we set $\rho$ (the persistence parameter of the technology process) to zero, the correlation between inflation and the dividend yield turns negative. Temporary technology shocks have a small transitory impact on the firm value, dividends and inflation, resulting in weak positive correlation between inflation and the dividend yield. When technology shocks are temporary,
monetary policy shocks play relatively larger roles. In particular, a one-standard-deviation increase in monetary policy shocks reduces both inflation and the firm value, pushing the dividend yield and inflation in opposite directions. Persistent technology shocks are essential in obtaining a positive correlation coefficient comparable to the data.

The parameter $\theta$ indexes the degree of labor supply elasticity. In the sensitivity analysis, we examine a case of highly inelastic labor supply by setting $\theta$ to 100. In this case the real wage is highly responsive to outside shocks. Although a fraction of firms cannot change prices, those that can change their prices do so drastically in response to fluctuations in the real marginal cost. As a result, inflation is no longer sticky when labor is highly inelastic. Technology shocks now have a transitory impact on the dividend yield and inflation. The regression coefficient of the dividend yield and inflation becomes very small, but still remains positive.\textsuperscript{11}

Varying $\eta$ (the parameter which governs the capital adjustment costs) does not seriously affect the correlation between inflation and the dividend yield. A higher $\eta$ implies higher adjustment cost, and consequently lower volatility of investment relative to output.

In the steady state of our model, the labor compensation as a fraction
of output is equal to $\alpha \gamma$. Since dividends are equal to consumption minus labor compensation, the log of consumption is closest to that of dividends when $\gamma$ approaches 0. Since labor compensation often covaries with inflation, the higher is $\gamma$, the stronger is the negative correlation between dividends and inflation, as compared with that between consumption and inflation. As a result, higher values of $\gamma$ lead to lower positive association between the dividend yield and inflation.

### 3.3.3 Policy Rule Parameters

With regard to monetary policy rules, a high $\rho_\pi$ represents strong inflation-stabilizing stance of the monetary authority. When $\rho_\pi$ is set to 0.6, a higher value than in the benchmark case, inflation declines only slightly in response to positive technology shocks. A stable inflation prevents the relative prices of some firms (in particular, those with sticky nominal prices) from rising too high. As a result, consumption, dividends, and especially firm value increase more in response to positive technology shocks as compared to the benchmark case. The regression coefficient of dividend yield on inflation is 8.16, much higher than in the benchmark case.$^{12}$

We further examine the case in which the central bank does not respond
to the inflation gap by setting $\rho_\pi$ close to 0. In this case, the regression coefficient of the dividend yield on inflation is slightly above zero. In our model, when the changes in inflation are not moderated by the monetary authority, drastically declining inflation can push up the relative prices of goods produced by firms with sticky prices, and possibly reduce the aggregate consumption in the short run. As consumption declines, the real discount rate increases temporarily, in an opposite direction to the movement of inflation. As a result, the correlation between inflation and the dividend yield is barely positive.

A high $\rho_y$ represents a strongly countercyclical monetary policy. When $\rho_y$ is set to 0.8, the monetary authorities raises the interest rate aggressively in response to positive deviations of output from its steady state. As a result, inflation declines more in response to positive technology shocks compared to the benchmark case. Dividends increase more due to the resulting reduction in labor cost. Despite an increase in firm value, the strong increase in dividends leads to smaller decrease in the dividend yield, and consequently, smaller positive correlation between the dividend yield and inflation.
3.3.4 The nominal rigidity parameter

The parameter $\varphi$ governs the degree of price stickiness. As $\varphi$ increases from zero to 1, prices change from extremely sticky to completely flexible. In the sensitivity analysis, we set the value of $\varphi$ to 0.05 and to 0.9. Table 4 shows that $\beta_\pi$, the regression coefficients of the log dividend yield on inflation in both cases are not only positive, but also close to the benchmark case in terms of the magnitude. The last column shows the values of both $\text{cov}(d_t - v_t, \pi_t)$ and $\text{var}(\pi_t)$, the numerator and denominator of equation (30), which gives the value of $\beta_\pi$.

Figure 2 compares the impulse responses of the dividend yield and inflation to technology shocks and sheds light on the sensitivity results. As shown in Figure 2, in the case of extreme price stickiness ($\varphi = 0.05$), the impulse responses of both inflation and the dividend yield are substantially smoothed over time. The magnitude of responses remain quite small due to this smoothing. The variance of inflation is less than half the magnitude as compared to the benchmark case. However, the dividend yield responds more strongly to technology shocks and moves in the same direction as inflation, thus resulting in a positive correlation between inflation and the dividend yield of a magnitude close to that observed in the data. In the case of more
flexible prices ($\varphi = 0.9$), Figure 2 shows larger but short-lived responses of inflation, as compared to the case of extreme price stickiness. Although the variance of inflation is larger compared to the benchmark case, the covariance of the dividend yield and inflation is larger as well, resulting in a relatively large $\beta_\pi$ as shown in Table 4. Thus the model results are robust to different degrees of nominal rigidities.

