Econ 306 (Spring 2009)
Problem Set 3

Instructor: Chao Wei

Problem 1 *Asset pricing in the consumption CAPM. (This follows from Hansen and Singleton JPE 1983)*

Consider the first order condition satisfied by optimal plans of consumers:

$$1 = \beta E_t \left[\frac{u'(C_{t+1})}{u'(C_t)} (1 + R_{it}) \right].$$

Assume that utility is CRRA, with a coefficient of relative risk aversion of γ. *Assume also that the rate of growth of consumption, defined as* $\log(C_{t+1}) - \log(C_t)$, *and the net rate of return on asset* i, *defined as* $\log(1 + R_{it})$, *are conditionally joint normally distributed, with conditional mean* g *and* x *and covariance matrix* V. *Derive the equilibrium rate of return* x *as a function of* β, γ, g, *and the elements of* V. *Interpret the economic intuition of the relationship bwtween* x *and* β, γ, g *and* V *respectively.*

Problem 2 *The term structure of interest rates. (This is a simplified version of Campbell QJE 1986)*

Consider the Lucas asset-pricing model, with only one asset. Assume that the dividend on the asset follows

$$\log(d_t) = g + \log(d_{t-1}) + e_t,$$

where e_t *is normally distributed white noise, with mean zero and variance* σ^2. *Assume that utility is CRRA, with a coefficient of relative risk aversion of* γ.

(a) Consider a n*-period pure discount bond at time* t, *a bond that pays one unit of the good at time* $t+n$ *and has price* $p_{n,t}$. *Derive the equilibrium values of* $p_{n,t}$.

(b) Define the yield to maturity on a n*-period pure discount bond as* $r_{n,t}$, *such that*

$$r_{n,t} = -\frac{\log(p_{n,t})}{n}.$$

Characterize the term structure of yields to maturity at time t. *Is it upward* *sloping or downward sloping? Explain.*

(c) How would we have to change the model to explain why the term structure is sometimes upward sloping and sometimes downward sloping?

Problem 3 *Suppose instantaneous utility is of the constant relative risk aversion form,* $u(c_t) = \frac{c_t^{1-\theta}}{1-\theta}, \theta > 0$. *Assume that the real interest rate,* r*, is constant but not necessarily equal to the subjective discount rate,* ρ.

(a) Find the Euler equation relating C_t *to expectations concerning* C_{t+1}.

(b) Suppose that the log of income is distributed normally, and that as a result the log of C_{t+1} *is distributed normally; let* σ^2 *denote its variance conditional on information available at time* t*. Rewrite the expression in part (a) in terms of* $\ln C_t, E_t \ln C_{t+1}, \sigma^2$*, and the parameters* r, ρ*, and* θ.

(c) Show that if r *and* σ^2 *are constant over time, the result in part (b) implies that the log of consumption follows a random walk with drift:*

$$\ln C_{t+1} = \alpha + \ln C_t + u_{t+1},$$

where u *is white noise.*

(d) How do changes in each of r *and* σ^2 *affect expected consumption growth,* $E_t [\ln C_{t+1} - \ln C_t]$*? Interpret the effect of* σ^2 *on expected consumption growth in light of the presence of precautionary saving.*