

# An Introduction to Punchscan

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## Abstract

Punchscan is a precinct-read optical-scan balloting system that allows voters to take their ballot with them after scanning. This does not violate the secret ballot principle because the ballots cannot be read without secret information held by the authority in charge of the election. In fact, this election authority will publish the ballots for everyone to see, allowing voters whose ballots were incorrectly omitted to complain. Punchscan vote-counting is performed in private by the Election Authority – who uses their secret information to decode the ballots – but is verified in public by an auditor.

In this paper we describe how and why PunchScan works. We have kept most of the description at an outline level so that it may be used as a straw model of a cryptographic voting system.

This paper was also presented in June 2006, at VSRW06 in Washington DC.

## 1 Motivation

The accurate results of a democratic election are at the heart of any modern society. Democracies are built throughout the world with the commitment to have elected individuals representing the entire population of a nation. To be able to record the wish of the people accurately we need to have a voting system that is transparent, reliable and verifiable. We need to be able to prove that the elections run correctly, that every vote counts, and that the every person going to the polls and exercising their right to vote can make a difference. At the same time, we have to respect the secret nature of any vote. Linking a voter to a vote should not be possible, with or without the complicity of the voter.

PunchScan is a novel voting system and extremely easy to use, both by the voter and by the people running the elections. It is transparent and reliable, and provides public verifiability, election integrity and enhanced voter privacy.

## 2 Key elements/Ideas

There are two key elements that make PunchScan work:

1. The ballot is made out of two separate pages. When the two pages are put together, the resulting ballot reveals the choices of the voter. When only one page is viewed, it gives no information – in the computational sense – about what candidates the voter chose. Thus, if one page of the ballot is destroyed, the voter can keep the other page, without violating ballot secrecy.
2. A mechanism that allows the recovery of the candidate choices from only one page of the ballot
3. A proof of the integrity of the election, through pre and post election audits.

These ideas are common both to PunchScan, and to a previous method of David Chaum's [?]. However, PunchScan is more practical, because it does not suffer from the perfect alignment problem of the previous method, because the cryptography used is simpler, and because the time required to find the result and obtain the integrity proof is smaller.

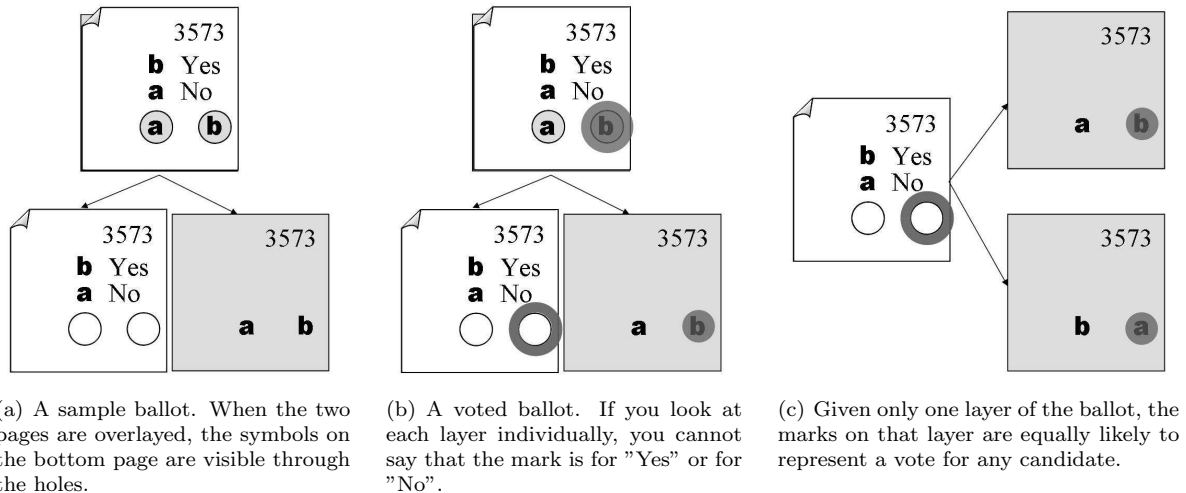


Figure 1: PunchScan's ballot

### 3 High-level system design

PunchScan achieves publicly verifiable integrity by a simple blend of paper voting and optical scan. It gives each voter the opportunity to take their vote home and check that it is counted in the final tally. In this section, we first describe the ballot itself, then we present all the phases of the voting process as seen by all the participants: voters, the election authority, and candidates.