### 3.3.5 The Three Key Parameters

The sensitivity analysis above indicates three crucial elements for the positive association between inflation and the dividend yield. First, $\sigma$, which represents the curvature of the utility function, should be sufficiently high for the real discount rate, and the resulting dividend yield, to be positively correlated with inflation. Second, the persistence parameter of technology shocks, $\rho$, should be high enough for the impact of technology shocks to dominate that of the monetary policy shocks. Third, the counter-inflationary measures of the monetary authorities, indexed by $\rho_\pi$, should enable the real interest rate to increase sufficiently in response to rising inflation. The absence of any of these three ingredients results in little or slightly negative correlation between inflation and dividend yield.
4 Conclusion

In this paper, we show that the positive association between dividend yields and inflation as observed in the data can be rationalized in a dynamic general equilibrium model with no inflation illusion involved. We find that a third factor, technology shocks, moves both inflation and the dividend yield in the same direction, resulting in a positive association between the two.
Notes

1 The money demand equation serves only to determine how much money the central bank needs to supply to clear markets given its interest rate target. This equation can be dropped when a monetary policy rule is present.

2 Although the output of wholesale goods, $Y_{w,t}$, is different from the aggregate output $Y_t$, they are the same in the log-linearized first order approximation, as demonstrated in Christiano, Eichenbaum and Evans (2001). We denote their log-linearized deviations from the steady state output as $y_t$.

3 The specification of convex adjustment cost is similar to Jermann (1998) and Boldrin, Christiano and Fisher (2001).

4 The derivation of the Phillips curve is identical to Christiano, Eichenbaum, and Evans (2001), except that in their model pricing decisions are made one period ahead of the realization of shocks.

5 We also work with a different measure of the output gap as in Wu (2006). In reality the Federal Reserve is unable to observe or accurately estimate a flexible-price output in real time, but rather may rely on some linear rule to update its estimates of potential output. Wu (2006) assumes a simple learning rule for the monetary authority to estimate the potential output and the output gap, defined as the percentage deviations of actual output from the potential output. Our results are robust under this alternative
monetary policy rule.

6We require the system to be stationary. Thus the characteristic roots of Λ have modulus less than or equal to 1.

7The details of the derivations are contained in Appendix A.

8We assume elastic labor supply to capture inflation persistence in our model. As a result, the implied equity premium is small. However, even in a model with a substantial equity premium, the channel described in this paper still applies. It is not our goal to address the equity premium puzzle in this fairly standard New-Keyneisan model. Wei (2009) discusses the features required to generate an equity premium in a model with nominal rigidities.

9We can also increase the volatility of dividend yields by introducing leverage into the model.

10We also regress the simulated expected one-period risk-free interest rate against current inflation, the regression coefficient remains positive, indicating a positive correlation between inflation and the risk-free component of the one-period required rate of return.

11We also examine the robustness of model results when frictions on labor demand are incorporated. The details are contained in Appendix B.

12An alternative interpretation is that a stronger policy response to infla-
tion means that the nominal interest rate increases much more than inflation, resulting in a stronger positive correlation between real interest rate and inflation, the key element behind the positive correlation between inflation and dividend yields.
References


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A Derivation of the Log-linearized Dividend

The log-linearized approximation to equation (24) can be written as

\[ \hat{dd}_t = \hat{cc}_t - \left( \hat{c} - \hat{d} \right) (w_t + l_t), \]  

(37)

where $\hat{c}$ and $\hat{d}$ are, respectively, the steady state ratio of consumption and dividend to output.

