

# Econ 306 (Spring 2009)

## Problem Set 4

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**Problem 1** *Home Production (Hansen and Wright 1992, Benhabib, Rogerson, and Wright 1991)*

Consider an economy with infinitely lived individuals. Individuals maximize

$$E_0 \sum_{t=0}^{\infty} \{ \beta^t [\log(C_t) + A \log(L_t)] \},$$

where

$$\begin{aligned} C_t &= [\alpha C_{Mt}^e + (1 - \alpha) C_{Ht}^e]^{\frac{1}{e}}, \\ L_t &= 1 - H_{Mt} - H_{Ht}. \end{aligned}$$

Here  $C_{Mt}$  is consumption of a market-produced good,  $C_{Ht}$  is consumption of a home-produced good,  $H_{Mt}$  is hours worked in the market sector, and  $H_{Ht}$  is hours worked in the home, all in period  $t$ .

The model has two technologies, one for market production and one for home production:

$$\begin{aligned} f(z_{mt}, K_{Mt}, H_{Mt}) &= \exp(z_{mt}) K_{Mt}^{\theta} H_{Mt}^{1-\theta}, \\ g(z_{Ht}, K_{Ht}, H_{Ht}) &= \exp(z_{Ht}) K_{Ht}^{\eta} H_{Ht}^{1-\eta}, \end{aligned}$$

where  $\theta$  and  $\eta$  are the capital share parameters. The two technology shocks follow the processes

$$\begin{aligned} z_{mt+1} &= \rho z_{mt} + \varepsilon_{Mt}, \\ z_{Ht+1} &= \rho z_{Ht} + \varepsilon_{Ht}, \end{aligned}$$

where the two innovations are normally distributed with standard deviations  $\sigma_M$  and  $\sigma_H$ , have a contemporaneous correlation  $\gamma = \text{cov}(\varepsilon_{Mt}, \varepsilon_{Ht})$ , and are independent over time. In each period, a capital constraint holds:

$$K_{Mt} + K_{Ht} = K_t,$$

where total capital evolves according to

$$K_{t+1} = (1 - \delta) K_t + I_t.$$

Finally, the constraints

$$\begin{aligned} C_{Mt} + I_t &= f(z_{mt}, K_{Mt}, H_{Mt}), \\ C_{Ht} &= g(z_{Ht}, K_{Ht}, H_{Ht}), \end{aligned}$$

imply that all new capital is produced in the market sector.

(a) What are the state variables in this model? What are the control variables?

(b) Derive the first order conditions of this model and interpret their economic implications.

(c) Intuitively, how does the labor supply elasticity of market activity depend upon the parameters  $e$  and  $\gamma$ ?

**Problem 2** *Variable Capacity Utilization (King and Rebelo 1999, Burnside and Eichenbaum 1996 AER)*

The economy is populated by a large number of infinitely lived agents whose expected utility is defined as

$$E_0 \sum_{t=0}^{\infty} \{ \beta^t [\log(C_t) + A \log(L_t)] \},$$

where  $C_t$  represents consumption and  $L_t$  represents leisure. Normalizing the total amount of time in each period to one, the time constraint is:

$$N_t + L_t = 1.$$

The production function is

$$Y_t = A_t (z_t K_t)^{1-\alpha} N_t^\alpha,$$

where  $A_t$  is a random productivity shock variable, which follows the following process:

$$\log(A_t) = \rho \log(A_{t-1}) + \epsilon_t,$$

and  $z_t$  denotes the utilization rate. For convenience, we assume that this economy does not grow over time. The costs of variable capital utilization are embedded in the following law of motion for the capital stock:

$$K_{t+1} = I_t + [1 - \delta(z_t)] K_t,$$

where  $\delta(\cdot)$  is a convex, increasing function of the utilization rate.

- (a) What are the state variables in this model? What are the control variables?
- (b) Derive the first order conditions and interpret their economic implications.
- (c) Intuitively why does the incorporation of variable capacity utilization enhance the amplification capability of the benchmark real business cycle model?

**Problem 3** Assume that there are only two trees in the economy, type A (for apple) and O (for orange). Fruits yielded each period,  $a_t$  (apples) and  $o_t$  (oranges) follow respectively a Markov process. These two fruits are nonstorable and are perfect substitutes. The household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to

$$z_{a,t}(p_{a,t} + a_t) + z_{o,t}(p_{o,t} + o_t) \geq c_t + z_{a,t+1}p_{a,t} + z_{o,t+1}p_{o,t},$$

where  $z_{a,t}$  and  $z_{o,t}$  represent respectively the quantity of the apple tree and orange tree the representative household holds between  $t - 1$  and  $t$ .

- (a) Write down the Bellman equation and derive the first order conditions and the envelope condition, and in the end derive a pair of Euler equations.
- (b) Determine the market clearing conditions and substitute into the Euler equations.
- (c) What is the expression for the stochastic discount factor in equilibrium? Suppose that the household is risk averse, is the stochastic discount factor more volatile when the apple crop and orange crop are positively correlated or negatively correlated? (For the latter part of (c), intuitive reasoning is sufficient.)