Problem 1 Asset pricing in the consumption CAPM. (This follows from Hansen and Singleton JPE 1983)

Consider the first order condition satisfied by optimal plans of consumers:

\[ 1 = \beta E_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} (1 + R_{it}) \right]. \]

Assume that utility is CRRA, with a coefficient of relative risk aversion of \( \gamma \). Assume also that the rate of growth of consumption, defined as \( \log(C_{t+1}) - \log(C_t) \), and the net rate of return on asset \( i \), defined as \( \log(1 + R_{it}) \), are conditionally joint normally distributed, with conditional mean \( \mu \) and \( \Sigma \) and covariance matrix \( V \). Derive the equilibrium rate of return \( x \) as a function of \( \beta, \gamma, \mu, \) and the elements of \( V \). Interpret.

Problem 2 The term structure of interest rates. (This is a simplified version of Campbell QJE 1986)

Consider the Lucas asset-pricing model, with only one asset. Assume that the dividend on the asset follows

\[ \log(d_t) = g + \log(d_{t-1}) + e_t, \]

where \( e_t \) is normally distributed white noise, with mean zero and variance \( \sigma^2 \). Assume that utility is CRRA, with a coefficient of relative risk aversion of \( \gamma \).

(a) Consider a n-period pure discount bond at time \( t \), a bond that pays one unit of the good at time \( t+n \) and has price \( p_{n,t} \). Derive the equilibrium values of \( p_{n,t} \).

(b) Define the yield to maturity on a n-period pure discount bond as \( r_{n,t} \), such that

\[ r_{n,t} = -\frac{\log(p_{n,t})}{n}. \]

Characterize the term structure of yields to maturity at time \( t \). Is it upward sloping or downward sloping? Explain.
(c) How would we have to change the model to explain why the term structure is sometimes upward sloping and sometimes downward sloping?

Problem 3 An analytical approach to the stochastic growth model. (This follows from Campbell JME 1994)

This problem is intended to encourage you to read through Campbell’s paper. Although most answers can be found in his paper, you should answer the questions using your own words and derivations. I use the same equation numbers as in Campbell (1994) in this problem. Lower case letters represent the logarithm of the corresponding variables.

(1) Conjecture $c_t$ as a function of $k_t$ and $a_t$, and use the method of undetermined coefficients to obtain the solutions for the model from equations (13), (17) and (18).

The following questions deal with variations of the benchmark model we have covered in class.

(2) An additively separable model. The model leaves the production function unchanged, but allows labor input to be variable rather than constant and normalized to one. The capital accumulation equation is also unchanged. However the objective function now has a period utility function involving both consumption and leisure, as in equation (34) of the paper.

a. Derive the first order conditions which solve the model.
b. Derive the steady state values of $\frac{A_t}{K_t}$, $\frac{Y_t}{K_t}$, $\frac{C_t}{Y_t}$, and $N_t$.
c. Derive equations (37), (38), and (41).
d. Use the method of undetermined coefficients to obtain explicit solutions for the model from equations (37), (38), (39) and (41).

(3) A nonseparable model. The model is the same as that in (2) except for the nonadditively separable Cobb-Douglas utility function as stated in (46).

a. Derive the first order conditions of the model.
b. Derive the steady state values of $\frac{A_t}{K_t}$, $\frac{Y_t}{K_t}$, $\frac{C_t}{Y_t}$, and $N_t$.
c. Derive equations (48) and (49).
d. Use the method of undetermined coefficients to obtain explicit solutions for the model.