Given the Cobb-Douglas structure of the production function, equation (9) implies that

\[ \mu_t = w_t + l_t - y_t, \]  

(38)

which yields

\[ \hat{dd}_t = \hat{cc}_t - \left( \hat{c} - \hat{d} \right) (\mu_t + y_t). \]  

(39)

Equation (26) can be obtained after we express $\mu_t$ as a function of current, past and expected inflation according to the Phillips curve (19), and substitute in the log-linearized output as given by

\[ y_t = \hat{cc}_t + (1 - \hat{c}) i_t. \]  

(40)
B Modelling Frictions on Labor Demand Side

In this appendix, we examine whether the model results are robust when frictions on labor demand side are incorporated. We follow Hall (2004) in modelling frictions on labor demand side as quadratic adjustment costs for labor. Specifically, the wholesale firm in the model faces the following maximization problem:

\[
\max_{L_t} \sum_{n=0}^{\infty} E_t \left\{ \beta^n \frac{\Delta_{t+n}}{\Lambda_t} \left[ \frac{P_{w,t+n}}{P_{t+n}} Y_{w,t+n} - W_{t+n} L_{t+n} - I_{t+n} - \frac{\theta}{2} \left( \frac{L_{t+n} - L_{t+n-1}}{L_{t+n-1}} \right)^2 L_{t+n-1} \right] \right\}.
\] (41)

After incorporating adjustment costs for labor, \( \mu_t \), the log-linearized deviation of the retailers’ real marginal cost (originally defined in Equation 10) is now given by

\[
\mu_t = \frac{\theta - \alpha}{\alpha} y_t - \frac{\theta}{\alpha} z_t - \frac{\theta (1 - \alpha)}{\alpha} k_t - \lambda_t + \frac{\vartheta}{W} (l_t - l_{t-1}) - \beta \frac{\vartheta}{W} E_t (l_{t+1} - l_t),
\] (42)

where \( W \) represents the steady-state real wage rate. The first four terms on the right hand side of the above equation are the same as in the model without labor adjustment costs, while the last two terms reflect the impact of
the labor adjustment costs on the real marginal cost of retailers. As discussed in the text, high frictions on labor supply side, as captured through inelastic labor supply, imply that the real marginal cost responds strongly to the movement in $l_t$, the labor employment itself. The above equation also shows that high costs of adjusting labor imply that the real marginal cost responds strongly to the deviation of labor employment from its value in the previous period. Both frictions on labor supply and demand side work through the marginal cost.

Hall (2004) presents a range of estimates of $\vartheta$ for industries, we choose the highest estimated value of 7.695 to examine the robustness of our model results. We find that the regression coefficient of dividend yield on inflation is 2.80 in this case, slightly smaller than the benchmark case in the text. The results show that the positive correlation between inflation and dividend yield remains robust under the assumption of adjustment costs for labor.
Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>$std(r)$</th>
<th>$std(\pi)$</th>
<th>$std(dy)$</th>
<th>$\frac{\sigma_{\Delta C}}{\sigma_{\Delta Y}}$</th>
<th>$\frac{\sigma_{\Delta I}}{\sigma_{\Delta Y}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>3.66</td>
<td>3.27</td>
<td>0.17</td>
<td>0.78</td>
<td>2.30</td>
</tr>
<tr>
<td>Data</td>
<td>3.47</td>
<td>2.95</td>
<td>0.43</td>
<td>0.51</td>
<td>2.65</td>
</tr>
<tr>
<td>$corr_1(r)$</td>
<td>0.97</td>
<td>0.89</td>
<td>0.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$corr_1(\pi)$</td>
<td>0.97</td>
<td>0.65</td>
<td>0.86</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The symbols have the following meaning: $dy$, the demeaned log dividend yield; $corr_1()$, the first order auto-correlation of the corresponding variable; $\frac{\sigma_{\Delta C}}{\sigma_{\Delta Y}}$, the ratio of the standard deviation of quarterly consumption (investment) growth rate over the standard deviation of quarterly output growth rate. Data on nominal interest rates are 3-month treasury bill rates from the Federal Reserve. Data on inflation are computed from the consumer price index for all urban consumers covering all items. The dividend yield is extracted from value-weighted equity returns from CRSP. Business cycle data is from Jermann (1998). Business cycle data are quarterly. Asset return and inflation data are annualized and in percentage terms except for the log dividend yield.
Table 2: Regression Results