We assume that the candidates are auditing the election, since they are the ones that should care most about a correct outcome; in particular, each candidate would want to check that his rightful votes were not given to another candidate.

#### 3.1 Ballot design

A ballot consists of a single sheet of paper, transversally folded in half to form two pages. The top page of the ballot has holes in it, and the information on the bottom page can be read through the holes. The top page also contains all the text needed on the ballot, such as contests (i.e.: ballot questions) and the candidates' names. Every answer has a symbol assigned to it and the assignment of symbols to answers varies from ballot to ballot. On the bottom page of the ballot, there is an (apparently) unordered list of symbols and their order differs from ballot to ballot. The top and the bottom ballot pages are aligned in such a way that when they are overlaid, for every question on the ballot, the symbols from the bottom page are visible through the holes made on the top page (see figure 1(a)).

In PunchScan, the voter uses a dauber to mark the selection of candidates. A dauber is a pen that leaves a disk of ink on the paper when it makes contact, just like the ones used by Bingo players to mark the numbers on their tickets. The diameter of the ink disc is greater than the diameter of the hole punched through the top page, which means the dauber leaves a mark on both the top and bottom ballot pages. Figure 1(b) contains a ballot voted for "Yes".

Because the order of the symbols on the two pages of a ballot is different (and independent), one cannot determine which mark is for which candidate by viewing only one page. We assume that the association of candidates with symbols and the order of the symbols on the bottom page are uniformly random. Figure 1(c) has the right answer selected on the top layer; depending on which possible bottom layer is this ballot's actual bottom layer, that mark could represent a vote for "Yes" or a vote for "No", both with a probability of 50%.

#### 3.2 Chronological description

There are three phases of the voting:

- the preelection phase (labeled B for *Before*)
- the election day (labeled E for *Election*)

- the post election phase (labeled A for *After*)

### 3.2.1 The preelection phase

The preelection phase is a preparatory one, allowing the setup of the election and allowing integrity proofs to be carried out. During the preelection phase, the ballots are generated, printed and audited. Also, the information that allows recovering the choice from one page of the ballot is generated and checked. The chronological order is the following:

- B.1 The election authority generates ballots and commits to them.
- B.2 The election authority generates and commits to the information necessary for decrypting one page of the ballot when the other one is destroyed.
- B.3 The candidates challenge the election authority and ask to see some of the ballots (say half), along with the information from B.2.
- B.4 The election authority provides the requested ballots, and opens the commitments associated with them, thus spoiling them.
- B.5 The candidates check to ensure that the commitments are consistent with the opened ballots.

### 3.2.2 Election day

On election day, the voters go to their assigned polling places, authenticate themselves as legitimate voters, and get a ballot from the election officials.

- E.1 The voter marks his or her favorite candidates on the ballot
- E.2 The voter chooses one ballot page to keep. The other one is destroyed
- E.3 The surviving page is scanned, and the marks are recorded and made public. Henceforth, all references to “ballot” will refer to this surviving page.

### 3.2.3 The post election phase

After all the polls close, the election is audited and proofs carried out to ensure the integrity of the election. The chronological order of the events following an election is as follows:

- A.1 Any voter can go to the election authority web site, enter a serial number for her ballot, check that the ballot is there and that it accurately resembles the page she possesses.
- A.2 The election authority processes all ballots to produce decrypted versions, along with an intermediary (partially decrypted) form of all the ballots
- A.3 The candidates ask to see some of the transformations from the original ballots to the intermediary forms, and some of the transformation from the intermediary form to the clear form.
- A.4 The election authority replies to the challenges made by the candidates in A.3
- A.5 The candidates check to see if the reply of the election authority is consistent with the commitments made in the preelection phase [B.2], and with the information made public in [A.2].

