A Dynamic General Equilibrium Model of Driving, Gasoline Use and Vehicle Fuel Efficiency

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Abstract

The paper constructs a dynamic general equilibrium model to study the endogenous determination of gasoline use, driving and vehicle fuel efficiency. Before vehicles are produced, their fuel efficiency can be chosen optimally. Once produced, their fuel efficiency cannot be changed. The model generates endogenously different short-run and long-run price elasticities of gasoline use, with their magnitudes well within the region of plausible estimates in the empirical literature. The paper shows that although raising gasoline taxes and tightening the CAFE standard both reduce gasoline use in the long run, they are different in terms of the transmission mechanism, magnitudes of responses and dynamic paths of key endogenous variables.

Key words: gasoline use, fuel efficiency, vehicle miles of travel, CAFE, gasoline taxes

Gasoline consumption accounts for 44% of the U.S. demand for crude oil. Reducing gasoline consumption has become part of the strategic efforts to protect the nation from the serious economic and strategic risks associated with the reliance on foreign oil and the possible destabilizing effects of a changing climate. There has been heated discussion on policy options to discourage gasoline consumption, including increasing the gasoline taxes and tightening Corporate Average Fuel Economy (CAFE) standards. In order to evaluate the merits of these policy options, we need a structural framework to understand how people decide on how much to drive and what types of vehicles to own.

Key to this evaluation is the fact that the decisions on how much to drive and what types of vehicles to own are dynamic in nature. Vehicles are important durable goods with embodied technological characteristics. The durable nature of vehicles implies that people are forward-looking in making decisions regarding vehicle choice and utilization. The embodiment of technological characteristics also implies that the characteristics of existing vehicles may have significant impact on the transition dynamics after any exogenous shocks. Despite the dynamic nature of the issue, static models of consumer and producer behavior are widely used in this literature.[1]

In this paper we construct a dynamic general equilibrium model to study the endogenous determination of gasoline use, driving and vehicle fuel efficiency by a representative household which takes utility in vehicle miles of

[1] For example, Parry and Small (2005), Bento et al (2009), Jacobsen (2007), and etc, as discusssed later in the introduction.

travel. The model captures the putty-clay nature of the way transportation capital (vehicles) and gasoline are combined to "produce" vehicle miles of travel. Before vehicles are produced, their fuel efficiency can be chosen optimally in anticipation of future gasoline prices and economic conditions. Once the vehicles are produced, their fuel efficiency cannot be changed ex post. Decisions can be made on whether or not, or how often to utilize a vehicle, but if the vehicle is utilized, the gasoline use required for a given mileage is determined by its fuel efficiency.

Under the putty-clay specification, gasoline price shocks affect gasoline use through two main channels. The first channel is the endogenous capacity utilization of pre-existing vehicles. Since the quantity and fuel efficiency of existing vehicles are pre-determined, the representative household can only alter its driving behavior in the short run. As gasoline prices increase, vehicles are driven less due to higher gasoline cost, thus creating immediate gasoline savings. The second channel is the substitution of more fuel efficient vehicles as new generations of vehicles are produced. This channel leads to continuing gasoline savings as fuel-inefficient vehicles are phased out gradually. The impact of a permanent change in gasoline prices is fully realized after all the pre-existing vehicles are replaced.

The putty-clay specification allows the model to capture different short-run and long-run price elasticities of gasoline demand. These elasticities are obtained from endogenous dynamic responses of key variables to gasoline price shocks as a result of households' optimal decisions. The magnitudes of those elasticities, generated by the model calibrated to U.S. fuel consumption and vehicle usage data, are well within the range of plausible estimates in the empirical literature.

We use this model to compare two policy options: gasoline taxes and CAFE standard. Gasoline taxes have a long history in the United States. The CAFE regulation was enacted after the 1973 oil embargo. It imposes a limit on the average fuel economy of new vehicles sold by a particular firm, with fines applied to violations of the standard. In a representative-agent model like ours, the impact of an increase in gasoline taxes on the model dynamic is the same as that of a corresponding increase in gasoline prices under the assumption of a lump-sum tax rebate. As a result, the short- and long-run price elasticities of gasoline use can also be used to characterize the effect of gasoline taxes.

Although there has been conjecture that the minimum CAFE standard may have contributed to reduced gasoline use, it has been difficult to conduct a "comprehensive assessment of what would have happened had fuel economy standards not been in effect"(NRC 2002). The structural model developed in this paper, however, allows us to quantify the effect on gasoline savings achieved through the minimum CAFE standard. In contrast to gasoline taxes, a tightening of the CAFE standard achieves the gasoline savings by raising the fuel efficiency of new vehicles, with little effect on miles driven by pre-existing vehicles. We find that in order to achieve the same amount of permanent gasoline savings as a one-percent permanent increase in gasoline prices, the CAFE standard has to increase by 0.68 percent from its initial level which is assumed to be binding to start with.[2] We also find that the

[2] Austin and Dinan (2005) estimate that a 3.8 miles per gallon increase in the standard reduces long-run gasoline consumption by 10%. According to our model calculations, it takes a 20 percent increase in the after-tax gasoline price or a 13.6 percent increase in the

presence of the minimum CAFE standard alone avoids around 60 percent of the increase in gasoline consumption, which would have occurred absent such a standard at the time of permanent price decreases.

Existing studies in this literature have typically adopted a static approach. Parry and Small (2005) adopt a structural approach to study the optimal gasoline tax. In their model the representative agent decides on its optimal driving and gasoline use in a one-period utility-maximizing framework, where vehicle miles of travel is related to gasoline consumption in a reduced form and vehicle fuel efficiency is not a choice variable. This approach does not examine the dynamic responses of driving and vehicle choice in a multi-period setting. Bento and et al. (2009) estimate the distributional and efficiency impacts of increased U.S. gasoline taxes using a large sample of household data. Jacobsen (2007) incorporates the producer's decision problem into Bento et al. (2009) framework to study the equilibrium effects of an increase in the U.S. CAFE standards. However, both papers impose a specific indirect utility function for households and use Roy's identity to derive household decisions on vehicle choices. While in this paper, the representative households make optimal decisions on driving and vehicle ownership based on the direct utility function.[3]

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minimum CAFE standard to reduce gasoline consumption by 10%. The latter is equivalent to a 3.4 miles per gallon increase in the CAFE standard, a magnitude within the ballpark of the estimation by Austin and Dinan (2005).

[3]The models in Bento and et al.(2009) and Jacobsen (2010) are solved period by period, different from a dynamic general equilibrium model with rational expectations, where current and future variables are jointly determined and future expectations of variables are consistent with one another. The agents are considered fully forward-looking in the latter setting.

In contrast with nearly all prior work, this study adopts the dynamic general equilibrium modelling approach. The advantages of this approach are threefold. First, the structural framework makes transparent the transmission mechanism from exogenous shocks to optimal decision-making on driving and vehicle fuel efficiency and makes it possible to analyze the roles played by deep structural parameters. Second, the model is internally consistent. Third, it is a dynamic model which is not only forward-looking, but also captures the dynamic paths of key economic variables over time. To our knowledge, this is the first paper that employs a dynamic general equilibrium approach to examine the determination of gasoline use, driving and vehicle fuel efficiency.

This paper is related to the literature on the relationship between energy price shocks and the macroeconomy. Wei (2003) utilizes a putty-clay model to study the impact of energy price shocks on the stock market. In that paper, the adverse impact of an oil price shock is limited by the small share of oil costs as a fraction of the total production costs in the aggregate economy. The present study focuses on the effect of gasoline price shocks on transportation. The importance of gasoline to transportation makes it possible to capture the significance of oil to the aggregate economy through the transportation sector. This paper also relates to Bresnahan and Ramey (1993) and Ramey and Vine (2009), which use industry data to examine the segment shifts and capacity utilization in the U.S. automobile industry. The household's optimal decisions on vehicle fuel efficiency, as captured in our model, play an important role in the misalignment between supply and demand across vehicle market as documented in the data.

The paper is organized as follows. Section 1 describes the model and examines the model dynamics. Section 2 examines the steady-state equilibrium of the model and long-run elasticities. Section 3 presents the benchmark calibration. Section 4 compares the policy effects of gasoline taxes and the CAFE standard. Section 5 concludes.

1 The Model

This section describes the putty-clay feature of the production technology of travel and presents the household's problem.

1.1 The Production Technology of Travel

Vehicle-miles of travel are "produced" according to a putty-clay production technology as described in Wei (2003) and Gilchrist and Williams (2000). Transportation capital and gasoline are the only production factors. The *ex ante* production technology is assumed to be Cobb-Douglas with constant returns to scale, but after the configuration is already embedded in the transportation capital, production possibilities take the Leontief form: there is no *ex post* substitutability of transportation capital and gasoline. Transportation capital goods require one period for configuration and remain productive for the next N periods. Once constructed, transportation capital goods cannot be converted into consumption goods or capital goods with different embodied characteristics.

According to the *ex ante* Cobb-Douglas function, vehicle miles of travel per unit of gasoline use is a function of the transportation capital-gasoline ratio. E*x ante* constant returns to scale implies an indeterminacy of scale at the level of "vehicles". Without loss of generality, all "vehicles" are normalized to use one unit of gasoline at full capacity. As a result, miles of travel per vehicle depends upon the capital-gasoline ratio.

At period $t-j$, the household decides on the fuel efficiency (similar to the MPG measure) of vintage $t-j$ vehicles by choosing k_{t-j}, the amount of transportation capital embodied in each vehicle. Here k_{t-j} can also be considered as the capital-gasoline ratio at full capacity. After the decision is made, an idiosyncratic productivity term, ς_i, is revealed for each vehicle i. Both k_{t-j} and ς_i are fixed during the life span of the vehicle.[4] Subject to the constraint that the gasoline used at time t on vehicle i of vintage $t-j$, $O_{i,t,j}$, is nonnegative, and less than or equal to 1, vehicle-miles of travel produced in period t by vehicle i of vintage $t-j$ is

$$M_{i,t,j} = \varsigma_i k_{t-j}^{\alpha} O_{i,t,j}, \tag{1}$$

where $1-\boldsymbol{\alpha}$ is the exponent for gasoline.[5]

Given k_{t-j}, the vehicle miles of travel are linear in the total amount of

[4] Since we assume that the idiosyncratic fuel efficiency term, ς_i, is revealed after the decisions on the transportation capital-gasoline ratio are made, all machines of vintage $t-j$ share the same k_{t-j}.

[5] Equation (1) looks different from a simple Cobb-Douglas form. However, once we expand the ex ante form of the right-hand side as

$$\varsigma_i \left(\frac{\text{Capital}}{\text{Gasoline Use}}\right)^{\alpha} \bullet \text{Gasoline Use},$$

one can recognize that the exponential coefficient for gasoline use becomes $1-\alpha$, as in a standard Cobb-Dougals specification.

gasoline use. The variable, X_{t-j} , defined as

$$X_{t-j} = k_{t-j}^{\alpha}, \tag{2}$$

can be considered approximately as the average fuel efficiency (similar to the miles per gallon) of the vintage $t-j$ vehicle.

The idiosyncratic term, ς_i, is lognormally distributed[6]:

$$\log \varsigma_i \sim N\left(-\frac{1}{2}\sigma^2, \sigma^2\right), \tag{3}$$

where the mean correction term $-\frac{1}{2}\sigma^2$ implies that the mean of ς_i is equal to 1.

In sum, the transportation capital goods owned by households are heterogeneous and are characterized by the transportation capital-gasoline ratio chosen at the time of installation and an idiosyncratic fuel efficiency term. The fuel efficiency of vehicle i of vintage $t-j, \varsigma_i k_{t-j}^{\alpha}$, is fixed *ex post*.

