Econ 306 (Spring 2009)
Problem Set 2

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Problem 1 *Consider an economy consisting of a constant population of infinitely lived individuals. The representative individual maximizes the expected value of* $\sum_{t=0}^{\infty} u(c_t) / (1+\rho)^t, \rho > 0$. *The instantaneous utility function,* $u(c_t)$, *is* $u(c_t) = c_t - \theta c_t^2, \theta > 0$. *Assume that* c *is always in the range where* $u'(c)$ *is positive.*

Output is linear in capital, plus an additive disturbance: $Y_t = AK_t + e_t$. *There is no depreciation; thus* $K_{t+1} = K_t + Y_t - c_t$, *and the interest rate is* A. *Assume* $A = \rho$. *Finally, the disturbance follows a first-order autoregressive process:* $e_t = \phi e_{t-1} + \varepsilon_t$, *where* $-1 < \phi < 1$ *and where the* $\varepsilon_t' s$ *are mean zero i.i.d. shocks.*

(a) Find the Euler equation relating c_t *and expectations of* c_{t+1}.

(b) Guess that consumption takes the form $c_t = \alpha + \beta K_t + \gamma e_t$. *Given this guess, what is* K_{t+1} *as a function of* K_t *and* e_t?

(c) What values must the parameters α, β, *and* γ *have for the first order condition in part (a) to be satisfied for all values of* K_t *and* e_t?

(d) What are the effects of a one-time shock to ε *on the paths of* Y, K, *and* C?

Problem 2 *(Optional midterm exam question in Spring 06) Consider an unemployed worker who samples wage offers on the following terms: When the economy is in expansion (denoted as* $S_t = 1$*), with probability* $\phi_h, 1 > \phi_h > 0$, *she receives an offer to work for* w *forever. With probability* $1 - \phi_h$, *she receives no offer (we may regard this as a wage offer of zero forever). When the economy is in recession (denoted as* $S_t = 0$*), with probability* $\phi_l, 1 > \phi_l > 0$, *she receives an offer to work for* w *forever. With probability* $1 - \phi_l$, *she receives no offer. The wage offer* w *is drawn from a cumulative probability distribution function* $F(w)$, *.with* $0 < w \leq B$. *Successive draws across periods are independently and identically distributed.*
Assume that the economy alternates between expansions and recessions following a Markov chain process, specifically,

$$\begin{aligned} prob\,(S_{t+1} = 1 \mid S_t = 1) &= p, \\ prob\,(S_{t+1} = 0 \mid S_t = 0) &= q. \end{aligned}$$

Since S *can only take two values, you should be able to figure out* $prob\,(S_{t+1} = 0 \mid S_t = 1)$ *and* $prob\,(S_{t+1} = 1 \mid S_t = 0)$.

The worker chooses a strategy to maximize $E\sum_{t=0}^{\infty}\beta^t y_t$, where $y_t = w$ or $y_t = c$, depending on whether she is employed or unemployed.
(a). Formulate the Bellman equation for the worker's problem. (You must expand the expectation term until the expectation is only with respect to the job offer in the next period.)
(b). Show that the optimal policy is to set a reservation wage that depends upon whether the economy is in expansion or recession.
(c). Derive the two equations which can be used to compute the above two reservation wages. Note that the unknowns in these two equations must be the two reservation wages only.

Problem 3 *A Time-to-build Model. Consider a social planner who faces the following problem:*

$$\max_{\{c_t,k_{t+2},l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

subject to

$$\begin{aligned} c_t + s_t &\leq A_t F(k_t, l_t), & (2)\\ k_{t+1} &= (1-\delta) k_t + I_t, & (3)\\ I_t &= s_{t-1}, & (4)\\ k_0 &> 0, 0 < l_t \leq 1, \forall k_t > 0. \end{aligned}$$

The production technology has the so-called time-to-build feature where investment made in period $t-1$, s_{t-1},becomes productive capital stock in period $t+1$.

(a) What are the state variables in this model? What are the control variables?

(b) Formulate the Bellman equation for the social planner's problem (Hint 1: Your life will be much easier later if you write the Bellman equation in such a way that k_{t+2} instead of investment is the control variable to optimize. Just like in a standard model, we choose the optimal decision rule for k_{t+1} instead of I_t. Hint 2: There are three state variables.)

(c) Derive the first order conditions, envelop theorem equations, and the Euler equation for the model.

(d) Use the Lagrange method to solve the model and verify with what you get in (c).

(3). Interpret the economic intuition of the Euler equation you get in (c) and (d).