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Coefficient on $\pi_t$</th>
<th>$R^2$(percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dy_t$</td>
<td>3.44</td>
<td>69</td>
</tr>
<tr>
<td>$-E_t \sum_{j=0}^{\infty} \beta^j \Delta d_{t+1+j}$</td>
<td>-1.03</td>
<td>65</td>
</tr>
<tr>
<td>$E_t \sum_{j=0}^{\infty} \beta^j r_{t+1+j}^s$</td>
<td>4.47</td>
<td></td>
</tr>
<tr>
<td>$r_{f,t}^{Long}$</td>
<td>4.33</td>
<td>79</td>
</tr>
<tr>
<td>$r_{f,t}^{Short}$</td>
<td>0.06</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes: The table shows the regression coefficients of the dependent variables in the first column on inflation ($\pi$) and the corresponding regression $R^2$ values. The variable $dy_t$ represents the demeaned log dividend yield, $E_t \sum_{j=0}^{\infty} \beta^j \Delta d_{t+1+j}$ is the demeaned long-term dividend growth, and $E_t \sum_{j=0}^{\infty} \beta^j r_{t+1+j}^s$ is the demeaned long-term real discount rate. The regression coefficient of $dy$ on $\pi$ is the average of 100 simulations each 1000 periods long. The regression coefficient of $E_t \sum_{j=0}^{\infty} \beta^j \Delta d_{t+1+j}$ is computed analytically as population moment. The variables, $r_{f,t}^{Long}$ and $r_{f,t}^{Short}$, represent respectively the expected long-term real risk-free interest rate and the expected one-period real risk-free interest rate.
Table 3: Sensitivity Analysis

<table>
<thead>
<tr>
<th>Alternative Calibrations</th>
<th>Regressions of Dividend Yield’s Component on Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$dy_t$</td>
</tr>
<tr>
<td>Category</td>
<td></td>
</tr>
<tr>
<td>Preference</td>
<td></td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td>0.79</td>
</tr>
<tr>
<td>$b = 0$</td>
<td>2.42</td>
</tr>
<tr>
<td>Production</td>
<td></td>
</tr>
<tr>
<td>$\rho = 0$</td>
<td>-0.43</td>
</tr>
<tr>
<td>$\theta = 100$</td>
<td>0.76</td>
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<tr>
<td>$\eta = 10$</td>
<td>3.74</td>
</tr>
<tr>
<td>$\gamma = 0.9$</td>
<td>2.01</td>
</tr>
<tr>
<td>Policy</td>
<td></td>
</tr>
<tr>
<td>$\rho_\pi = 0.6$</td>
<td>8.16</td>
</tr>
<tr>
<td>$\rho_\pi = 0.0001$</td>
<td>0.07</td>
</tr>
<tr>
<td>$\rho_y = 0.8$</td>
<td>2.54</td>
</tr>
</tbody>
</table>

Notes: The table shows the regression coefficients of the dependent variables in the second row on inflation $\pi$. The variable $dy_t$ represents the demeaned log dividend yield, $Et \sum_{j=0}^{\infty} \beta^j \Delta d_{t+1+j}$ is the demeaned long-term dividend growth rate, and $Et \sum_{j=0}^{\infty} \beta^j r^{*}_{t+1+j}$ is the demeaned long-term real discount rate. The regression coefficients of $dy$ on $\pi$ ($\beta_\pi$) computed analytically using log-linear approximation are very close to the average of regression coefficients computed from 100 simulations each 1000 periods long. In the table we report the former.
Table 4: Sensitivity to degrees of Nominal Rigidities

<table>
<thead>
<tr>
<th>Alternative Calibrations</th>
<th>Regressions of Dividend Yield’s Component on Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category</td>
<td>$dy_t - E_t \sum_{j=0}^{\infty} \beta^j \Delta d_{t+1+j}$</td>
</tr>
<tr>
<td>Nominal Rigidity $\phi = 0.05$</td>
<td>2.93</td>
</tr>
<tr>
<td></td>
<td>3.44</td>
</tr>
<tr>
<td></td>
<td>2.80</td>
</tr>
</tbody>
</table>

Notes: The variable $dy_t$ represents the demeaned log dividend yield, $E_t \sum_{j=0}^{\infty} \beta^j \Delta d_{t+1+j}$ is the demeaned long-term dividend growth rate, and $E_t \sum_{j=0}^{\infty} \beta^j r^s_{t+1+j}$ is the demeaned long-term real discount rate. The regression coefficients of $dy_\pi (\beta_\pi)$ computed analytically using log-linear approximation are very close to the average of regression coefficients computed from 100 simulations each 1000 periods long. In the table we report the former.
Figure 1: Impulse Responses of Key Variables
Figure 1: Impulse Responses of Key Variables

The left and right columns display the impulse responses of six macroeconomic variables to a 1% positive monetary policy shock and a 1% positive technology shock respectively.
Figure 2: Sensitivity Analysis of Nominal Rigidity
Figure 2: Sensitivity Analysis of Nominal Rigidity

The left, middle and right columns display the impulse responses of inflation and the dividend yield to a 1\% positive technology shock in the case of high degree of nominal rigidity ($\varphi = 0.05$), the benchmark case ($\varphi = 0.5$), and the case of low degree of nominal rigidity ($\varphi = 0.9$) respectively.