1.2 Vehicle Miles of Travel and Gasoline Usage

There are no costs for taking vehicles on- or off-road. At each period, the only variable costs to operate one unit of vintage $t-j$ vehicles are the gasoline costs, which are proportional to the gasoline use at time t. As shown in equation (1), the vehicle miles of travel by vehicle i of vintage $t-j$ are proportional to its gasoline use as well. As a result, the net gain of operating each vehicle is linear in gasoline used. There is an endogenous cutoff value

[6] As emphasized in Gilchrist and Williams (2005), the log-normal distribution facilitates the analysis of aggregate quantities while preserving the putty-clay characteristics of the microeconomic structure.

for the minimum fuel efficiency of vintage $t-j$ vehicles used in travel at any time period t. Those with fuel efficiency higher than this cutoff value are run at full capacity at period t, while those less fuel efficient are left idle. In other words, the gasoline used at time t on vehicle i of vintage $t-j$ is equal to 1 when the vehicle is in operation and equal to 0 otherwise.[7]

Since k_{t-j} is pre-determined and common to all vintage $t-j$ vehicles, the endogenous cutoff value at period t for the minimum fuel efficiency corresponds to a cutoff value for the idiosyncratic term, denoted as $\underline{\varsigma}_t^{t-j}$. When ς_i is higher than $\underline{\varsigma}_t^{t-j}$, the vehicle is in operation and vice versa. For convenience we define z_t^{t-j} to be equal to $\frac{\log\left(\underline{\varsigma}_t^{t-j}\right)+\frac{1}{2}\sigma^2}{\sigma}$. Under the log-normal distribution of the idiosyncratic productivity term, the fraction of vintage $t-j$ vehicles that are run at full capacity at period t is now $1-\Phi\left(z_t^{t-j}\right)$, while vehicle miles of travel produced by vintage $t-j$ vehicles run at full capacity are $\left[1-\Phi\left(z_t^{t-j}-\sigma\right)\right]Q_{t-j}X_{t-j}$,[8] where Q_{t-j} is the quantity of new vehicles produced in period $t-j$.

Accordingly, the total vehicle-miles of travel at period t, M_t, and the total

[7]Since there are no costs for taking vehicles on- or off-road, a vehicle which is below cutoff for utilization in a given year sits idle for that year and then return for consideration in the next year.

[8]The proof makes use of the fact that under a log-normal distribution Γ, $\int_{\underline{\varsigma}_t^{t-j}}^{\infty} \varsigma_i X_{t-j} d\Gamma\left(\varsigma_i\right)=\left[1-\Phi\left(z_t^{t-j}-\sigma\right)\right] X_{t-j}$.

amount of gasoline usage, O_t, are given by

$$M_t = \sum_{j=1}^{N} \left\{ \left[1 - \Phi\left(z_t^{t-j} - \sigma\right)\right] (1-\delta)^{j-1} Q_{t-j} X_{t-j} \right\}, \tag{4}$$

$$O_t = \sum_{j=1}^{N} \left\{ \left[1 - \Phi\left(z_t^{t-j}\right)\right] (1-\delta)^{j-1} Q_{t-j} \right\} \tag{5}$$

where δ reflects that a subset of vehicles has depreciated completely each period. The summation of gasoline usage derives from the fact that in equilibrium each vehicle in operation uses one unit of gasoline.

1.3 The Penalty for Violating the CAFE Standard

Introducing the CAFE regulation into the model involves modeling the penalty for violating the standard. According to the CAFE regulation's official penalties, the fine can be described by[9]

$$F_t = \mathbf{I}\left(\underline{X} > \widehat{X}_t\right) f\left(\underline{X} - \widehat{X}_t\right) Q_t, \tag{6}$$

where $\mathbf{I}\left(\underline{X} > \widehat{X}_t\right)$ is an indicator function which is equal to 1 when the harmonian average of the fuel efficiency measure of new vehicles[10], $\widehat{X}_t$, is lower than the CAFE standard $\underline{X}$. From now on, we define $\widehat{\bullet}$ as the harmonian

[9] The indicator function below is defined for a single year whereas the regulation allows banking or borrowing of credits for up to three years. Such an approximation provides a tractable model.

[10] The violation of the CAFE standard is determined using the harmonian average of fuel efficiency for the entire fleet. A harmonic mean is not a simple arithmetic mean. It is the reciprocal of the average of the reciprocals of the fuel economies of the vehicles in the fleet. Given the log-normal distribution of the idiosyncratic term, the harmonian average of the fuel efficiency, $\widehat{X}_t$, is equal to $X_t \exp\left(-\sigma^2\right)$.

average of the corresponding variable. The parameter f represents the proportional penalty on the deviation from the CAFE standard. The penalty is imposed on all new vehicles, similar to the structure of the official penalty imposed by the CAFE regulation.

We approximate the indicator function $\mathbf{I}\left(\underline{X} > \widehat{X}_t\right)$ with a smooth transition function of a logistic type.[11] A smooth transition eliminates the kinks brought by abrupt transition, and thus facilitates the numerical analysis.

1.4 The Household's Problem

The economy consists of many identical, infinitely-lived households which derive utility from consumption of goods, vehicle-miles of travel, and leisure. The utility from travel comes from mobility provided to the household. The representative household maximizes the following lifetime utility:

$$\max \sum_{t=0}^{\infty} \left\{ \beta^t U\left(C_t, M_t, 1 - L_t - T_t\right) \right\}, \tag{7}$$

where C is the quantity of a numeraire consumption good, M is vehicle-miles of travel,[12] T is time spent driving, and L represents labor. The household

[11] The function we use is:

$$\mathbf{I}\left(\underline{X} > \widehat{X}_t\right) \approx H\left(\underline{X}, \widehat{X}_t\right) = \frac{1}{1 + \exp\left[-\gamma\left(\underline{X} - \widehat{X}_t\right)\right]},$$

where the parameter γ determines the abruptness of the transition of the indicator from 0 to 1 as $\underline{X} - \widehat{X}_t$ changes from negative to positive. For example, for a very large γ, a small negative value of $\underline{X} - \widehat{X}_t$ results in $H\left(\underline{X}, \widehat{X}_t\right)$ being very close to 0.

[12] Note that vehicle miles of travel, M, is different from person miles of travel. A decline in the vehicle miles of travel can be achieved by a carpool of multiple people, which

has a fixed time endowment of 1. Accordingly, $1 - L_t - T_t$ represents the amount of leisure.[13] The driving time is determined as follows:

$$T_t = \omega M_t, \tag{8}$$

where ω is the inverse of the average travel speed.[14]

We assume that the household produces output using a linear production technology with labor as the only production input. The household production function is

$$Y_t = AL_t, \tag{9}$$

where A represents the level of aggregate productivity.

At the beginning of each period t, there are N vintages of transportation capital in existence. Each vintage is identified by the capital-gasoline ratio, k_{t-j}, and the quantity of vehicles produced per vintage, Q_{t-j}, $j = 1, \cdots, N$. At each period t, the representative household takes the vintage structure of vehicles $\{Q_{t-j}, k_{t-j}\}_{j=1}^{N}$, the future paths of real after-tax gasoline prices

generates the same person miles of travel.

[13]Abstraction from utility in leisure does not change the model results qualitatively, but quantitatively vehicle miles of travel and gasoline use respond much more strongly than in the data. Detailed results in this case are available from the author upon request.

[14]We also work with an alternative version of the model where the driving time is determined by

$$T_t = \psi\left(\overline{M}_t\right) M_t.$$

Here $\psi(\bullet)$ is the inverse of the average travel speed and $\overline{M}_t$ is aggregate miles driven. An increase in the aggregate vehicle-miles of travel leads to more congestion on roads, so $\psi'(\bullet) > 0$. Agents take $\psi\left(\overline{M}_t\right)$ as fixed. They do not take into account of their own impact on congestion. This setup generates quanlitatively similar predictions of the model as the benchmark setting.

$\{P_{t+s}\}_{s=0}^{\infty}$ and minimum CAFE standards $\{\underline{X}_{t+s}\}_{s=0}^{\infty}$as given. The household then chooses k_t, the capital-gasoline ratio to be embedded in the new vehicles, Q_t, the quantity of new vehicles, and $\{z_t^{t-j}\}_{j=1}^{N}$, the cutoff value for vehicle utilization, to maximize its lifetime utility as described in equation (7), subject to equations (4), (5), an exogenous deterministic gasoline price process[15] and the aggregate resource constraint[16]:

$$C_t + P_t O_t + Q_t k_t + \frac{\theta}{2} Q_t (k_t - k_{t-1})^2 + F_t = AL_t + \zeta_t. \tag{10}$$

The left hand side of the budget constraint represents the total spending, including consumption C, after-tax gasoline expenses $P_t O_t$, investment in new vehicles, $Q_t k_t$, the adjustment cost of changing the fuel efficiency configuration from that of the previous period, and possible penalty on CAFE standard violations F_t. The cost of adjusting the fuel efficiency is convex at $\frac{\theta}{2}(k_t - k_{t-1})^2$ per vehicle. We assume that all the revenues or expenses from the energy policy, including gasoline taxes and penalty for violating the CAFE standard are redistributed to the household in lump-sum amount of ζ_t.[17]

[15]The model focuses on the demand for gasoline use, taking the gasoline price as exogenously given. Exogenous gasoline price shocks can be understood as increases in gasoline taxes, change in the technology for turning output into gasoline, or term-of-trade shocks.

[16]The representative household makes vehicle production decisions in this model, and as a result, pays for the penalty for CAFE violations. The household also owns all vintages of vehicles. In equilibrium the value of vehicles enters symmetrically on both sides of the budget constraint. We omit them for the sake of simplicity.

[17]We assume that the fines collected from the CAFE violation are redistributed in a lump-sum fashion. By doing this, we not only abstract from the income effect, but also partially account for the credits (debits) the firm is allowed to accumulate according to the CAFE regulations.

1.4.1 Decisions on Fuel Efficiency and Quantity of New Vehicles

The first-order conditions with respect to k_t and Q_t are:

$$\begin{aligned} & 1+\theta\left(k_t-k_{t-1}\right)+f\left(\underline{X}-\widehat{X}_t\right)\frac{\partial H\left(\underline{X},\widehat{X}_t\right)}{\partial k_t}+fH\left(\underline{X},\widehat{X}_t\right)\frac{\partial\left(\underline{X}-\widehat{X}_t\right)}{\partial k_t} \\ = \; & E_t\sum_{s=1}^{N}\left\{\beta^s\left(1-\delta\right)^{s-1}\frac{\mu_{t+s}}{\lambda_t}\left[1-\Phi\left(z_{t+s}^t-\sigma\right)\right]\alpha k_t^{\alpha-1}\right\}+\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\theta\left(k_{t+1}-k_t\right)\frac{Q_{t+1}}{Q_t}\right]. \end{aligned} \tag{11}$$

$$\begin{aligned} & k_t+\frac{\theta}{2}\left(k_t-k_{t-1}\right)^2+f\left(\underline{X}-\widehat{X}_t\right)H\left(\underline{X},\widehat{X}_t\right) \\ = \; & E_t\sum_{s=1}^{N}\left\{\beta^s\left(1-\delta\right)^{s-1}\left[\frac{\mu_{t+s}}{\lambda_t}\left[1-\Phi\left(z_{t+s}^t-\sigma\right)\right]X_t-\frac{\lambda_{t+s}}{\lambda_t}P_{t+s}\left[1-\Phi\left(z_{t+s}^t\right)\right]\right]\right\} \end{aligned} \tag{12}$$

The left-hand sides of the above equations are respectively the marginal costs of increasing the capital-gasoline ratio and the quantity of new vehicles. The marginal costs include the impact on the resources, the adjustment cost and the CAFE penalty from marginal variations in the fuel efficiency or the quantity of new vehicles. The right-hand side are the corresponding marginal benefits.

