Econ 306 (Spring 2008)
Final Exam

Instructor: Chao Wei

The exam starts at 6:10 pm and ends at 9:00 pm.

There are 50 points on this exam.

There are four questions. You are REQUIRED to answer question 1, but can choose to answer TWO out of the three remaining questions. Please specify which question you choose to answer. If you fail to do that and answer all the questions, I will grade the first three questions. The first question is worth 20 points. The other three questions are worth 15 points each.

Read through the entire test before answering any questions. Do not use your book or notes.

Please keep your answers clear and concise. Be sure to identify important results. Put a box around any critical mathematical results.

Correct but irrelevant material will generate no credit.

Problem 1 *Indivisible Labor Real Business Cycle Model (20 points).*
The economy is populated by a large number of infinitely lived agents whose expected utility is defined as

$$E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \left[\log (C_t) - \theta N_t \right] \right\},$$

where C_t represents consumption and N_t represents hours worked per capita. The production function is

$$Y_t = A_t^{\alpha} K_t^{1-\alpha} N_t^{\alpha},$$

where A_t stands for technology, Y_t for output and K_t for capital. The capital accumulation process is:

$$K_{t+1} = (1-\delta) K_t + Y_t - C_t.$$

We now look for a steady-state or balanced growth path of this model, in which technology, capital, output, and consumption all grow at a constant common rate. We use the notation G for this growth rate: $G \equiv \frac{A_{t+1}}{A_t}$ in the steady state.

(a) Derive the first order conditions of the model and interpret their economic implications.

(b) Derive the steady state values of $\frac{A_t}{K_t}$, $\frac{Y_t}{K_t}$, $\frac{C_t}{Y_t}$, and N_t.

(c) Log-linearize the intertemporal first-order condition with respect to K_{t+1}, that is, write $E_t \triangle c_{t+1}$ as a function of $E_t a_{t+1}, E_t n_{t+1}$, and $E_t k_{t+1}$, where lower-case letters represent the log deviations from the steady state of the corresponding variables.

(d) Describe the heuristic labor supply curve and the labor demand curve generated by the model.

(e) Use the graphic analysis of the labor market behavior to describe the responses of the real wage and labor hours to a bad technology shock.

(f) The indivisible labor model generates considerably more volatility of output than the standard model, and slightly lower covariance between real wage and labor hours than the standard model. Explain why this is the case.

Problem 2 *The term structure and regime switching. Consider a pure exchange economy where the stochastic process for consumption is given by*

$$c_{t+1} = c_t \exp\left(\alpha_0 - \alpha_1 s_t + \varepsilon_{t+1}\right),$$

where
(i) $\alpha_0 > 0, \alpha_1 > 0,$ *and* $\alpha_0 - \alpha_1 > 0.$
(ii) ε_t *is a sequence of i.i.d. random variables distributed* $N\left(\mu, \tau^2\right)$, *where* $\mu > 0$. *Note: given this specification, it follows that* $E\left(e^{\varepsilon}\right) = \exp\left(\mu + \tau^2/2\right).$
(iii) s_t *is a Markov process independent from* ε *that can take only two values,* $\{0, 1\}$. *The transition probability matrix is completely summarized by*

$$\begin{aligned} prob\left(s_{t+1} = 1 | s_t = 1\right) &= \pi_1, \\ prob\left(s_{t+1} = 0 | s_t = 0\right) &= \pi_0. \end{aligned}$$

(iv) The information set at time $t, \Omega_t,$ *contains* $\left\{c_{t-j}, s_{t-j}, \varepsilon_{t-j}, j \geq 0\right\}.$

There is a large number of individuals with the following utility function

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right),$$

where $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. *Assume that* $\sigma > 0$ *and* $0 < \beta < 1$. *Hint: the one-period interest rate,* $r_{n,t}$ *is equal to* $-\frac{\log(P_{n,t})}{n}$, *where* $P_{n,t}$ *denote the price of the n-period discount bond.*

(a). Compute the "short-term" (one-period) interest rate, i.e, the interest rate on a discount bond which promises one unit of consumption good in one

period.

(b). Compute the "long-term" (two-period) interest rate, i.e, the interest rate on a discount bond which promises one unit of consumption good in two periods.