## 4 Description by roles

### 4.1 The voter

On Election Day, the voter comes to the assigned polling place and authenticates herself as a legitimate voter. She gets a ballot and a dauber and enters a private polling booth. She chooses her favorite candidates by making a mark with the dauber on the symbol associated with the candidate. She then shreds one of the pages of the ballot, and keeps the other one. Then, she scans this page. She may walk out of the polling place with this page, which serves as her (encrypted) receipt. Later, she can go to a web site, type in the serial number of her ballot and check that the ballot is there. No other checks are required from the voter.

$\oplus$	Non Flipped	Flipped
Non Flipped	Non Flipped	Flipped
Flipped	Flipped	Non Flipped

Table 1: Flipped / Non Flipped logic

## 4.2 The election authority (EA)

In the preelection phase, EA decides the format of a canonical ballot. This is the one from which all the other ballot variants will be generated. Also, the canonical ballot is used to recover the choices of the voters, after one page of the ballot has been destroyed.

EA generates at least twice the number of ballots needed in the election, and commits to them (making the commitment public; the ballots themselves remain secret). It also generates and commits to information necessary to recover the intent of a voter from one page of the ballot.

In response to the preelection challenge, [B.3], EA discloses all the information about half of the ballots (thus spoiling them). This allows the candidates to check the commitments and ensures, with high probability, that all the ballots have been correctly generated.

After the election, EA posts partially decrypted ballots and clear text ballots. To prove that the decryption (the partial one and the final one) was done correctly, for each vote EA will reveal either how it transformed the voted ballot into a partially decrypted one, or how it transformed a partial decrypted ballot into a clear text one, but not both for the same ballot. The auditors choose which part will be revealed, and the chances of a cheating EA being detected grow exponentially with the number of votes cheated on.

## 4.3 The candidates

We assume that the candidates are competing in an election. Because of this, we can safely allow the candidates also to play the role of auditors. As auditors, the candidates challenge EA during preelection and post election and check that the replies are consistent with the commitments.

## 5 An Example

We describe a minimal example: the election consists of a single binary contest; the voters vote “Yes” or “No”. The EA decides that, in the canonical ballot, the symbol “a” is associated with “Yes” and the symbol “b” with “No” on the top page. The EA also decides that the order is “a” “b” on the bottom page. The canonical ballot is presented in figure 2. The EA defines what is a shift of one from the canonical form on top and bottom pages. The canonical ballot corresponds to a shift of 0 (call it a non-flipped ballot) and the non-canonical ballot corresponds to a shift of one (call it a flipped ballot). Figure 3(a) contains all the possible top and bottom pages. Any top page can be combined with any bottom page to give out a ballot as seen in figure 3(b). The four types of ballots are equally likely.

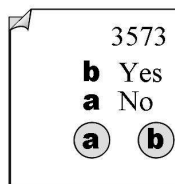


Figure 2: The canonical ballot on a Yes/No contest

A non-flipped top page combined with a flipped bottom page results in a flipped ballot. All the possibilities are in table 1. Note that we are only interested in knowing if the entire ballot is flipped or not, not individual pages.