1.4.2 Driving Decisions

The first-order condition for the capacity utilization rate $\left\{z_t^{t-j}\right\}_{j=1}^N$ is given by

$$\mu_t\phi\left(z_t^{t-j}-\sigma\right)X_{t-j}=\lambda_tP_t\phi\left(z_t^{t-j}\right), \tag{13}$$

where $\phi(\bullet)$ is the probability density function of a standard normal random variable, μ_t, the Lagrange multiplier for equation (4), is the marginal value of travel, and λ_t, the Lagrange multiplier for equation (10), is the marginal utility of consumption. Here μ_t and λ_t are respectively given by

$$\mu_t = U_2(C_t, M_t, 1 - L_t - T_t) - U_3(C_t, M_t, 1 - L_t - T_t)\,\omega, \tag{14}$$

$$\lambda_t = U_1(C_t, M_t, 1 - L_t - T_t). \tag{15}$$

The left- and right- hand sides of equation (13) represent respectively the marginal value of extra travel and the marginal gasoline cost due to higher vehicle utilization.

Simple algebra manipulations of equation (13) also yields

$$z_t^{t-j} = \frac{1}{\sigma}\left(\log\lambda_t + \log P_t - \log\mu_t - \log X_{t-j} + \frac{1}{2}\sigma^2\right), \tag{16}$$

which shows that the lower fuel efficiency (the lower X_{t-j}), the lower capacity utilization rate among vehicles of vintage $t - j$. In addition to the pre-determined fuel efficiency X_{t-j}, the exogenously given gasoline price, P_t, and endogenously determined λ_t and μ_t also affect the decision on vehicle utilization. Higher gasoline prices affect the capacity utilization of vehicles through its impact on the cut-off value of fuel efficiency.

In equilibrium, all produced goods are either consumed, invested, or used to pay for gasoline expenses and cost of adjusting the fuel efficiency from that of the previous generation of vehicles.

2 Long-Run Elasticities

In this section we characterize the steady-state solutions of the model and derive long-run elasticities of fuel efficiency and gasoline use in response to permanent changes in gasoline prices or the minimum CAFE standard.

2.1 The Case of A Nonrestrictive CAFE Standard

When there is no minimum CAFE standard, or $\underline{X}$ is lower than the harmonian mean of the unconstrained optimal choice of fuel efficiency, the minimum CAFE standard is not restrictive. Any changes in $\underline{X}$ in the region where the minimum CAFE standard is not restrictive have no impact on equilibrium solutions. As a result, we focus on the price elasticities of fuel efficiency and gasoline use instead.

Proposition 1 *Under a nonrestrictive CAFE standard, the steady-state values of z and k are given by*

$$
\begin{aligned}
(1-\alpha)\frac{1-\Phi\left(z_{ss}^{u}-\sigma\right)}{\phi\left(z_{ss}^{u}-\sigma\right)} &= \frac{1-\Phi\left(z_{ss}^{u}\right)}{\phi\left(z_{ss}^{u}\right)}, && (17)\\
k_{ss}^{u} &= \frac{\alpha}{1-\alpha}\widetilde{d}P\left[1-\Phi\left(z_{ss}^{u}\right)\right]. && (18)\\
\textit{where } \widetilde{d} &= \sum_{s=1}^{N}\left[\beta^{s}\left(1-\delta\right)^{s-1}\right]. && (19)
\end{aligned}
$$

Here the superscript u represents the nonrestrictive case, and the subscript ss represents the steady state.

Proof. The proof is contained in the appendix. ■

The steady-state capacity utilization depends only upon α and σ.[18] Once z_{ss}^{u} is determined, the steady-state capital-gasoline ratio, k_{ss}^{u}, and the resulting fuel efficiency of vehicles, X_{ss}^{u}, can be easily derived from equation (18). Equation (18) shows that the ratio of expected gasoline cost over the value

[18] Gilchrist and Williams (2005) show that the solution of z_{ss}^{u} is unique for this class of functions.

of the transportation capital embodied in this new vehicle is proportional to $\frac{(1-\alpha)}{\alpha}$. This result is consistent with the ex ante Cobb-Douglas specification of the production technology of travel.

Panel A of Figure 1 illustrates the determination of long-run equilibrium solutions of z^u_{ss} and X^u_{ss} in the case of a nonrestrictive CAFE standard. The vertical axes of Figure 1 represent the harmonian means of fuel efficiency for a vintage in the steady state, and the horizontal axes represent z, which determines the capacity utilization. In the panel, the downward sloping curve represents equation (18), which has the interpretation that the optimal fuel efficiency of vehicles decreases with z and increases with the capacity utilization. The curve shifts rightward when the gasoline price increases. The vertical line represents equation (17), which determines the long-run steady-state solution of z. The fuel efficiency of vehicles determined at the intersection of these two curves is X^u_{ss}, whose harmonian mean is assumed to be above the minimum CAFE standard.

The steady-state values of z^u_{ss} and X^u_{ss}, and the real shadow price of travel, $\frac{\mu^u_{ss}}{\lambda^u_{ss}}$, are all determined in a self-contained system consisting of the steady-state versions of equations (16), (11) and (12). Given z^u_{ss}, X^u_{ss} and $\frac{\mu^u_{ss}}{\lambda^u_{ss}}$, the steady-state quantity of vehicles, Q^u_{ss}, is determined by the aggregate resource constraint (10). The vehicle miles of travel, M^u_{ss}, and the total gasoline usage, O^u_{ss} are determined accordingly.

Proposition 2 characterizes long-run price elasticities of fuel efficiency. It defines a price level, P_H, which makes $\underline{X}e^{\sigma^2}$ an optimal choice of fuel efficiency in the absence of a minimum CAFE standard.

Proposition 2 *There exists a threshold price level, P_H, given by,*

$$P_H = \frac{\left(\underline{X} e^{\sigma^2}\right)^{\frac{1}{\alpha}}}{\frac{\alpha \widetilde{d}}{1-\alpha}\left[1-\Phi\left(z_{ss}^u\right)\right]}, \tag{20}$$

such that

(i) When $P \geq P_H$, the minimum CAFE standard is not restrictive. The threshold price level, P_H, is an increasing and convex function of the minimum standard $\underline{X}$.

(ii) As long as $P \geq P_H$, any permanent changes in the gasoline price level, P, have no impact on the long-run capacity utilization of vehicles. Within this price range, the long-run elasticity of vehicle fuel efficiency with respect to gasoline prices, $\frac{\partial \ln X_{ss}^u}{\partial \ln P}$, is equal to α.

Proof. The proof is self-evident from equations (17) and (18). ■

2.2 The Case of A Restrictive CAFE Standard

Now we consider the case of a restrictive minimum CAFE standard in the steady state. In this case, the minimum CAFE standard $\underline{X}$ is higher than the harmonian mean of the unconstrained optimal choice of fuel efficiency. This case can occur when the gasoline price P is lower than the threshold level P_H. In this section we first characterize equilibrium solutions, and then examine the long-run elasticities with respect to gasoline prices and the minimum CAFE standard.

When the minimum CAFE standard is restrictive, equations (18) and (17) no longer hold. We use k^c, X^c and z^c to denote the corresponding variables under the restrictive minimum CAFE standard. Corresponding to equations (18) and (17), we have[19],

$$
\begin{aligned}
k^c &= \frac{\alpha}{1-\alpha}\left\{\widetilde{d}P\left[1-\Phi\left(z^c\right)\right]+fH\left(\underline{X},\widehat{X}^c\right)\underline{X}\right\}, && (21)\\
fH\left(\underline{X},\widehat{X}^c\right)\left[\underline{X}-(1-\alpha)\widehat{X}^c\right] &= \widetilde{d}P\left[1-\Phi\left(z^c\right)\right]\left\{(1-\alpha)\frac{\phi\left(z^c\right)\left[1-\Phi\left(z^c-\sigma\right)\right]}{\phi\left(z^c-\sigma\right)\left[1-\Phi\left(z^c\right)\right]}-1\right\}. && (22)
\end{aligned}
$$

As shown in Panels B to D of Figure 1, equation (21) characterizes two segments of a downward sloping curve. The upper segment, which corresponds to the region in which $\widehat{X}$ is higher than $\underline{X}$, coincides with the corresponding segment in Panel A. However, the lower segment of the downward sloping curve shifts to the right when $\widehat{X}<\underline{X}$. The curve characterized by equation (22) demonstrates a similar pattern. The upper segment of the curve is vertical, corresponding to the region in which $\widehat{X}$ is higher than $\underline{X}$, but the lower segment shifts to the left and becomes upward sloping when $\widehat{X}<\underline{X}$.[20]

2.2.1 Long-Run Elasticity with respect to Gasoline Prices

Proposition 3 characterizes the steady-state equilibrium solutions in the presence of the CAFE standard.

Proposition 3 *Define P_L as the price level that solves the following equa-*

[19] We omit the term involving $\left(\underline{X}-\widehat{X}\right)\frac{\partial H(\underline{X},\widehat{X})}{\partial k}k$ since this term converges to zero numerically for large enough γ over the entire range of $\widehat{X}$.

[20] The curve is upward sloping because the right hand side of equation (22) is decreasing in z, as shown by Gilchrist and Williams (2005) for this class of functions.

tion,

$$\left(\underline{X}e^{\sigma^2}\right)^{\frac{1}{\alpha}} = \frac{\alpha}{1-\alpha}\left\{\widetilde{d}P_L\left[1-\Phi\left(\widetilde{Z}\left(f,P_L,\underline{X},\Theta\right)\right)\right]+f\underline{X}\right\}. \qquad (23)$$

where $\widetilde{Z}\left\{f,P_L,\underline{X},\Theta\right\}$ *is defined by the following implicit function*

$$f\alpha\underline{X} = \widetilde{d}P_L\left[1-\Phi\left(\widetilde{z}\right)\right]\left\{(1-\alpha)\frac{\phi\left(\widetilde{z}\right)\left[1-\Phi\left(\widetilde{z}-\sigma\right)\right]}{\phi\left(\widetilde{z}-\sigma\right)\left[1-\Phi\left(\widetilde{z}\right)\right]}-1\right\}. \qquad (24)$$

Here $\widetilde{z}$ *is the value of* z^c *that satisfies equation (22) when* $\widehat{X}$ *becomes infinitely close to* $\underline{X}$ *from below, and* Θ *represents the rest of the model parameters.*
(i) P_L *is the threshold price level at which the household chooses to violate the minimum CAFE standard by choosing* $\widehat{X}$ *to be infinitely close to* $\underline{X}$ *from below. If* $P_L < P < P_H$, *the household complies with the minimum CAFE standard exactly by setting* X^c_{ss} *to* $\underline{X}e^{\sigma^2}$ *and setting* z^c_{ss} *to satisfy the following equation*[21]

$$\left(\underline{X}e^{\sigma^2}\right)^{\frac{1}{\alpha}} = \widetilde{d}P\left[1-\Phi\left(z^c_{ss}\right)\right]\left\{\frac{\phi\left(z^c_{ss}\right)\left[1-\Phi\left(z^c_{ss}-\sigma\right)\right]}{\phi\left(z^c_{ss}-\sigma\right)\left[1-\Phi\left(z^c_{ss}\right)\right]}-1\right\}. \qquad (25)$$

(ii) If $P \leq P_L$, *the optimal values of* X^c_{ss} *and* z^c_{ss} *satisfy equations (21) and (22). The optimal fuel efficiency thus chosen is below that corresponding to the minimum CAFE standard, but above the nonrestrictive optimal choice* X^u_{ss}.
(iii) The penalty threshold P_L *is a function of* $\left\{f,\underline{X},\Theta\right\}$. P_L *depends negatively upon the unit penalty* f *and positively on the minimum CAFE standard* $\underline{X}$. *For a given* $\underline{X}$, P_L *is lower than* P_H *by definition.*

[21] The value of z^c_{ss} depends upon f in approximation but not theoretically.

