

Econ 306 Problem Set 1

Spring 2009

Due February 5, 6:10 pm

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Problem 1 Consider a social planner who faces the following problem:

$$\max_{\{c_t, k_{t+1}, l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t), \quad (1)$$

subject to

$$\begin{aligned} 0 &\leq k_{t+1} \leq (1 - \delta)k_t + F(k_t, l_t) - c_t, \\ k_0 &> 0 \end{aligned} \quad (2)$$

Here l_t represents the labor supply and other notations are standard. Assume that $\delta = 1$ and

$$\begin{aligned} u(c_t, 1 - l_t) &= \ln c_t + b \ln(1 - l_t), \\ y_t &= F(k_t, l_t) = Ak_t^\alpha L_t^{1-\alpha}, 0 < \alpha < 1. \end{aligned}$$

1. Derive the Bellman equation of this social planner's problem.

2. First start with iterations on the Bellman equation. Start with $v_0(k) = 0$ (assuming the end-of-the-world scenario), and solve the one period problem to obtain $v_1(k)$, the value function. Then plug your $v_1(k)$ into the Bellman equation, solve for optimal k' and obtain $v_2(k)$. (Note that here $v_n(k)$ does NOT represent the n th derivative of the value function, but the n th iteration of the value function.)

3. Then solve this problem using the Euler equations. First find the first order conditions for k_{t+1} and l_t respectively and then derive the Envelop condition. Explain the economic intuition of the first order conditions.

4. Now solve the problem using the Guess and Verify method. Guess that $V(\cdot)$ takes the form $V(k) = G + H \ln k$, and solve for G and H .

5. What are the implied values of $\left\{ \frac{c_t}{y_t}, l_t \right\}_{t=0}^{\infty}$? Show that these two data series do not depend upon k_t .

Problem 2 Consider a social planner who faces the following problem:

$$\max_{\{c_t, k_{t+1}, l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t), \quad (3)$$

subject to

$$\begin{aligned} 0 &\leq k_{t+1} \leq (1-\delta)k_t + F(k_t, l_t) - c_t, \\ k_0 &> 0, 0 < l_t \leq 1 \end{aligned} \tag{4}$$

where $u(c) = c - \theta c^2$, $\theta > 0$. Assume that c is always in the range where $u'(c)$ is positive. Output is linear in capital, $y = Ak$, and $\delta = 0$.

(1) Find the Euler equation relating c_t and expectations of c_{t+1} ;

(2) Guess that consumption takes the form $c_t = \alpha + Fk_t$. Given this guess, what is k_{t+1} as a function of k_t ?

(3) What values must the parameters α and F have for the first order condition in part (1) to be satisfied for all values of capital stock?

Problem 3 A household seeks to maximize

$$-\sum_{t=1}^{\infty} \beta^t \left\{ (c_t - b)^2 + \gamma i_t^2 \right\}$$

subject to

$$\begin{aligned} c_t + i_t &= ra_t + y_t \\ a_{t+1} &= a_t + i_t \\ y_{t+1} &= \rho_1 y_t + \rho_2 y_{t-1}. \end{aligned}$$

Here c_t, i_t, a_t, y_t are the household's consumption, investment, asset holdings and exogenous labor income at t ; while $b > 0, \gamma > 0, r > 0, \beta \in (0, 1)$, and ρ_1 and ρ_2 are parameters, and y_0, y_{-1} are initial conditions. Assume that ρ_1 and ρ_2 are such that $(1 - \rho_1 z - \rho_2 z^2) = 0$ implies that $|z| > 1$.

(1) Specify state and control variables.

(2) Write down the Bellman equation.