To decrypt one page of the ballot, it is necessary to know if it came from a flipped or non-flipped ballot, to know if it should be flipped or not to get the canonical ballot. In Punchscan, this information is split into two flip/non-flip operations (flip1 and flip2) for each ballot. These operations, combined, will take the ballot page to the canonical ballot. The information is split so that one half of the split information can be

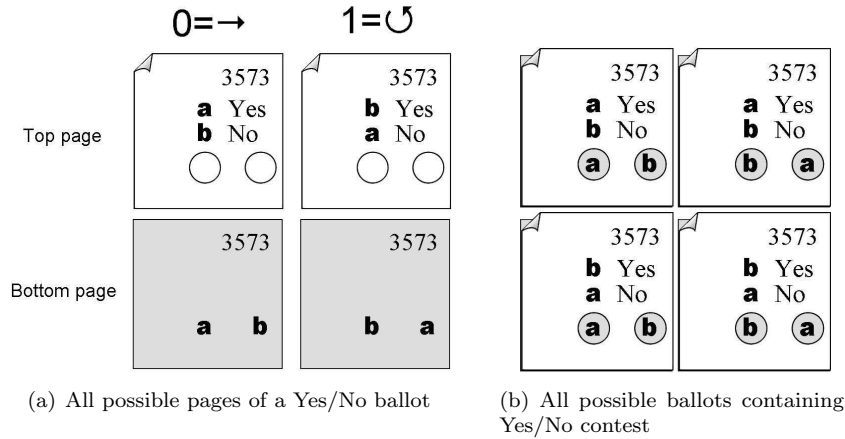


Figure 3: PunchScan's ballot

made public for auditing purposes. The relation that has to hold between the pages of the ballot and the information used for recovering is: top + bottom = flip1 + flip2.

The EA makes public commitments to the ballots and to flip1 and flip2. The candidates choose half the ballots at random and the election authority makes public the requested ballots along with the flip1 and flip2 for each ballot. Anyone can check that the equation top + bottom = flip1 + flip2 holds. Only the ballots that were not made public in this phase (pre election) will be further used in the election.

During the election phase, the EA publishes all the marked pages (half ballots) as voted on by voters. After the election, it publishes the intermediary state of the ballots (ballots + flip1) and the decrypted ballots (ballots + flip1 + flip2). These are commitments to the values of flip1 and flip2 used in the decryption of the voted half ballots.

Post election, the EA is asked to open either flip1 or flip2, but not both, since opening both would allow the linking of a voted ballot to the corresponding decrypted one. Also, it is necessary that the intermediary state ballots and the decrypted ones be in a random order (distinct from the order of the voted ballots).

The election authority defines the following tables:

- $P$  (for **Print**)
- $D$  (for **Decrypt**)
- $R$  (for **Results**)

The  $P$  table is indexed by ballot serial number and contains the top page ( $P_1$ ), bottom page ( $P_2$ ), and space for the filled-in vote (to be entered after the election). It also contains commitments to  $P_1$  and  $P_2$ .

The  $D$  table contains the first ( $D_2$ ) and second ( $D_4$ ) mark permutations (flips), the intermediary vote ( $D_3$ ) to be filled in during decryption, and information to connect it with the  $P$  table ( $D_1$ ) and the  $R$  table ( $D_5$ ). It also contains a commitment for each row of  $D$ , as well as a commitment for columns  $D_1$  and  $D_2$ , and another commitment for columns  $D_4$  and  $D_5$ .

The  $R$  table contains the clear text votes (after post-election decryption).

Consider further, for the purposes of illustration, an election with only six votes. The clear data in all the tables is in table 2. Before the election, but after the EA made the commitments, the tables look as in table 3

The candidates challenge the election authority to open a random half of the ballots, say the ones numbered 2, 4 and 5. The EA reveals the requested information, and the tables look as in table 4. Ballots 2, 4 and 5 cannot be used in the election and are excluded from any further representation of the tables (see table 7).

Assume that the choices of the voters are as follows. On ballot 1, the leftmost mark was marked, and the top page was chosen. On ballot 3, the rightmost mark and the bottom page were chosen, and on ballot 6, the leftmost mark and the top page were chosen. Because the canonical ballot is "ab", "ab" (that is, "ab" on top and bottom pages), left is associated with "a", and right with "b". The voter choices eventually end up in  $P_3$ , and when they do, each row describes what can be learned through knowledge of the ballot page chosen by the voter.