Proof. The proof is contained in the Appendix. ∎

According to Proposition 3, the representative household chooses to violate the minimum CAFE standard and pay the penalty when P is lower than and equal to P_L, but complies with the standard if otherwise. Since P_L depends negatively upon f, the household's optimal decision at a given level P can be interpreted as violating the CAFE standard when f is sufficiently small, but complies with it when the penalty f is sufficiently large.

In Panel B to D of Figure 1, we vary f to generate different values of P_L. Panel B of Figure 1 demonstrates the case where $P < P_L$ (f sufficiently small). In this case, the lower segments of the two curves intersect below $\underline{X}$, but above $\widehat{X}^u_{ss}$. Panel C of Figure 1 demonstrates the case where the lower segments of the two curves intersect at a value marginally below $\underline{X}$ when $P = P_L$. In Panel D where $P_L < P < P_H$ (f sufficiently large), the two curves no longer intersect.

The negative dependence of P_L on both the unit penalty f and its positive dependence upon the minimum CAFE standard $\underline{X}$ are intuitive since when the unit penalty is higher, the threshold price level needs to be relatively lower to entice the household to violate the minimum CAFE standard. Similarly, when the minimum CAFE standard is higher but the unit penalty remains fixed, the household has more incentives to violate the minimum CAFE standard, thus leading to a higher P_L.

Proposition 4 characterizes the long-run price elasticities of fuel efficiency in the presence of the CAFE standard.

Proposition 4 *Holding all the parameters in* Θ *fixed,*

(i) Given $\underline{X}$, *as long as the gasoline price varies in the region of* $[P_L, P_H]$,

the elasticity of fuel efficiency with respect to gasoline prices is zero.
(ii) Given $\underline{X}$, any changes in the gasoline price level from the region of $[P_L, P_H]$ to the region below P_L or above P_H and vice versa result in a positive elasticity of fuel efficiency below α.
(iv) An increase in f lowers P_L but has no impact on P_H, thus resulting in an expanded region within which optimal fuel efficiency does not respond to fluctuations in gasoline prices for any given $\underline{X}$.

Proof. *The proof is contained in the appendix.* ■

In all, there exists a region in which the optimal fuel efficiency is not responsive to changes in the gasoline price level in the long run. The width of the region depends upon the level of the minimum CAFE standard and the severity of the unit penalty f. Since the long-run impact of gasoline price shocks on gasoline use is transmitted through the gradual adjustment of vehicle fuel efficiency, the existence of such a region alters the price elasticities of gasoline use and other endogenous variables both in the short run and in the long run.

2.2.2 Long-Run Elasticities with respect to CAFE Standard

Proposition 3 shows that for a given minimum CAFE standard, there exists a price level P_L, which is the lower bound of a price region within which the price elasticities of fuel efficiency is zero. Given the monotone correspondence, equation (23) also implies that for a given price level P, there exists a minimum CAFE standard $\underline{X}_L$, such that for the given $\underline{X}_L$, the given price level P becomes the lower bound. For any given P, $\underline{X}_L$ is higher than $\widehat{X}^u_{ss}$

by definition.

Proposition 5 describes the long-run elasticities with respect to changes in the minimum CAFE standard.

Proposition 5 *For a given price level P,*
(i) As long as the minimum CAFE standard $\underline{X}$ varies in the range of $[\widehat{X}^u_{ss}, \underline{X}_L]$, the household chooses to comply with the minimum CAFE standard exactly by setting X^c_{ss} to $\underline{X}e^{\sigma^2}$. The elasticity of fuel efficiency with respect to changes in the minimum CAFE standard within this range is 1.
(ii) Any variations in the minimum CAFE standard in the region below $\widehat{X}^u_{ss}$ have no impact on the optimal fuel efficiency.
(iii) Any variations in the minimum CAFE standard from the region $[\widehat{X}^u_{ss}, \underline{X}_L]$ to above $\underline{X}_L$ result in an elasticity which is positive but less than 1, *with the magnitude of elasticity depending inversely upon the distance between the new minimum CAFE standard and $\underline{X}_L$.*

Proof. The proof is contained in the appendix. ■

In the next two sections, we first calibrate the model and then use the analytically derived long-run elasticities and numerically computed short-run dynamics to characterize the dynamic paths of key endogenous variables after permanent changes in gasoline prices or the CAFE standard.

3 Calibration

This section describes the benchmark calibration. There are four categories of parameters. The first category contains parameters related to the travel technology and gasoline use. The second category relates to the CAFE standard. The third category relates to the preference specification. The fourth category contains parameters which specify the mean of gasoline prices and the level of aggregate productivity. Table 1 summarizes the benchmark calibration for all parameters.

3.1 Parameters Related to Travel and Gasoline Use

A model period corresponds to a calendar year. We set the maximum life span of vehicles, N, to be equal to 15. Vehicles have 15 years of life span. The depreciation rate δ is set at the annual rate of 0.1.

The production technology of travel is ex ante Cobb-Douglas. We calibrate the parameter α to 0.42 to match the ratio of real gasoline expenditure over the expenditure on new vehicles, which is equal to 1.68 according to BEA (2006) data[22].

The standard deviation of idiosyncratic uncertainty, σ, is set to 0.3403 to

[22] According to the steady state equilibrium of the economy,

$$\begin{aligned} & \frac{\text{real expenditure on gasoline}}{\text{real expenditure on new vehicles}} \\ = & \frac{1-\alpha}{\alpha} \times \frac{\sum_{s=1}^{N}\left[(1-\delta)^{s-1}\right]}{\sum_{s=1}^{N}\left[\beta^{s}(1-\delta)^{s-1}\right]}. \end{aligned}$$

According to the BEA (2006) data, the real expenditure on gasoline and new vehicles respectively make up for 1.63% and 0.97% of GDP. The data on real personal expenditure on new autos (Line 4, 109.1 billions of chained 2000 dollars) and gasoline (line 75, 184.2 billions of chained 2000 dollars) are obtained from Table 2.4.6. The data on real GDP (11294.8 billions of chained 2000 dollars) is obtained from Table 1.1.6. Both tables are available at the BEA website: http://www.bea.gov/national/nipaweb/SelectTable.asp?Selected=N#S2 match the fraction of vehicles in use out of all available vehicles. According to 2001 National Household Travel Survey (NHTS) , there are about 204 million personal vehicles available for regular use in the United States.[23] Also, according to the National Transportation Statistics, there are 228 million registered vehicles in 2001.[24] The ratio of the two is 89%, which corresponds to the fractions of vehicle in use in the model.

The parameter θ indexes the cost of adjusting the fuel efficiency k. This parameter governs the evolution of fuel efficiency of new generations of vehicles. According to the estimate of the National Research Council (2002), an extra \$1000 could increase the fuel efficiency of a conventional gas-powered vehicle by between 15 and 25 percent. We calibrate θ to 0.16 to match the adjustment cost in relative terms.[25]

The parameter ω measures time spent on one unit of travel. We calibrate ω to 3.843 so that the travel time makes up for 4% of the household's

[23] The number of personal vehicles available for regular use is obtained from Highlights of the 2001 National Household Travel Survey, p8.

[24] I subtract trucks with 6 or more tires and buses from the total number of registered vehicles. The data are obtained from Table 1-11 of the National Transportation Statistics at the Bureau of Transportation Statistics. The table can be downloaded from http://www.bts.gov/publications/national_transportation_statistics/html/table_01_11.html

[25] According to the model, the marginal cost of adjusting the fuel efficiency is $\theta\left(k'-k\right)$. Assuming that the increase in the fuel efficiency, $\frac{k'^{\alpha}-k^{\alpha}}{k^{\alpha}}$, is 1%, the model specificiation implies that $\frac{\theta\left(k'-k\right)}{k}$, which is the marginal adjustment cost of 1% increase in the fuel efficiency as a fraction of the value of a new vehicle, is 0.02θ. If we use the estimate that an extra \$1000 could increase the fuel effiiciency by 15%, the average adjustment cost of 1% increase in the fuel efficiency as a fraction of the value of a new vehicle (assumed to be \$25000) would be $\frac{1000/15}{25000}$. Equating this value with 0.02θ yields the calibrated value of θ around 0.16.

discretionary time in the steady state.[26]

3.2 Parameters Related to the CAFE Standard

According to the NHTSA, the penalty for failing to meet CAFE standards is \$55 for each mile per gallon below the standard multiplied by the total volume of those vehicles manufactured for a given model year. In relative terms, the penalty amounts to a fraction of $\frac{55}{25000}$ of the value of a new vehicle per 4% of the standard[27]. We set f to 0.15 to match the fine in relative terms.[28]

3.3 Preference Specifications

We set β, the subjective discount rate, to 0.97. In the benchmark framework, we assume log-preferences for consumption, travel and leisure:

$$U\left(C_t, M_t, 1 - L_t - T_t\right) = \varphi_1 \log C_t + \varphi_2 \log M_t + (1 - \varphi_1 - \varphi_2) \log\left(1 - L_t - T_t\right).$$

[26] According to *Highlights of the 2001 National Household Travel Survey* (p.11), overall for all adults, including nondrivers and those who may not have driven in a given day, 55 minutes are spent behind the wheel a day. We can infer from Juster and Stafford (1991) that the total weekly discretionary time endowment is 146.67 hours. Based on the two estimates above, travel time makes up for 4.38 percent of the total discretionary time endowment.

[27] The current CAFE standard is around 25 miles per gallon for a combination of passenger cars and light trucks. 4% of 25 miles per gallon is exactly equal to 1 mile per gallon.

[28] Specifically, the normalized penalty for violating the CAFE standard in the model is $\frac{55}{25000} k_{ss} \times \frac{25}{\underline{X}} \left(\underline{X} - \widehat{X}\right) Q$, where k_{ss}, the steady-state capital-gasoline ratio, also represents the steady-state value of new vehicle in the model. When k_{ss} is equal to 25000 and $\underline{X}$ is equal to 25, we have the penalty standard applied in reality. The normalized penalty implies that the value of f is equal to $\frac{55}{25000} k_{ss}$ divided by $0.04\underline{X}$.

Such a specification implies that these two types of goods are not fully substitutable. The parameters φ_1 and φ_2 are calibrated to 0.34 and 0.05 respectively to match the fraction of time spent on market activities, which is 0.35,[29] and the fraction of gasoline expenditures out of output, which is 0.0163.

3.4 Gasoline Price and Productivity Level

The gasoline price is computed as the ratio of the implicit deflator for gasoline to the GDP deflator[30]. The estimation of annual real gasoline prices yields an estimated mean value of 1.0872. We use this value for the level of gasoline price in the initial steady state. The level of aggregate productivity, A, is normalized to 1.

4 Gasoline Taxes versus CAFE Standard

In this section, we use the model to compare two policy options to reduce the gasoline demand. The policy options are: gasoline taxes and the minimum

[29] Ghez and Becker (1975) and Juster and Stanfford (1991) have found that households allocate about one third of their discretionary time-i.e, time not spent sleeping or in personal maintenance-to market activities.

[30] The data series for nominal retail gasoline prices are downloaded from the web site http://www.eia.doe.gov/emeu/steo/pub/fsheets/real_prices.xls. We compute the implicit deflator for gasoline using year 2000 as the base year. The GDP implicit price deflator series are from the Department of Commerce, Bureau of Economic Analysis. The data on deflator are quarterly and seasonally adjusted. We convert the quarterly deflator series into an annual series and use year 2000 as the base year as well. The real gasoline prices are constructed as the ratio of the implicit deflator for gasoline to the GDP deflator.

