Hints on Final Exam Econ 306 Questions (Spring 2008)

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problem 1.

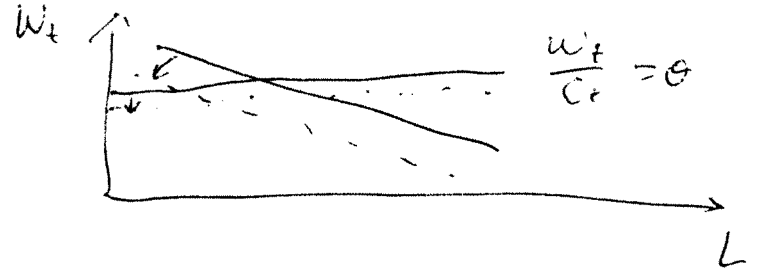
Basically Campbell's paper. I do not have anything to add except that

(a special case)

(d) Labor Supply: $\frac{w_t}{C_t} = \theta$

Labor Demand: $w_t = x \frac{Y_t}{N_t}$.

where w_t is shadow real wage.

(e) 

(f) Given that the labor supply curve is horizontal in this case, the implication is straight forward.

problem 2.

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problem 3.

(a)

$$V(K, Z) = \max_{k'} \left\{ u(c) + \beta E V(K', Z' | K, Z) + \mu [K' - (1-\delta)K] \right\}$$

$$\Rightarrow u'(C_t) = \beta E_t \left\{ u'(C_{t+1}) \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right) - \mu_{t+1}(1-\delta) \right\}$$
$$+ \mu_t$$

(b) ~~Not binding~~ $\cancel{\Rightarrow \mu_t = 0}$

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binding $\Rightarrow \begin{cases} k_{t+1} = (1-\delta) k_t \\ C_t = Z_t k_t^{\alpha} \end{cases}$

(c)

$$V(k,z) = \max_{k'} \left\{ u(c) + \beta V(k', z') + \mu (k' - (1-\delta)k) \right\}$$

where $c = zk^{\alpha} - [k' - (1-\delta)k]\left[1 + h\left(\frac{k' - (1-\delta)k}{k}\right)\right]$

Derive the Euler equation

$\Rightarrow$

$$u'(c_t)\left[1 + h_t + h_t' \frac{i_t}{k_t}\right]$$

$$= \beta E_t \left\{ u'(c_{t+1})\left[\alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta)(1+h) + \frac{i_t}{k_t} h'(1-\delta) + h' \frac{i_t^2}{k_t^2}\right] - (1-\delta)\mu_{t+1} \right\} + \mu_t$$

(d) obvious from (c).

problem 4

(a) $C_t^{-\gamma} = \beta E_t \left\{ C_{t+1}^{-\gamma} [1 - \delta + (1-\tau) A_{t+1}] \right\}$

(b) Given the guessed solution of K_{t+1}, derive C_t by this relation

$$C_t = y_t + (1-\delta) K_t - K_{t+1}$$

Guess and Verify:

$$\Rightarrow \eta = \beta^{\frac{1}{\gamma}} \left[E_t \left[(A_{t+1} + 1 - \delta)^{-\gamma} \left[(1-\delta) + (1-\tau) A_{t+1} \right] \right] \right]^{\frac{1}{\gamma}}$$

Since A_t is independent, E_t is equivalent to unconditional expectation, thus η is a constant

(c) Easy to prove $\frac{\partial \eta}{\partial \tau} < 0$

As $\tau \uparrow$, $K_{t+1} \downarrow$ for given K_t and A_t

High tax rate reduces investment.

(d) $\frac{\partial \log C_t}{\partial \log A_t} = \frac{A_t}{A_t + 1 - \delta}$, no change

$$\frac{\partial \log I_t}{\partial \log A_t} = \frac{\eta A_t}{\eta [A_t + 1 - \delta] - (1-\delta)}$$

(e) $\frac{\partial X_{1,t}}{\partial \tau} = 0$, $\frac{\partial X_{2,t}}{\partial \tau} > 0$