(c) Note that the log of the growth rate of consumption is given by

$$\log c_{t+1} - \log c_t = \alpha_0 - \alpha_1 s_t + \varepsilon_{t+1}.$$

Thus, the conditional expectations of this growth rate is just $\alpha_0 - \alpha_1 s_t + \mu$. *Note that when* $s_t = 0$, *growth is high, and when* $s_t = 1$, *growth is low. thus, loosely speaking, we can identify* $s_t = 0$ *with the peak of the cycle (or good times) and* $s_t = 1$ *with the trough of the cycle (or bad times). Given the above conditions, are short term rates pro- or countercyclical?*

(d) Are long rates pro- or countercyclical? If you cannot give a definite answer to this question, find conditions under which they are either pro- or countercyclical, and interpret your conditions in terms of the "permanence" (you get to define this) of the cycle.

Problem 3 *The real business cycle model assumes that investment can be negative. This implies that existing capital can be costlessly converted into the consumption good. In this problem, we impose a non-negativity constraint on investment. Assume that households maximize*

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \tag{1}$$

subject to

$$\begin{aligned} y_t &= z_t k_t^{\alpha} n_t^{1-\alpha}, && (2)\\ k_{t+1} &= (1-\delta) k_t + i_t, && (3)\\ y_t &= c_t + i_t, && (4)\\ i_t &\geq 0. && (5) \end{aligned}$$

(a) Derive the Bellman equation, the first-order conditions and the Envelop condition when there is a non-negativity constraint on investment.

(b) What is the Euler equation when the non-negativity constraint is not binding? What are the solutions to k_{t+1} and c_t in periods when the non-negativity constraint is binding?

(c) Now we further assume that

$$c_t + i_t \left[1 + h\left(\frac{i_t}{k_t}\right)\right] \leq z_t k_t^{\alpha} n_t^{1-\alpha}, \tag{6}$$

where $h\left(\frac{i_t}{k_t}\right)$ describes the resources required to install new capital, where $h(0) = 0, h' > 0$,and $2h'(x) + xh''(x) > 0$ for all $x > 0$, Write down the Bellman equation, first-order conditions and the envelop condition in this case.

(d) Define $x_t = \frac{i_t}{k_t}$, show that if $1 + h(x) + xh'(x) = 1$ and $h'(x) = 0$, the solutions of the model are equivalent to those of (a).

Problem 4 *Consider a representative household who faces the following problem:*

$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}, \gamma > 1 \tag{7}$$

subject to

$$\begin{aligned} 0 &\leq K_{t+1} \leq (1-\delta) K_t + (1-\tau) F(K_t) - C_t + \psi_t, \\ K_0 &> 0 \end{aligned} \tag{8}$$

Here τ is the tax rate levied on all gross output. We assume that the government gives back all the tax revenues in the form of lump-sum rebate ψ_t, which is equal to $\tau F(K_t)$. Note that by the definition of lump-sum rebate, the derivative of ψ_t with respect to control variables are zero. In other words, the representative household does not take into account the impact of their decisions on ψ_t.
Assume that

$$y_t = F(K_t) = A_t k_t,$$

where A_t is an identical and independently distributed random variable across time. Define

$$I_t = K_{t+1} - (1-\delta) K_t.$$

(a) Derive the Euler equation and interpret its economic intuition.

(b) Show that the first order conditions are satisfied when $K_{t+1} = \eta H_t K_t$, where $H_t = A_t + (1-\delta)$.

(c) Derive η and prove that $\frac{\partial \eta}{\partial \tau} < 0$. Explain the economic implication of this inequality.

(d) Derive $x_{1,t} = \frac{\partial \log C_t}{\partial \log A_t}$ and $x_{2,t} = \frac{\partial \log I_t}{\partial \log A_t}$. How does $x_{2,t}$ depend upon the investment-output ratio $\frac{I_t}{Y_t}$?

(e) Derive $\frac{\partial x_{1,t}}{\partial \tau}$ and $\frac{\partial x_{2,t}}{\partial \tau}$. Are they positive or negative? What do they say about the distortionary impact of taxes on the gross output?