CAFE standard. In the U.S., gasoline taxes are in the form of tax liabilities per unit of gasoline purchase. To relate to gasoline price elasticities, we capture changes in gasoline taxes as changes in the after-tax gasoline price. Given that gasoline taxes make up for around 11 percent of after-tax gasoline prices, a 10 percent increase in gasoline liabilities per unit of gasoline purchase is equivalent to around 1 percent increase in the after-tax gasoline price.[31]

We first examine the long-run outcome by locating the regions of prices and the minimum CAFE standards within which the minimum CAFE standard is exactly binding. We then compare the short-run and long-run dynamic paths of key endogenous variables.

4.1 The Regions of Binding CAFE Standard

The regions of binding CAFE standard are defined in Propositions 3 and 5. In Figure 2 we plot P_L and P_H respectively as a function of $\underline{X}$ under the benchmark calibration (or equivalently, $\underline{X}_L$ and $\widehat{X}^u_{ss}$ as a function of P). The vertical distance between the two curves represents the region of gasoline prices, $[P_L, P_H]$, within which the price elasticity of fuel efficiency is 0 in the long run, for a corresponding $\underline{X}$. Correspondingly, the horizontal distance between the two curves represents the region of minimum CAFE standards, $[\widehat{X}^u_{ss}, \underline{X}_L]$, within which the elasticity of fuel efficiency with respect to changes in CAFE standards is 1 in the long run, for a corresponding P.

[31] According to data from the U.S. Energy Information Administration, 11 percent out of a retail price of \$3.90 per gallon are for federal, state and local gasoline taxes. A 9.09 percent increase in tax liabilities per unit implies a 1 percent increase in the after-tax gasoline price.

For convenience, in the following numerical calculations we assume that the economy starts from point G in Figure 2. Point G represents a steady state consistent with a price level p_h, which is equal to the estimated mean value of gasoline prices, 1.087. We set the initial minimum CAFE standard $\underline{x}$ to 1.68, so that the harmonian mean of the optimal fuel efficiency at the price level p_h coincides with the minimum CAFE standard. As a result, p_h corresponds to the threshold price level P_H evaluated at the minimum CAFE standard $\underline{x}$.

As described in Propositions 3 and 5, the long-run impact of changes in gasoline prices and the minimum CAFE standard is asymmetric and nonlinear depending upon the initial starting point. For example, starting from point G in Figure 2, increases in gasoline prices have larger impact on fuel efficiency while decreases in gasoline prices may have no impact. From the same starting point, increases in the minimum CAFE standard have one-on-one effect on the fuel efficiency, while decreases in the minimum CAFE standard have no effect. However, if we start from Point R, increases in gasoline prices may have no effect on the fuel efficiency, while decreases do. From the same starting point, decreases in the minimum CAFE standard may have one-on-one effect on fuel efficiency, but increases have smaller effect.

Under the benchmark calibration, the price region of binding CAFE standard at point G is $[1.032, 1.087]$ for the given $\underline{x}$, while the corresponding region for the minimum CAFE standard is $[1.68, 1.71]$. Here p_l is lower than p_h by 5.12 percent, while $\underline{x}_l$ is higher than the initial optimal choice of $\underline{x}$ by 2.14 percent.

With the region of binding CAFE standard in mind, we study the short-run and long-run dynamic paths of key endogenous variables in response to permanent changes in gasoline prices or the minimum CAFE standard. One of the advantages of a dynamic model in contrast to a static one is its ability to generate dynamic responses of endogenous variables in response to shocks. The elasticities of key variables demonstrate important features. They are not only different in the short run versus the long run, but also are nonlinear and asymmetric, especially when the minimum CAFE standard is binding from below.

4.2 Comparison of Dynamic Paths

In this section, we compare the short-run and long-run dynamic paths of key endogenous variables after increases in gasoline prices or the minimum CAFE standard. We ask how much percentage change in the CAFE standard is required in order to reduce gasoline use by the same magnitude in the long run as a 1% permanent increase in the after-tax gasoline price. We then examine the dynamic responses of key variables under these two scenarios.

We find that given the benchmark calibration, the minimum CAFE standard $\underline{X}$ has to be increased by 0.68 percent to reduce the gasoline use by 0.5 percent in the long run, the same amount of permanent gasoline savings as a permanent 1 percent gasoline price hike would have achieved. According to the calculations in the previous section, an increase of the standard by 0.68 percent is well within the region of $[\widehat{X}^u_{ss}, \underline{X}_L]$, while a *decrease* of gasoline price by 1 percent is within the region of $[P_L, P_H]$.

As illustrated in our model, vehicle miles of travel are produced with two inputs: transportation capital and gasoline. By levying taxes on gasoline use, the gasoline tax policy raises after-tax prices and encourages higher fuel efficiency for new vehicles. CAFE standard, on the other hand, encourages higher fuel efficiency of new vehicles by imposing penalty on new vehicles with lower fuel efficiency than the minimum standard. Figure 3 sheds light on the different mechanisms through which the two policy options affect key endogenous variables. Those variables are: gasoline use, capacity utilization rate, fuel efficiency, vehicle miles of travel and household welfare.

As shown in Figure 3, gasoline use declines by 0.2 percent instantly in response to a 1 percent permanent gasoline price hike. Gasoline use continues to decline till reaching the new steady state, which represents a 0.5 percent decline from that before the gasoline price hike. These numbers correspond to a short-run price elasticity of -0.2 and a long-run price elasticity of -0.5 of gasoline demand, well within the region of plausible estimates in the empirical literature. Using a large sample of household data, Bento and et al (2009) estimate that each percentage point increase in gasoline prices leads to a reduction of between 0.25 and 0.30 percent in the equilibrium demand for gasoline. The U.S. Department of Energy (USDOE, 1996) proposes an estimate of price elasticities of -0.38. It is reassuring that our stylized model is able to endogenously generate magnitudes of price elasticities close to those empirical estimates both in the short run and in the long run.[32]

The differences in the short-run and long-run price elasticities of gasoline use stem from the putty-clay nature of the production technology of travel.

[32] Goodwin (1992) categorizes estimates of the elasticity of gasoline consumption with respect to fuel prices. He finds the average of short term price elasticities to be -0.27, and the average of the long term price elasticities to be -0.71 when time series data are used. However, much lower values are found when using data after 1990.

In the period of the gasoline price shock, both the quantity and the fuel efficiency of all existing vehicles are pre-determined, the household can only adjust the capacity utilization of these vehicles in response to the gasoline price shock. Figure 3 also displays the utilization rate of the oldest vintage in each period. Since all pre-existing vehicles have the same fuel efficiency to start with, Figure 3 shows that the capacity utilization of these vehicles decline by 0.2 percent at the period of gasoline price increase, which leads to a short-run price elasticity of -0.2. The immediate gasoline savings come from lower capacity utilization of pre-existing vehicles.

In the periods following the gasoline price shock, the household can substitute more fuel efficient vehicles to replace those obsolete, less fuel-efficient vehicles. Figure 3 shows that a one-percent gasoline price increase raises the fuel efficiency of new vehicles by around 0.2 percent in the first period, and after four periods, the fuel efficiency of new vehicles increases permanently to 0.4 percent (i.e., the value of α) above the pre-shock value. The slow adjustment reflects the cost of adjusting the fuel efficiency over time. As a result, the long-run price elasticity of gasoline use settles at -0.5 after all the pre-existing vehicles at the time of the gasoline price hike become obsolete. After N periods, the capacity utilization of the oldest vintage, which is the vintage produced at the period of the shock, increases as a result of higher fuel efficiency. The capacity utilization eventually reverts to the pre-shock steady-state value.

By contrast, the immediate gasoline savings from a higher CAFE standard is close to zero. Increases in the minimum CAFE standard have little impact on the capacity utilization of pre-existing vehicles, which primarily depends upon gasoline costs. In response to the elevation of the minimum CAFE standard by 0.68 percent, the fuel efficiency of new vehicles rises to its new permanent level immediately, reflecting that the penalty for violating the CAFE standard is a more important consideration than the cost of adjusting the fuel efficiency. As the pre-existing vehicles gradually phase out, gasoline use is reduced to its new permanent level around N years after the increase in the CAFE standard.

Higher fuel efficiency of new vehicles increases the marginal benefit of driving, while the marginal cost of gasoline remains barely changed. As a result, the capacity utilization of newly produced vehicles (which are shown in Figure 3 as the oldest vintages after the first N years) increases by 0.4 percent. This is the so-called "rebound effect". Empirical estimates by Jones (1993) and Greene et al. (1999) suggest that this "rebound" effect offsets 10 to 20 percent or more of the initial fuel reduction from tighter CAFE standard. The rebound effect explains that in order to achieve the same amount of gasoline savings in the long run, CAFE standard has to exert stronger impact on the fuel efficiency of new vehicles, since it does not have the extra channel of reducing the capacity utilization of pre-existing vehicles as a corresponding increase in gasoline prices.

The different impact of these two policy options on the capacity utilization can explain their different effects on vehicle miles of travel. Increases in gasoline prices reduce the vehicle miles of travel by around 0.1 percent. By contrast, a tightening of the CAFE standard has very little impact on vehicle miles of travel in the long run, as increases in vehicle utilization offset the adverse effect of the CAFE standard on the quantity of new generation of vehicles.

Since we abstract from the income effect by assuming lump-sum rebates of both tax revenues and the CAFE penalty, the welfare loss is entirely due to distortions in optimal decisions. In the short run, household welfare declines in response to both a gasoline price hike and an increase in the minimum CAFE standard, with the former mainly due to the decline in utility from reduced vehicle miles of travel, while the latter due to declines in consumption and leisure as a result of higher expenditure on fuel efficient vehicles and longer travel time. The magnitude of the negative welfare effect is slightly larger in the latter case due to larger weight on consumption and leisure in the benchmark utility function. However, as all endogenous variables adjust to their long-run equilibrium values, the impact of these two policy options on the welfare converges to zero.

Although increases in gasoline prices and the CAFE standard have marked effect on variables closely related to households' driving behavior, such as gasoline use and vehicle miles of travel, the effect of these two policy options on aggregate household welfare is not of the same order of magnitude (in fact, less than 0.003 percent deviation from the steady state). Given that gasoline cost and total vehicle purchase make up for respectively 1.63 and 0.97 percent of GDP, and travel time makes up for 4 percent of the household's discretionary time, it is not surprising that changes in gasoline prices and CAFE standard do not result in large welfare losses at the aggregate level.

4.3 Impact of CAFE Standard on Gasoline Price Elasticities

Not only increases in the minimum CAFE standard reduce the gasoline use, the presence of the standard alone may lead to reduced gasoline use. In this section, we consider the impact of the minimum CAFE standard on gasoline savings by comparing the dynamic effect of gasoline price decreases under two policy environments. In the first environment, there exists no minimum CAFE standard, while in the second environment, the minimum CAFE standard coincides with the optimal choice of fuel efficiency by the household, such that any decreases in gasoline prices would lead to a restrictive minimum CAFE standard.

Figure 4 compares dynamic paths of fuel efficiency of new vehicles and total gasoline usage in two policy environments after a one-percent permanent decrease in gasoline prices. As shown in the first column, the responses of gasoline use and fuel efficiency are approximately symmetric with respect to price increases or decreases when there exists no minimum CAFE standard. The nonlinearity and asymmetry of price elasticities, however, are striking in the case of permanent decreases in gasoline prices in the presence of the minimum CAFE standard. Since the minimum CAFE standard is binding from below at p_h, a decrease in the gasoline price does not lead to any changes in fuel efficiency of new vehicles in the long run unless the price level falls below p_l, which is lower than p_h by more than 5 percent under the benchmark calibration.

