

# Answer Key to Problem Set 1

Econ 121  
Instructor: Chao Wei

**Total: 15 points**

*Grading guidelines in General:*

1. *Be fair, but be generous, give benefit of doubt when you can.*
2. *Clearly mark the errors, with comments when necessary.*
3. *If the student writes down correct formulas, but the final numerical answer is wrong, he gets partial credit for the formulas. However, if someone does not write down formulas, and the numerical answer is wrong, he gets no partial credit.*
4. *Give full credit to students who do not write down formulas, but provide the correct answer.*
5. *In question 3, it is possible that one numerical error can lead to another numerical error. For example, if the student gets a wrong answer for bond prices, he will get wrong answers on the rate of return as well. In the case, only deduct points at one place.*

1. Suppose you are the agent for a baseball pitcher. Suppose he is offered the following contract by the New York Yankees: a signing bonus of \$3,000,000 (to be received immediately), a first year's salary of \$6,000,000 (to be received one year from today), a second year's salary of \$7,000,000 (to be received two years from today), and a third year's salary of \$8,000,000 (to be received three years from today). Suppose he is offered the following contracts by the San Francisco Giants: a signing bonus of \$6,000,000, a first year's salary of \$5,500,000, a second year's salary of \$6,000,000, and a third year's salary of \$6,000,000. If you believe the interest rate is 10%, which offer would you advise the pitcher to accept? Would your advice change if you believed the relevant discount rate were 5%? **(4 points)**

If  $i = 10\%$ , the present value of Yankee's offer is the following:

$$\begin{aligned} PV^Y &= 3,000,000 + \frac{6,000,000}{1.1} + \frac{7,000,000}{1.1^2} + \frac{8,000,000}{1.1^3} && \leftarrow \mathbf{0.5 \text{ point}} \\ &= 20,250,188. && \leftarrow \mathbf{0.5 \text{ point}} \end{aligned}$$

The present value of Giant's offer is:

$$\begin{aligned} PV^G &= 6,000,000 + \frac{5,500,000}{1.1} + \frac{6,000,000}{1.1^2} + \frac{6,000,000}{1.1^3} && \leftarrow \mathbf{0.5 \text{ point}} \\ &= 20,466,566. && \leftarrow \mathbf{0.5 \text{ point}}. \end{aligned}$$

So, based on the present value criterion, the Giants are making the more valuable offer.

With an interest rate of 5%, the present value of Yankee's offer is the following:

$$\begin{aligned} PV^Y &= 3,000,000 + \frac{6,000,000}{1.05} + \frac{7,000,000}{1.05^2} + \frac{8,000,000}{1.05^3} && \leftarrow \mathbf{0.5 \text{ point}} \\ &= \$21,974,193 && \leftarrow \mathbf{0.5 \text{ point}} \end{aligned}$$

The present value of Giant's offer is

$$\begin{aligned} PV^G &= 6,000,000 + \frac{5,500,000}{1.05} + \frac{6,000,000}{1.05^2} + \frac{6,000,000}{1.05^3} && \leftarrow \mathbf{0.5 \text{ point}} \\ &= 21,863,298 && \leftarrow \mathbf{0.5 \text{ point}} \end{aligned}$$

So Yankee's offer is more valuable.

2. Suppose that your roommate wants to borrow money from you. He offers three repayment plans. According to plan A, you lend him \$800 now, and he repays you \$1000 in two years. According to plan B, you lend him \$800 now, and he repays you \$500 in one year, and another \$500 in two years. According to Plan C, you lend him \$800 now, and he repays you \$900 in one year. Which plan do you prefer? (or equivalently, which plan gives you the highest yield to maturity?) (**3 points, 1 point for each plan**)

For plan A,

$$\begin{aligned} 800 &= \frac{1000}{(1+i)^2} \\ \implies i &= 11.80\% \end{aligned}$$

For plan B,

$$\begin{aligned} 800 &= \frac{500}{(1+i)} + \frac{500}{(1+i)^2} \\ \implies i &= 16.26\% \end{aligned}$$

For plan C,

$$\begin{aligned} 800 &= \frac{900}{(1+i)} \\ \implies i &= 12.50\% \end{aligned}$$