Ballot ID	$P_1$	$P_2$	$P_3$	$CP_1$	$CP_2$
1	ab	ab		$C_{1,1}$	$C_{1,2}$
2	ab	ba		$C_{2,1}$	$C_{2,2}$
3	ba	ab		$C_{3,1}$	$C_{3,2}$
4	ba	ba		$C_{4,1}$	$C_{4,2}$
5	ab	ba		$C_{5,1}$	$C_{5,2}$
6	ba	ab		$C_{6,1}$	$C_{6,2}$

$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$DC$
6	$\rightarrow$		$\odot$	5	$C_A$
5	$\odot$		$\rightarrow$	4	$C_B$
2	$\odot$		$\rightarrow$	1	$C_C$
1	$\odot$		$\odot$	3	$C_D$
4	$\rightarrow$		$\rightarrow$	2	$C_E$
3	$\rightarrow$		$\odot$	6	$C_F$
$CD_{1,2}$			$CD_{4,5}$		

$R_{id}$	$R_1$
1	
2	
3	
4	
5	
6	

Table 2: *PDR* tables as the Election Authority sees them, with all the information available. The tables are properly formed, because, for all the ballots,  $D_2 + D_4$  correctly represents whether  $P_2$  is a flipped version of  $P_1$  or not. For example, for ballot number 3, on the top page, “a” is associated with “Yes”, and b with “No”. On the bottom page, the order is “ba”, thus  $P_2$  is a flipped version of  $P_1$ . In the  $D$  table, in the row corresponding to 3, we have  $\rightarrow + \odot = \text{flip}$ . For ballot 1,  $C_{1,1}$  is a commitment to  $P_1$ ,  $C_{1,2}$  is a commitment to  $P_2$  and so on.

Ballot ID	$P_1$	$P_2$	$P_3$	$CP_1$	$CP_2$
1				$C_{1,1}$	$C_{1,2}$
2				$C_{2,1}$	$C_{2,2}$
3				$C_{3,1}$	$C_{3,2}$
4				$C_{4,1}$	$C_{4,2}$
5				$C_{5,1}$	$C_{5,2}$
6				$C_{6,1}$	$C_{6,2}$

$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$DC$
					$C_A$
					$C_B$
					$C_C$
					$C_D$
					$C_E$
					$C_F$
$CD_{1,2}$			$CD_{4,5}$		

Table 3: *PD* tables in the Preelection phase, as the public sees them.

The EA performs the first flip to ballots 1,3 and 6 to obtain the partially decrypted ballots as in  $D_3$ , and the totally decrypted ballots as in  $R_1$  (see table 6). The ballots in both  $D$  and  $R$  are shuffled, so it is not possible to link rows among Tables  $P$ ,  $R$  and  $D$ . Post election, the auditor asks the EA to open either the left or the right side of  $D$  (but not both). If the election authority cheats, the auditor will catch it with probability 0.5 (for a higher probability see section 6.4). In our example, suppose the auditor chooses the right hand side. The EA then reveals  $D_4$  and  $D_5$ . The auditor can now check that  $D_3 + D_4 = R_1$ , and that the commitment  $CD_{4,5}$  to the columns  $D_4$  and  $D_5$  is valid.

## 6 A more technical description

### 6.1 The ballot

Let  $S$  be a set of symbols. The symbols in  $S$  will appear on both the top and bottom page. We assume that  $S$  is sorted and the order is fixed. We denote by “canonical ballot” a ballot that will have  $S$  (ordered) on both the top and bottom page. Let  $T_p$  (top permutation),  $B_p$  (bottom permutation), and  $D_2$  be three random, independent permutations of  $S$ .

Compute  $D_4$  such that  $B_p \circ T_p = D_2 \circ D_4$ . Therefore,  $D_4 = D_2^{-1} \circ B_p \circ T_p$ .