Since the fuel efficiency of new vehicles remain essentially unchanged when the level of gasoline prices declines by only 1 percent, the only channel through which gasoline price decreases can affect gasoline use in both the short and long run is through increasing utilization of pre-existing vehicles. As a result, in response to a 1 percent decline in gasoline prices, the gasoline use increases by approximately 0.2 percent on the initial impact and barely increases beyond this level in the long run. Considering that gasoline use would have increased by 0.5 percent absent of the minimum CAFE standard, it is fair to say that the presence of the minimum CAFE standard avoids around 60 percent of the increase in gasoline consumption which would have occurred absent of such a standard. The total amount of gasoline savings decreases as more drastic decreases in gasoline prices would lead to larger adjustments of vehicle fuel efficiency.

5 Conclusion

By incorporating a putty-clay specification in the "production" technology of travel, the dynamic general equilibrium model developed in this paper provides a rich framework for analyzing the endogenous determination of gasoline use, driving and vehicle fuel efficiency. The model is able to generate endogenously different short-run and long-run price elasticities of gasoline use, with their magnitudes well within the region of plausible estimates in the empirical literature. The model also demonstrates that when the minimum CAFE standard is binding from below, decreases in gasoline prices lead to muted or dampened responses in vehicle fuel efficiency, which results in gasoline savings in the long run.

The paper shows that gasoline taxes and increases in the CAFE standard achieve gasoline savings through different mechanisms, and thus have different impact on vehicle utilization, fuel efficiency and gasoline use. The model shows that the effectiveness of raising the minimum CAFE standard not only depends upon the region of its variations, but also upon the current and expected level of gasoline prices. As compared to gasoline taxes, increases in the CAFE standard may not lead to a reduction in vehicle miles of travel. As a result, it may have different environmental impact.

The paper features a representative household which is both a consumer and a vehicle producer. It would be fruitful to decentralize the roles of consumers and vehicle producers, and examine the implications of various policy options. The findings in this paper provide a benchmark framework for the ongoing research.

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References

[1] Atkeson, A. and P. Kehoe, 1999, Models of Energy Use: Putty-Putty versus Putty-Clay, American Economic Review, 89 (4), 1028-1043.

[2] Austin, D. and T. Dinan, 2005, Clearing the Air: The Costs and Consequences of Higher CAFE Standards and Increasing Gasoline Taxes, Journal of Environmental Economics and Management 50: 562-582.

[3] Bento, A., L. Goulder, M. Jacobsen, and R. Von Haefen, 2009, Distributional and Efficiency Impacts of Increased U.S. Gasoline Taxes, American Economic Review, 99 (3), 667-699.

[4] Bresnahan, Timothy and Valerie Ramey, 1993, Segment Shifts and Capacity Utilization in the U.S. Automobile Industry, American Economic Review, 83, 213-218

[5] Congressional Budget Office, 2002, Reducing Gasoline Consumption: Three Policy Options, Washington, DC.

[6] Ghez and Becker, 1975, The Allocation of Time and Goods over the Life Cycle, published by the NBER.

[7] Gilchrist, S.and J.Williams, 2000, Putty-Clay and Investment: A Business Cycle Analysis, Journal of Political Economy, 108 (5), 928-960.

[8] Gilchrist, S.and J.Williams, 2005, Investment, Capacity and Uncertainty: A Putty-Clay Approach, Review of Economic Dynamics 8.

[9] Goldberg, P. 1998. The Effects of the Corporate Average Fuel Economy Standards in the U.S., Journal of Industrial Economics, 46: 1-33.

[10] Goodwin, P.B. 1992, A Review of New Demand Elasticities with Special with Special Reference to Short and Long Run Effects of Price Changes, Journal of Transportation Economics and Policy, 26 (2) 155-169.

[11] Greene, D.L., J.R. Kahn, and R.C. Gibson, 1999, Fuel Economy Rebound Effect for U.S. Household Vehicles, The Energy Journal 20: 1-31.

[12] Hamilton, James, 2009, Causes and Consequences of the Oil Shock of 2007-2008, Working Paper, UCSD.

[13] Highlights of the 2001 National Household Travel Survey, U.S. Department of Transportation, Bureau of Transportation Statistics

[14] Jacobsen, Mark, 2007, Evaluating U.S. Fuel Economy Standards in a Model with Producer and Household Heterogeneity, Ph.D. Dissertation, Stanford University, Stanford, CA.

[15] Jones, C. 1993, Another Look at U.S. Passenger Vehicle Use and the "Rebound Effect" From Improved Fuel Efficiency, The Energy Journal 14: 99-110.

[16] Juster and Stafford, 1991, The Allocation of Time: Empirical Findings, Behavioral Models and Problems of Measurement, Journal of Economic Literature, June 1991, 29(2), 471-522.

[17] Li, Shanjun, Roger Haefen and Christopher Timmins, 2008, How Do Gasoline Prices Affect Fleet Fuel Economy? NBER Working Papers 14450.

[18] National Research Council, 2002, Effectiveness and Impact of Corporate Average Fuel Economy (CAFE) Standards, Washington, DC: National Academy Press

[19] National Transportation Statistics, 2009, Bureau of Transportation Statistics, U.S. Department of Transportation.

[20] Parry, Ian and Kenneth Small, 2005, Does Britain or the United States Have the Right Gasoline Tax? American Economic Review, 95 (4)..

[21] Ramey, Valerie, and Daniel Vine, 2010, Oil Shocks, Segment Shifts and Capacity Utilization in the U.S. Automobile Industry: What Has Changed in 30 Years?", NBER Macroeconomic Annual, Vol 25..

[22] Small, K., and K. Van Dender. 2007. Fuel Efficiency and Motor Vehicle Travel: The Declining Rebound Effect, Energy Journal 28: 25-51.

[23] U.S. Department of Energy, 1996, Policies and Measures for Reducing Energy Related Greenhouse Gas Emissions: Lessons from Recent Literature, Report No. DOE/PO-0047.

[24] Wei, Chao, 2003, Energy, the Stock Market, and the Putty-Clay Investment Model, American Economic Review, 93 (1), 311-323.

Appendix

A Proof for Proposition 1

In the steady state, equation (11) becomes

$$k_{ss}^{u}=\alpha\frac{\mu_{ss}^{u}}{\lambda_{ss}^{u}}\widetilde{d}\left[1-\Phi\left(z_{ss}^{u}-\sigma\right)\right]X_{ss}^{u},\text{ where }\widetilde{d}=\sum_{s=1}^{N}\left[\beta^{s}\left(1-\delta\right)^{s-1}\right].\qquad\text{(A.1)}$$

Equation (A.1) states that k, the value of the transportation capital embodied in each vehicle, is proportional to the present discounted value of the expected travel during the vehicle's life span.

Combining equations (11) and (12) in the steady state, we have equation (18). Combining equations (13), (A.1) and (18), we can obtain equation (17).

B Proof for Propositions 3 and 4

We first prove the following proposition.

Proposition 6 *Define $\overline{f}$ as the unique value of f that solves the following equation,*

$$\left(\underline{X}e^{\sigma^{2}}\right)^{\frac{1}{\alpha}}=\frac{\alpha}{1-\alpha}\left\{\widetilde{d}P\left[1-\Phi\left(\widetilde{Z}\left(\overline{f},P,\underline{X},\Theta\right)\right)\right]+\overline{f}\,\underline{X}\right\}.\qquad\text{(B.1)}$$

where $\widetilde{Z}\left\{f,P,\underline{X},\Theta\right\}$ is defined by the following implicit function

$$f\alpha\underline{X}=\widetilde{d}P\left[1-\Phi\left(\widetilde{z}\right)\right]\left\{\left(1-\alpha\right)\frac{\phi\left(\widetilde{z}\right)\left[1-\Phi\left(\widetilde{z}-\sigma\right)\right]}{\phi\left(\widetilde{z}-\sigma\right)\left[1-\Phi\left(\widetilde{z}\right)\right]}-1\right\}.\qquad\text{(B.2)}$$

Here $\widetilde{z}$ is the value of z^{c} that satisfies equation (22) when $\widehat{X}$ becomes infinitely close to $\underline{X}$ from below, and Θ represents the rest of the model parameters.

(i) If $f \leq \overline{f}$, the optimal values of X^c_{ss} and z^c_{ss} satisfy equations (21) and (22). The optimal fuel efficiency thus chosen is below that corresponding to the minimum CAFE standard, but above the nonrestrictive optimal choice X^u_{ss}.

(ii) If $f > \overline{f}$, the household chooses to comply with the minimum CAFE standard by setting X^c_{ss} to $\underline{X}e^{\sigma^2}$ and setting z^c_{ss} to satisfy the following equation

$$\left(\underline{X}e^{\sigma^2}\right)^{\frac{1}{\alpha}} = \widetilde{d}P\left[1 - \Phi\left(z^c_{ss}\right)\right]\left\{\frac{\phi\left(z^c_{ss}\right)\left[1 - \Phi\left(z^c_{ss} - \sigma\right)\right]}{\phi\left(z^c_{ss} - \sigma\right)\left[1 - \Phi\left(z^c_{ss}\right)\right]} - 1\right\}. \qquad \text{(B.3)}$$

(iii) The penalty threshold $\overline{f}$ is a function of $\{P, \underline{X}, \Theta\}$. It depends negatively upon the gasoline prices P and positively on the minimum CAFE standard $\underline{X}$. That is, $\frac{\partial \overline{f}}{\partial P} < 0, \frac{\partial \overline{f}}{\partial \underline{X}} > 0$.

Proof. Equation (B.1) is obtained by substituting $X^c = \underline{X}e^{\sigma^2}$ and $z^c = \widetilde{Z}\left(\overline{f}, P, \underline{X}, \Theta\right)$ into equation (21). Thus when f is equal to $\overline{f}$, the lower segments of equations (21) and (22) intersect at X^c_{ss}, which is infinitely close to $\underline{X}e^{\sigma^2}$ from below. Since the right hand side of equation (B.2) is decreasing in $\widetilde{z}$, an increase in f reduces the value of $\widetilde{z}$, and shifts the lower segment of the upward sloping curve to the left. In the meantime, an increase in f also pushes the lower segment of the downward sloping curve to the right. When f is higher than $\overline{f}$, these two curves no longer intersect with each other. Thus $\overline{f}$ is the minimum unit penalty required for compliance with the minimum CAFE standard.

If $f > \overline{f}$, the household chooses to comply with the minimum CAFE standard by setting X_{ss}^c to $\underline{X}e^{\sigma^2}$. The binding constraint implies that equation (11), the first order condition with respect to k, no longer holds with equality. However, equations (13) and (12) still hold in the steady state. When we substitute the steady-state value of $\frac{\mu_{ss}^c}{\lambda_{ss}^c}$ as defined in the steady-state version of (13) into its steady-state counterpart of (12), and set the level of fuel efficiency to $\underline{X}e^{\sigma^2}$, we obtain equation (B.3). ■

The minimum price level $P_L(f, \underline{X}, \Theta)$ as defined in Proposition 3 is effectively the gasoline price level that solves the implicit function

$$f = \overline{f}\left(P_L, \underline{X}, \Theta\right),$$

where $\overline{f}\left(\cdot\right)$ is the implicit function of $\overline{f}$ defined in equation (B.1). The proposition that P_L is an increasing function of $\underline{X}$ follows directly from the results $\frac{\partial \overline{f}}{\partial P} < 0$ and $\frac{\partial \overline{f}}{\partial \underline{X}} > 0$, as proved in Appendix D.
Since $\overline{f}$ depends negatively upon P, the region $P \leq P_L$ corresponds to $f \leq \overline{f}$, $P = P_L$ corresponds to $f = \overline{f}$, and $P > P_L$ corresponds to $f > \overline{f}$. Proposition 3 follows once the correspondence is established.

Since $\frac{\partial \overline{f}}{\partial P} < 0$, an increase in f lowers P_L. The rest of proofs for Proposition 4 follows directly from Proposition 3.

