Econ 306 (Spring 2009)
Midterm Exam

Instructor: Chao Wei

The exam starts at 6:10 pm and ends at 9:00 pm.

There are four questions, 10 points for each question. Only answer 3 of 4 questions. Please specify which three questions you choose to answer. If you fail to do that and answer all the four questions, I will assume that you choose to answer the first three questions.

Read through the entire test before answering any questions. Do not use your book or notes.

Please keep your answers clear and concise. Be sure to identify important results. Put a box around any critical mathematical results.

Correct but irrelevant material will generate no credit.

Problem 1 *Consider a social planner who faces the following problem:*

$$\max_{\{C_t,K_{t+1},l_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \ln C_t, \tag{1}$$

subject to

$$\begin{aligned} C_t + I_t &\leq Y_t, & (2)\\ K_{t+1} &= (1-\delta) K_t + Q_t I_t, & (3)\\ k_0 &> 0 \end{aligned}$$

Here Q_t represents the level of investment-specific technology which follows the following stochastic process:

$$\ln Q_{t+1} = \rho_Q \ln Q_t + \epsilon_{Q,t+1}.$$

The production function is given by

$$Y_t = F(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}, 0 < \alpha < 1,$$

where L represents the labor input in production. The level of aggregate productivity, A_t, follows the following process:

$$\ln A_{t+1} = \rho_A \ln A_t + \epsilon_{A,t+1}.$$

Assume that $\delta = 1$,

(a) Derive the Bellman equation of this social planner's problem.

(b)First solve this problem using the Euler equations. Find the first order condition for K_{t+1} and derive the Envelop condition.

(c)Now solve the problem using Guess and Verify method. Guess that $V(\cdot)$ takes the form $V(K, A, Q) = G + H \ln K + F \ln A + D \ln Q$, and solve for G,H, F and D.

(d)What are the implied values of $\left\{ \frac{C_t}{Y_t}, \frac{K_{t+1}}{Y_t} \right\}_{t=0}^{\infty}$*? Explain the intuition behind the relationship between* $\frac{K_{t+1}}{Y_t}$ *and* α, β, Q_t *respectively.*

(e)What are the effect of a one-time shock to ϵ_A *on the paths of* $\ln K, \ln C$*, and* lnY*? Will the effects die out over time? What are the effect of a one-time shock to* ϵ_Q *on the paths of* $\ln K, \ln C$*, and* lnY*? Will the effects die out over time?*

Problem 2 *An unemployed worker receives each period a wage offer* w *drawn from the distribution* $F(w)$*. The worker has to choose whether to accept the job–and therefore to work forever–or to search for another offer and collect* c *in unemployment compensation. The worker who decides to accept the job must choose the number of hours to work in each period. The worker chooses a strategy to maximize*

$$
\begin{aligned}
& E \sum_{t=0}^{\infty} \beta^t u(y_t, l_t), \quad \textit{where} \\
u(y_t, l_t) &= \ln y_t + b \ln l_t \\
0 &< \beta < 1, b > 0.
\end{aligned}
$$

and $y_t = c$ *if the worker is unemployed, and* $y_t = w(1 - l_t)$ *if the worker is employed and works* $(1 - l_t)$ *hours;* l_t *is leisure with* $0 \leq l_t \leq 1$*.*
(a). Formulate the Bellman equation for the worker's problem.
(b). Show that the number of hours worked is the same in every period.
(c) Use graphical analysis to show that the optimal policy is to set a reservation wage.
(d). Derive the equation which can be used to compute the reservation wage. Note that in this equation the unknown must be the reservation wage only.

Problem 3 *The representative consumer has preferences given by*

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\sum_{t=0}^{\infty} \beta^t \theta_t \log c_t,
$$

where $0 < \beta < 1$, c_t *is consumption,* $\theta_t = \theta^*$ *for* $t = 0, 2, 4, ...,$ *and* $\theta_t = \theta^{**}$ *for* $t = 1, 3, 5, ...,$ *where* $\theta^* > \theta^{**}$. *The production technology is given by*

$$y_t = k_t,$$

where y_t *is output and* k_t *is capital, with* $k_0 > 0$ *given. There is* 100% *depreciation, and as a result the aggregate resource constraint is*

$$c_t + k_{t+1} = y_t.$$

(a) Let $v(k_t)$ *and* $w(k_t)$ *denote the value functions in even periods and odd periods, respectively. Using guess-and-verify methods, determine* $v(\bullet)$ *and* $w(\bullet)$, *and the decision rules for consumption and investment.*
(b) This is a type of "cake-eating" problem, where it is as if k_0 *is some initial quantity of food that the consumer has, food can be stored, and the consumer needs to determine how much to eat each period. He or she is more hungry in even periods than in odd periods. Explain your results in part (a) in light of this interpretation.*

Problem 4 *Consider a consumer with initial assets* A_0 *and preferences*

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma} - 1}{1-\gamma}, \gamma > 0$$

where $0 < \beta < 1, c_t$ *is consumption. The consumer's budget constraint is*

$$A_{t+1} = (1 + r)(A_t - c_t + w_t), \tag{4}$$

for $t = 0, 1, 2, ...,$ *where* r *is the one-period interest rate (assumed constant over time) and* w_t *is income in period* t, *where income is exogenous. We also assume the no-Ponzi-scheme condition*

$$\lim_{t\to\infty} \frac{A_t}{(1+r)^t} = 0. \tag{5}$$

(a) Derive the lifetime budget constraint.

(b) Formulate the Bellman equation for the consumer's problem.

(c) Find the first order conditions for each control variable respectively and derive the Envelop condition.

(d) Without assuming $\beta(1+r) = 1$*, derive the solution for* c_0*, and the entire optimal consumption path.*

(e) Derive the Euler equation when there is a no-borrowing constraint $A_{t+1} \geq 0$*. Interpret the economic intuition behind this Euler equation.*