Ballot ID	$P_1$	$P_2$	$P_3$	$CP_1$	$CP_2$
1				$C_{1,1}$	$C_{1,2}$
2	ab	ba		$C_{2,1}$	$C_{2,2}$
3				$C_{3,1}$	$C_{3,2}$
4	ba	ba		$C_{4,1}$	$C_{4,2}$
5	ab	ba		$C_{5,1}$	$C_{5,2}$
6				$C_{6,1}$	$C_{6,2}$

$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$DC$
					$C_A$
5	$\odot$		$\rightarrow$	4	$C_B$
2	$\odot$		$\rightarrow$	1	$C_C$
					$C_D$
4	$\rightarrow$		$\rightarrow$	2	$C_E$
					$C_F$
$CD_{1,2}$			$CD_{4,5}$		

Table 4: *PD* tables after the election authority has replied to the request to open ballots 2,4 and 5

Ballot ID	$P_1$	$P_2$	$P_3$
1			
3			
6			

$D_1$	$D_2$	$D_3$	$D_4$	$D_5$
$CD_{1,2}$			$CD_{4,5}$	

Table 5: Ballots that can be used by voters in the election day. The other ballots were spoiled during the pre election phase. The row commitments are not shown anymore because they won't be checked, since no other complete row will ever be opened.

Ballot ID	$P_1$	$P_2$	$P_3$
1	ab		a
3		ab	b
6	ba		a

$D_1$	$D_2$	$D_3$	$D_4$	$D_5$
		a		
		b		
		b		
$CD_{1,2}$			$CD_{4,5}$	

$R_{id}$	$R_1$
3	a
5	b
6	a

Table 6:  $PDR$  snapshot after the polls close. One cannot say what row in the  $D$  table corresponds to what row in the  $P$  or  $R$  table, because the rows are permuted. Thus, the secret ballot principle is satisfied.

## 6.2 The tables

We describe the  $PDR$  tables using notation from relational algebra. In databases relational algebra is heavily used. It has the notions of relations (tables), projections ( $\pi$  - SQL SELECT), selection ( $\sigma$  - SQL WHERE) and join ( $\bowtie$ ). In a relation  $R(A, B)$ ,  $A \rightarrow B$  means that  $A$  implies  $B$  (given  $A$ ,  $B$  is uniquely identified).  $A$  is called a key of relation  $R$ .

Let  $P$  (print) be the following relation:

$$P(P_{id}, P_1, P_2, P_3, CP_1, CP_2), P_{id} \rightarrow (P_1, P_2, P_3, CP_1, CP_2)$$

where  $P_{id}$  is the ballot id (the serial number of the ballot),  $P_1$  is  $T_p$ ,  $P_2$  is  $B_p$ ,  $P_3$  is a projection of  $T_p \circ B_p$  (voter choices),  $CP_1$  is a commitment to  $P_1$ , and  $CP_2$  is a commitment to  $P_2$ . The commitments are cryptographically computed.  $P$  contains  $2n$  records.

Let  $D$  (decrypt) be the following relation:

$$D(D_1, D_2, D_3, D_4, D_5, DC), D_1 \rightarrow (D_2, D_3, D_4, D_5, DC)$$

where  $D_1$  is a foreign key pointing to the  $P_{id}$  attribute of  $P$ ,  $D_5$  is a foreign key pointing to the  $R_{id}$  attribute of  $R$  (see below),  $D_2$  and  $D_4$  are permutations of  $S$  described above,  $D_3$  is  $P_3 \circ D_2$ , and  $DC$  is a commitment to the tuple  $(D_1, D_2, D_4, D_5)$ .  $D$  contains  $2n$  records.

Let  $CD$  (commitments to the columns of  $D$ ) be the following relation:

$$CD(CD_{1,2}, CD_{3,4})$$

This relation has only one record.  $CD_{1,2}$  is a commitment to  $D_1$  and  $D_2$ ;  $CD_{4,5}$  is a commitments to  $D_4$  and  $D_5$ .