3. Compute the rate of return. (**7 points**)

1. (a) Denote the price of consol and the yield to maturity at the beginning of the year respectively as  $P_t$  and  $i_t$ , and denote the annual coupon payment as  $C$  :**(1 point)**

$$\begin{aligned} P_t &= \frac{C}{i_t} \\ &= \frac{200}{0.2} \\ &= 1000 \end{aligned}$$

Denote the price of consol and the yield to maturity at the end of the year respectively as  $P_{t+1}$  and  $i_{t+1}$ , and denote the annual coupon payment as  $C$  :**(1 point)**

$$\begin{aligned} P_{t+1} &= \frac{C}{i_{t+1}} \\ &= \frac{200}{0.1} \\ &= 2000 \end{aligned}$$

The return for the year,  $R_{t+1}$ , can be written as **(1 points. In the equations below, either the first line or the second line qualifies for 0.5 point, the final answer qualifies for 0.5 point.)**

$$\begin{aligned} R_{t+1} &= \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t} \\ &= \frac{200}{1000} + \frac{2000 - 1000}{1000} \\ &= 120\%. \end{aligned}$$

- (b) The price of the coupon bond at the beginning of the year,  $P_t$ , is equal to its face value, 1000.

Denote the price of the coupon bond and the yield to maturity at the end of the year respectively as  $P_{t+1}$  and  $i_{t+1}$ , and denote the annual coupon payment as  $C$ . Note that at the end of the year, the coupon bond will mature in another year. We compute  $P_{t+1}$  as the present value of the last coupon payment and principal, both due at maturity.**(1 points. In the equations below, either the first line or the second line qualifies for 0.5 points, the final answer**

qualifies for 0.5 point. )

$$\begin{aligned} P_{t+1} &= \frac{C}{(1+i_{t+1})} + \frac{F}{(1+i_{t+1})} \\ &= \frac{200}{(1+0.1)} + \frac{1000}{(1+0.1)} \\ &\approx 1091 \end{aligned}$$

The return for the year,  $R_{t+1}$ , can be written as **(1 points. In the equations below, either the first line or the second line qualifies for 0.5 points, the final answer qualifies for 0.5 point. )**

$$\begin{aligned} R_{t+1} &= \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t} \\ &= \frac{200}{1000} + \frac{1091 - 1000}{1000} \\ &= 29.1\% \end{aligned}$$

- (c) The price of the coupon bond at the beginning of the year,  $P_t$ , is equal to its face value, 1000.

Denote the price of the coupon bond and the yield to maturity at the end of the year respectively as  $P_{t+1}$  and  $i_{t+1}$ , and denote the annual coupon payment as  $C$ . Note that at the end of the year, the coupon bond will mature in another two years. We compute  $P_{t+1}$  as the present value of the last two coupon payments, due respectively one year and two years later and principal due at maturity. **(1 points. In the equations below, either the first line or the second line qualifies for 0.5 points, the final answer qualifies for 0.5 point. )**

$$\begin{aligned} P_{t+1} &= \frac{C}{(1+i_{t+1})} + \frac{C}{(1+i_{t+1})^2} + \frac{F}{(1+i_{t+1})^2} \\ &= \frac{200}{(1+0.1)} + \frac{200}{(1+0.1)^2} + \frac{1000}{(1+0.1)^2} \\ &\approx 1173.6 \end{aligned}$$

The return for the year,  $R_{t+1}$ , can be written as **(1 points. In the equations below, either the first line or the second line qualifies for 0.5 points, the final answer qualifies for 0.5 point.**

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$$\begin{aligned}R_{t+1} &= \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t} \\ &= \frac{200}{1000} + \frac{1173.6 - 1000}{1000} \\ &= 37.4\%\end{aligned}$$

4. If the interest rate is 10%, what is the present value of a security that pays you \$1,100 next year, \$1,210 the year after, and \$1,331 the year after that? **(1 point)**

The present value of the security is:

$$\begin{aligned}P &= \frac{1100}{1 + 10\%} + \frac{1210}{(1 + 10\%)^2} + \frac{1331}{(1 + 10\%)^3} \\ &= 3000.\end{aligned}$$