C Proof for Proposition 5

The minimum price level $\underline{X}_L(f, P, \Theta)$ as defined in Proposition 5 is effectively the minimum CAFE standard that solves the implicit function

$$f = \overline{f}\left(P, \underline{X}_L, \Theta\right),$$

where $\overline{f}\,(\cdot)$ is the implicit function of $\overline{f}$ defined in equation (B.1). Since $\overline{f}$ depends positively upon $\underline{X}$, the region $\underline{X} \geq \underline{X}_L$ corresponds to $f \leq \overline{f}$, $\underline{X} = \underline{X}_L$ corresponds to $f = \overline{f}$, and $\underline{X} < \underline{X}_L$ corresponds to $f > \overline{f}$. Proposition 5 follows from Proposition 3 once the correspondence is established.

D Proof for $\frac{\partial \overline{f}}{\partial P} < 0$ and $\frac{\partial \overline{f}}{\partial \underline{X}} > 0$

A. Proofs for $\frac{\partial \overline{f}}{\partial P} < 0$.

The proofs below make use of Result 4 in the appendix of Gilchrist and Williams (2005), which shows that

$$\frac{\partial \left\{ \frac{h(z)}{h(z-\sigma)} \right\}}{\partial z} = -\frac{\left\{ h\left(z-\sigma\right) - \left(z-\sigma\right) - \left[h\left(z\right) - z\right] \right\}}{h\left(z-\sigma\right)/h\left(z\right)} < 0, \tag{D.1}$$

where $h\left(z\right) = \frac{\phi(z)}{1-\Phi(z)}$. Applying the implicit function theorem to equation (B.1), we have

$$\frac{\partial \overline{f}}{\partial P} = -\frac{\left\{ \widetilde{d} \left[1 - \Phi\left(\widetilde{z}\right) - P\phi\left(\widetilde{z}\right) \frac{\partial \widetilde{z}}{\partial P} \right] \right\}}{\left[-\widetilde{d} P \phi\left(\widetilde{z}\right) \frac{\partial \widetilde{z}}{\partial f} + \underline{X} \right]}. \tag{D.2}$$

Since the right hand side of equation (B.2) decreases with $\widetilde{z}$ as a result of equation (D.1), we have $\frac{\partial \widetilde{z}}{\partial f} < 0$. In order for $\frac{\partial \overline{f}}{\partial P}$ to be negative, we only need to prove that $1 - \Phi\left(\widetilde{z}\right) - P\phi\left(\widetilde{z}\right) \frac{\partial \widetilde{z}}{\partial P} > 0$.

Applying the implicit function theorem to equation (B.2) and again making use of equation (D.1), we have

$$P\phi\left(\widetilde{z}\right) \frac{\partial \widetilde{z}}{\partial P} = -\frac{\left[1 - \Phi\left(\widetilde{z}\right)\right] \left\{ \left(1-\alpha\right) \frac{\phi(\widetilde{z})\left[1-\Phi(\widetilde{z}-\sigma)\right]}{\phi(\widetilde{z}-\sigma)\left[1-\Phi(\widetilde{z})\right]} - 1 \right\}}{\alpha - \left(1-\alpha\right) \sigma \frac{1-\Phi(\widetilde{z}-\sigma)}{\phi(\widetilde{z}-\sigma)}}. \tag{D.3}$$

Since $\frac{\partial\widetilde{z}}{\partial P} > 0$ as a result of equation (D.1), and since the numerator of equation (D.3) is positive, the denominator, $\alpha - (1-\alpha)\,\sigma\frac{1-\Phi(\widetilde{z}-\sigma)}{\phi(\widetilde{z}-\sigma)}$, is negative. After algebraic manipulations, we have

$$1-\Phi\left(\widetilde{z}\right)-P\phi\left(\widetilde{z}\right)\frac{\partial\widetilde{z}}{\partial P}=(1-\alpha)\left[1-\Phi\left(\widetilde{z}\right)\right]\frac{\left\{\frac{\phi(\widetilde{z})[1-\Phi(\widetilde{z}-\sigma)]}{\phi(\widetilde{z}-\sigma)[1-\Phi(\widetilde{z})]}-1-\sigma\frac{1-\Phi(\widetilde{z}-\sigma)}{\phi(\widetilde{z}-\sigma)}\right\}}{\left[\alpha-(1-\alpha)\,\sigma\frac{1-\Phi(\widetilde{z}-\sigma)}{\phi(\widetilde{z}-\sigma)}\right]}. \tag{D.4}$$

Since the denominator is negative, in order for $1-\Phi\left(\widetilde{z}\right)-P\phi\left(\widetilde{z}\right)\frac{\partial\widetilde{z}}{\partial P}$ to be positive, we only need to prove that $\frac{\phi(\widetilde{z})[1-\Phi(\widetilde{z}-\sigma)]}{\phi(\widetilde{z}-\sigma)[1-\Phi(\widetilde{z})]}-1-\sigma\frac{1-\Phi(\widetilde{z}-\sigma)}{\phi(\widetilde{z}-\sigma)} < 0$. This proof follows from Result 4 of GW (2005), which states that

$$h\left(\widetilde{z}\right)-\left(\widetilde{z}\right)<h\left(\widetilde{z}-\sigma\right)-\left(\widetilde{z}-\sigma\right). \tag{D.5}$$

B. Proofs for $\frac{\partial\overline{f}}{\partial\underline{X}} > 0$.

Applying the implicit function theorem to equation (B.1), we have

$$\frac{\partial\overline{f}}{\partial\underline{X}}=-\frac{\frac{\alpha}{1-\alpha}\left[-\widetilde{d}P\phi\left(\widetilde{z}\right)\frac{\partial\widetilde{z}}{\partial\underline{X}}+\overline{f}\right]-\frac{1}{\alpha}\left(\underline{X}e^{\sigma^2}\right)^{\frac{1}{\alpha}-1}e^{\sigma^2}}{\frac{\alpha}{1-\alpha}\left[-\widetilde{d}P\phi\left(\widetilde{z}\right)\frac{\partial\widetilde{z}}{\partial\overline{f}}+\underline{X}\right]}. \tag{D.6}$$

Applying the implicit function theorem to equation (B.2) and making use of equation (D.3), we have

$$\frac{\partial\widetilde{z}}{\partial\underline{X}}=\frac{\overline{f}\,\alpha}{\widetilde{d}P\phi\left(\widetilde{z}\right)\left[\alpha-(1-\alpha)\,\sigma\frac{1-\Phi(\widetilde{z}-\sigma)}{\phi(\widetilde{z}-\sigma)}\right]}. \tag{D.7}$$

Substituting this expression into the numerator of $\frac{\partial\overline{f}}{\partial\underline{X}}$ and considering that the denominator of $\frac{\partial\overline{f}}{\partial\underline{X}}$ is positive as shown above, we need to prove

$$\frac{1}{\alpha}\left(\underline{X}e^{\sigma^2}\right)^{\frac{1}{\alpha}-1}e^{\sigma^2}-\frac{\overline{f}\,\alpha}{1-\alpha}\left\{1-\frac{\alpha}{\alpha-(1-\alpha)\,\sigma\frac{1-\Phi(\widetilde{z}-\sigma)}{\phi(\widetilde{z}-\sigma)}}\right\}>0 \tag{D.8}$$

for $\frac{\partial \overline{f}}{\partial \underline{X}}$ to be positive. Multiplying both sides of equations (B.1) and (B.2) by $\underline{X}$ and substituting the right hand sides of those two equations respectively into the first and second terms on the left hand side of (D.8), we have

$$\begin{aligned}
& \frac{1}{\alpha}\left(\underline{X}e^{\sigma^2}\right)^{\frac{1}{\alpha}-1} e^{\sigma^2} - \frac{\overline{f}\,\alpha}{1-\alpha}\left\{1 - \frac{\alpha}{\alpha-(1-\alpha)\,\sigma\frac{1-\Phi(\widetilde{z}-\sigma)}{\phi(\widetilde{z}-\sigma)}}\right\} \\
= \;& \frac{\widetilde{d}P\left[1-\Phi\left(\widetilde{z}\right)\right]}{(1-\alpha)}\left\{1+\left[(1-\alpha)\frac{h\left(\widetilde{z}\right)}{h\left(\widetilde{z}-\sigma\right)}-1\right]\left[\frac{1}{\alpha}-1+\frac{\alpha}{\alpha-(1-\alpha)\,\sigma\frac{1-\Phi(\widetilde{z}-\sigma)}{\phi(\widetilde{z}-\sigma)}}\right]\right\} \\
= \;& \frac{\widetilde{d}P\left[1-\Phi\left(\widetilde{z}\right)\right]\left[\alpha h\left(\widetilde{z}\right)+(1-2\alpha)\,\sigma-(1-\alpha)^2\,\sigma\frac{h(\widetilde{z})}{h(\widetilde{z}-\sigma)}-\alpha h\left(\widetilde{z}-\sigma\right)\right]}{(1-\alpha)\,\alpha\left[\alpha h\left(\widetilde{z}-\sigma\right)-(1-\alpha)\,\sigma\right]}. \qquad \text{(D.9)}
\end{aligned}$$

Since the denominator is negative as shown above, we only need to prove

$$\alpha h\left(\widetilde{z}\right)+(1-2\alpha)\,\sigma-(1-\alpha)^2\,\sigma\frac{h\left(\widetilde{z}\right)}{h\left(\widetilde{z}-\sigma\right)}-\alpha h\left(\widetilde{z}-\sigma\right)<0 \qquad \text{(D.10)}$$

for $\frac{\partial \overline{f}}{\partial \underline{X}}$ to be positive. We proceed in two steps. First, we show that if we substitute z_{ss}^u for $\widetilde{z}$, the left hand side of (D.10) is negative. Then we prove that the left hand side of (D.10) is monotonically increasing. Since any $\widetilde{z}$ is lower than z_{ss}^u by definition, we can prove the inequality in (D.10) after completion of those two steps.

Now we proceed to step one by substituting z_{ss}^u for $\widetilde{z}$. Given equation (17), the left hand side of (D.10) becomes $\alpha^2 h\left(z_{ss}^u\right)-\alpha\sigma$. Gilchrist and Williams (2005) prove in their proposition 2 that $\alpha^2 h\left(z_{ss}^u\right)-\alpha\sigma$ is negative. Now we take derivative with respect to the left hand side of (D.10). The derivative is given by $\alpha\left[h'\left(\widetilde{z}\right)-h'\left(\widetilde{z}-\sigma\right)\right]-(1-\alpha)^2\,\sigma\frac{\partial\left[\frac{h(\widetilde{z})}{h(\widetilde{z}-\sigma)}\right]}{\partial\widetilde{z}}$. Results 3 and 4 of Gilchrist and Williams (2005) respectively state that the function $h\left(z\right)$ is convex and $\frac{\partial\left[\frac{h(z)}{h(z-\sigma)}\right]}{\partial z}$ is negative, consequently the derivative of the left hand side of (D.10) is positive.