Let  $R$  (results) be the following relation:

$$R(R_{id}, R_1), R_{id} \rightarrow (R_1)$$

where  $R_{id}$  is a unique identifier and  $R_1$  is  $P_3 \circ D_2 \circ D_4$ .  $R$  contains  $2n$  records.

Ballot ID	$P_1$	$P_2$	$P_3$
1	ab		a
3		ab	b
6	ba		a

$D_1$	$D_2$	$D_3$	$D_4$	$D_5$
		a	○	5
		b	○	3
		b	○	6
$CD_{1,2}$			$CD_{4,5}$	

$R_{id}$	$R_1$
3	a
5	b
6	a

Table 7:  $PDR$  snapshot after the post election audit. The election authority was asked to open the right hand side of the  $D$  table. Anyone can check that the intermediary result transformed by  $D_4$  gives the result in  $R_4$  ( $D_3 + D_4 = R$ ), thus the election authority did not cheat. Also  $CD_{4,5}$ , the commitment to  $D_4$  and  $D_5$  is checked. Note that there is still no link between  $P$  and  $R$ , thus the privacy

To select all the information for a ballot, we do:

$$(P \bowtie_{P_{id}=D_1} D) \bowtie_{D_5=R_{id}} R$$

### 6.3 The time line

Before the election the election authority(EA) computes  $P(P_{id}, P_1, P_2, CP_1, CP_2)$ ,  $D(D_1, D_2, D_4, D_5, DC)$ ,  $CD(CD_{1,2}, CD_{4,5})$  and makes public  $P(P_{id}, CP_1, CP_2)$ ,  $D(DC)$  and  $CD(CD_{1,2}, CD_{4,5})$ .

In the preelection audit, the auditor randomly selects half of the records in  $P$ . The election authority (EA) reveals  $P \bowtie_{P_{id}=D_1} D$  for all the requested records. The auditor can check that  $P_1 o P_2 o D_2 o D_4 = S$  and that the commitments  $CP_1$ ,  $CP_2$ , and  $DC$  are valid.

During the election, the voters fill in  $P_3$ .

After the election, EA computes  $D_3 = P_3 o D_2$  and  $R_1 = D_3 o D_4$  and makes  $D_3$  and  $R_1$  public.

In the post election audit, the auditor asks EA to either reveal  $(D_1, D_2)$  or  $(D_4, D_5)$ , but not both. EA reveals the requested information. The auditor can either check that  $P_3 o D_2 = D_3$  (using  $P \bowtie_{P_{id}=D_1} D$ ) or  $D_3 o D_4 = R_1$  (using  $D \bowtie_{D_5=R_{id}} R$ ). The chance of EA cheating and not being caught is 50% (see section 6.4).  $CD_{1,2}$  and  $CD_{4,5}$  are also checked.

### 6.4 Multiple instances of $D$

Because EA can cheat and not get caught with 50% probability, we introduce multiple instances of  $D$ . Thus we modify the relation  $D$  as follows: Let  $D$  (decrypt) be the following relation:

$$D(i, D_1, D_2, D_3, D_4, D_5, DC), (i, D_1) \rightarrow (D_2, D_3, D_4, D_5, DC)$$

where  $i$  is the instance number and the rest are as described in section 6.2

Let  $CD$  (commitments to the columns of  $D$ ) be the following relation:

$$CD(i, CD_{1,2}, CD_{3,4}), i \rightarrow (CD_{1,2}, CD_{3,4})$$

where  $i$  is a foreign key pointing to the  $i$  attribute of  $D$ .

In the post election audit, we can now make  $k$  challenges, where  $k$  is the number of  $D$  instances. The auditor will ask to open either  $(D_1, D_2)$  or  $(D_4, D_5)$  for each instance of  $D$ . The chance that EA cheats and does not get caught is one out of  $2^k$ . Thus we can make it as low as we want by increasing  $k$ .

## 7 Acknowledgments

We would like to thank David Chaum, Poorvi Vora, Rick Carback, Jeremy Robin and Ben Adida, for the vibrant discussions and insightful comments.