Econ 306 (Spring 2006)
Midterm Exam

Instructor: Chao Wei

The exam starts at 6:10 pm and ends at 9:00 pm.

There are four questions, 10 points for each question. Only answer 3 of 4 questions. Please specify which three questions you choose to answer. If you fail to do that and answer all the four questions, I will assume that you choose to answer the first three questions.

Read through the entire test before answering any questions. Do not use your book or notes.

Please keep your answers clear and concise. Be sure to identify important results. Put a box around any critical mathematical results.

Correct but irrelevant material will generate no credit.

Problem 1 *Consider a social planner who faces the following problem:*

$$\max_{\{c_t,k_{t+1},l_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1-l_t\right), \tag{1}$$

subject to

$$\begin{aligned} 0 &\leq k_{t+1} \leq (1-\delta) k_t + y_t - c_t, \\ k_0 &> 0 \end{aligned} \tag{2}$$

Here l_t represents the labor supply and other notations are standard. Assume that $\delta = 1$ and

$$\begin{aligned} u\left(c_t, 1-l_t\right) &= \ln c_t + b \ln\left(1-l_t\right), \\ y_t &= F\left(k_t, l_t\right) = A_t k_t^{\alpha} l_t^{1-\alpha}, 0 < \alpha < 1, \end{aligned}$$

where

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1},$$

is a stochastic first order autoregressive process.

(a) Derive the Bellman equation of this social planner's problem.

(b)First solve this problem using the Euler equations. Find the first order conditions for k_{t+1} and l_t respectively and derive the Envelop condition.

(c)Now solve the problem using Guess and Verify method. Guess that $V(\cdot)$ takes the form $V(k, A) = G + H \ln k + F \ln A$, and solve for G,H and F.

(d)What are the implied values of $\left\{\frac{c_t}{y_t}, l_t\right\}_{t=0}^{\infty}$? Show that these two data series do not depend upon k_t or A_t.

(e)What are the effect of a one-time shock to ϵ on the paths of $\ln k, \ln c$, and lny? Will the effects die out over time?

Problem 2 *Consider an unemployed worker who samples wage offers on the following terms: When the economy is in expansion (denoted as $S_t = 1$),* *with probability* $\phi_h, 1 > \phi_h > 0$, *she receives an offer to work for* w *forever. With probability* $1 - \phi_h$, *she receives no offer (we may regard this as a wage offer of zero forever). When the economy is in recession (denoted as* $S_t = 0$*), with probability* $\phi_l, 1 > \phi_l > 0$, *she receives an offer to work for* w *forever. With probability* $1 - \phi_l$, *she receives no offer. The wage offer* w *is drawn from a cumulative probability distribution function* $F(w)$ *, .with* $0 < w \leq B$. *Successive draws across periods are independently and identically distributed. Assume that the economy alternates between expansions and recessions following a Markov chain process, specifically,*

$$\begin{aligned} prob\,(S_{t+1} = 1 \mid S_t = 1) &= p, \\ prob\,(S_{t+1} = 0 \mid S_t = 0) &= q. \end{aligned}$$

Since S *can only take two values, you should be able to figure out* $prob\,(S_{t+1} = 0 \mid S_t = 1)$ *and* $prob\,(S_{t+1} = 1 \mid S_t = 0)$.
The worker chooses a strategy to maximize $E \sum_{t=0}^{\infty} \beta^t y_t$, *where* $y_t = w$ *or* $y_t = c$, *depending on whether she is employed or unemployed.*
(a). Formulate the Bellman equation for the worker's problem. (You must expand the expectation term until the expectation is only with respect to the job offer in the next period.)
(b). Show that the optimal policy is to set a reservation wage that depends upon whether the economy is in expansion or recession.
(c). Derive the two equations which can be used to compute the above two reservation wages. Note that the unknowns in these two equations must be the two reservation wages only.

Problem 3 *Suppose the representative household maximizes*

$$E_0 \sum_{t=0}^{\infty} \frac{u(c_t)}{(1+\rho)^t},$$

where $u(c_t) = \frac{c_t^{1-\theta}}{1-\theta}, \theta > 0$. *Assume that the real interest rate,* r, *is constant but not necessarily equal to the subjective discount rate,* ρ.

(a) Find the Euler equation relating C_t *to expectations concerning* C_{t+1}.

(b) Suppose that the log of income is distributed normally, and that as a

result the log of C_{t+1} *is distributed normally; let* σ^2 *denote its variance conditional on information available at time* t. *Rewrite the expression in part (a) in terms of* $\ln C_t, E_t \ln C_{t+1}, \sigma^2$, *and the parameters* r, ρ, *and* θ.

(c) Show that if r *and* σ^2 *are constant over time, the result in part (b) implies that the log of consumption follows a random walk with drift:*

$$\ln C_{t+1} = \alpha + \ln C_t + \epsilon_{t+1},$$

where ϵ *is white noise.*

(d) How do changes in each of r *and* σ^2 *affect expected consumption growth,* $E_t [\ln C_{t+1} - \ln C_t]$? *Interpret the effect of* σ^2 *on expected consumption growth in light of the presence of precautionary saving.*

Problem 4 *Assume that there are only two trees in the economy, type A (for apple) and O (for orange). Fruits yielded each period,* a_t *(apples) and* o_t *(oranges) follow respectively a Markov process. The household maximizes*

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to

$$z_{a,t}(p_{a,t} + a_t) + z_{o,t}(p_{o,t} + o_t) \geq c_t + z_{a,t+1}p_{a,t} + z_{o,t+1}p_{o,t},$$

where $z_{a,t}$ *and* $z_{o,t}$ *represent respectively the quantity of the apple tree and orange tree the representative household holds between* $t-1$ *and* t.

(a) Write down the Bellman equation and derive the first order conditions and the envelop condition, and in the end derive a pair of Euler equations.

(b) Determine the market clearing conditions and substitute into the Euler equations.

(c) What is the expression for the stochastic discount factor in equilibrium? Suppose that the household is risk averse, is the stochastic discount factor more volatile when the apple crop and orange crop are positively correlated or negatively correlated? (For the latter part of (c), intuitive reasoning is sufficient.)