Table 1: Benchmark Calibration

Category	*Parameter Definition*	*Notation*	*Value*
Travel and Gasoline Use	Maximum life span of vehicles	N	15
	Depreciation rate	δ	0.10
	Exponent of transportation capital	α	0.42
	std. of idiosyncratic uncertainty	σ	0.3403
	adjustment cost index	θ	0.16
	Inverse of travel speed	ω	3.84
CAFE Standard	Unit penalty	f	0.15
Preferences	Utility weight on consumption	φ_1	0.34
	Utility weight on mobility (VMT)	φ_2	0.05
	Subjective time preference	β	0.97
Initial Gasoline Price			1.0872

Figure 1: Comparison of Long-Run Equilibria

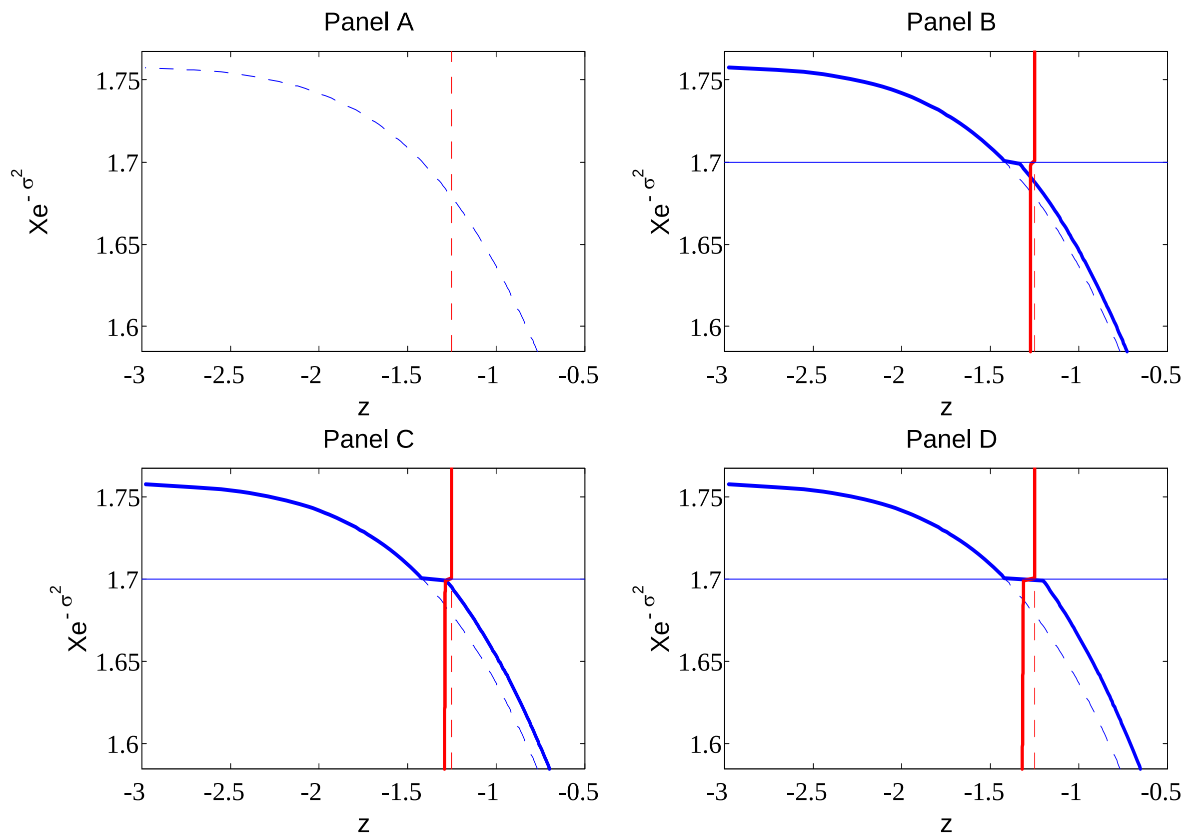
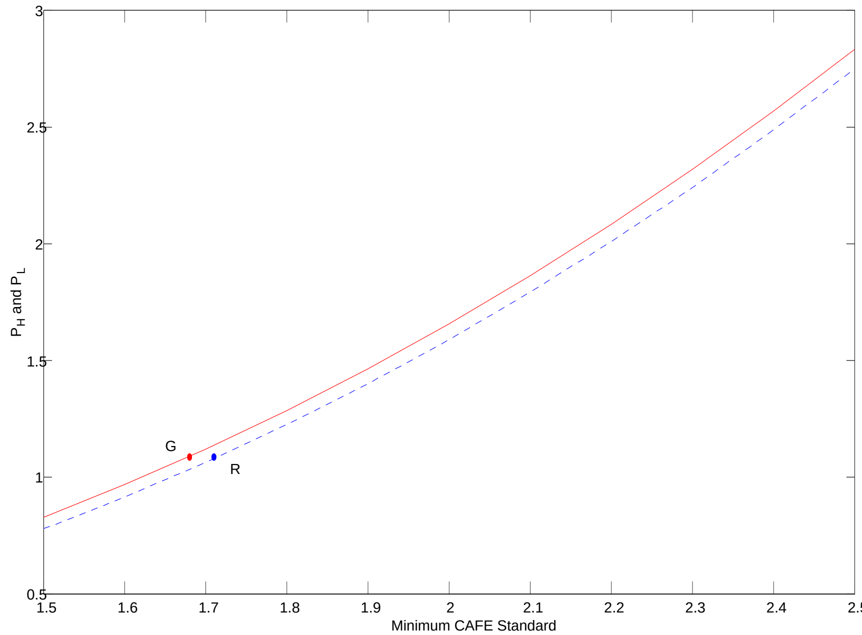


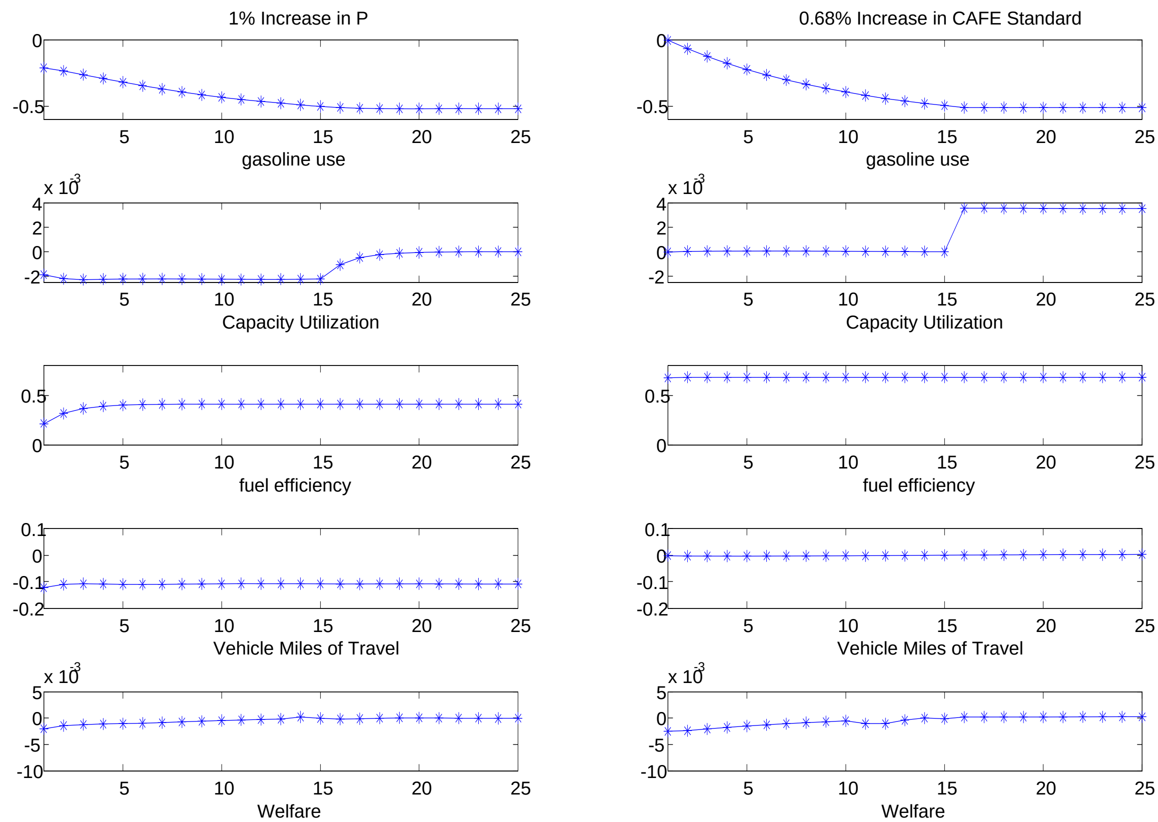
Figure 1 compares long-run equilibria with or without the minimum CAFE standard. Panel A represents the case where there is no minimum CAFE standard. Panel B to D represent the case where there exists a minimum CAFE standard. In Panel B, $P < P_L$. In Panel C, $P = P_L$. In Panel D, $P_L < P < P_H$. In all the panels, dashed lines demonstrate where the lower segments of the two curves would be without the CAFE standard, while solid lines demonstrate the case where the minimum CAFE standard is restrictive. The minimum CAFE standard $\underline{x}$ is set at 1.7 in Panels B to D.

Figure 2: The Region of Binding CAFE Standard



The solid curve represents P_H and the dashed curve represents P_L for the corresponding minimum CAFE standard.

Figure 3: Comparison of Two Policy Options



The left and right columns display respectively the effect of a one-percent increase in gasoline prices and a 0.68-percent increase in the minimum CAFE standard on key endogenous variables. The first to the fifth rows display respectively the dynamic paths of gasoline use (O), capacity utilization of the oldest vintage $(1\text{-}\Phi(z_t^{t-N}))$, fuel efficiency (x), vehicle miles of travel (M), and household welfare.

Figure 4: Asymmetric Dynamic Paths

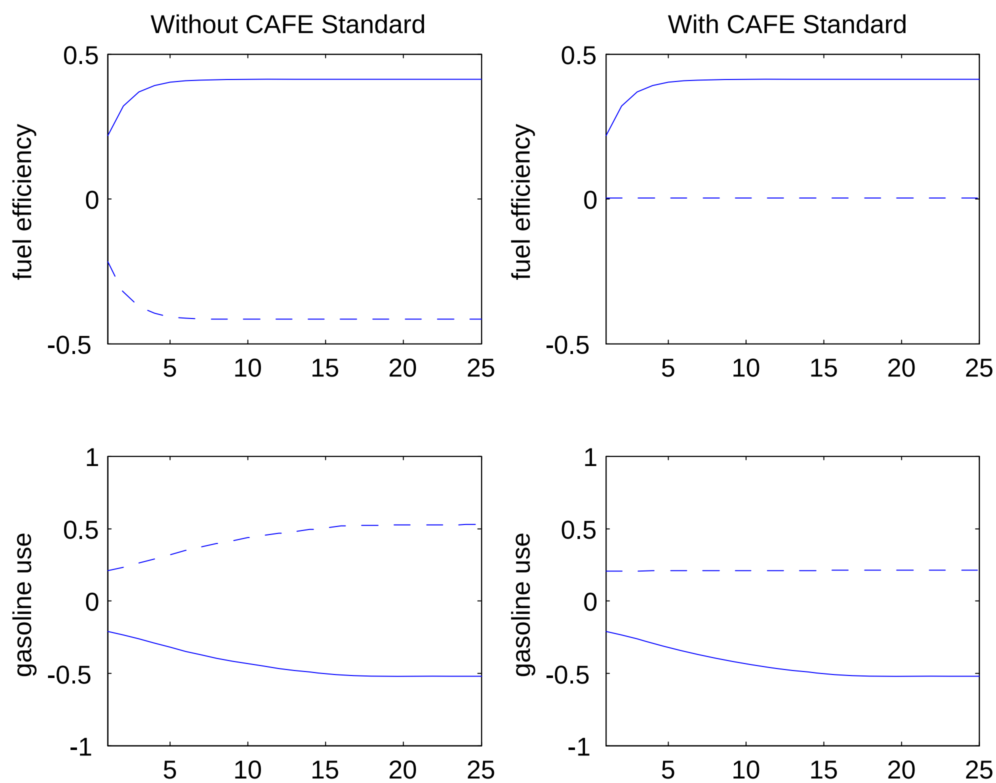


Figure 4 displays dynamic responses of fuel efficiency (the first row) and gasoline use (the second row) to permanent increases (solid line) or decreases (dashed line) in gasoline prices by 1% under two policy environments. The first column represents the case without a minimum CAFE standard, while the second column represents the case where a minimum CAFE standard is binding from below.