The George Washington University Combinatorics Seminar

Wednesday, November 19, 2008, 4:15 - 5:15 p.m.

Monroe Hall, Room 267
2115 G Street, N.W., Washington, D.C.

Fundamental transversal matroids and lattices of cyclic flats

Ken Shoda (George Washington University)

In the axiom scheme for matroids by cyclic flats, the rank of a set X is given by $r(X) = \min\{r(Z) + |X - Z| : Z \in \mathcal{Z}(M)\}$, where $\mathcal{Z}(M)$ is the set of cyclic flats of M. Thus the set of cyclic flats with $r(X) = r(Z) + |X - Z|$ determines the rank of X. The set $R(X)$ of such cyclic flats has a number of interesting properties, some of which I explore in this talk.

Joseph Bonin and Anna de Mier showed that, given an abstract lattice L, there is a matroid M whose lattice of cyclic flats, $\mathcal{Z}(M)$, is isomorphic to L. The matroid M they construct is a fundamental transversal matroid with a fundamental basis B; each cyclic flat of M is spanned by a subset of B. I noticed that M is a fundamental transversal matroid if and only if $\mathcal{Z}(M) = R(X)$, and we can always choose X to be a fundamental basis. Thus, for given lattice L, there is a matroid M such that $L \cong \mathcal{Z}(M) = R(X)$. I extend this result in two directions.

First, for a given abstract lattice L, I determined all integer-valued functions ρ on L such that there is a fundamental transversal matroid M with a lattice isomorphism $\varphi : L \to \mathcal{Z}(M)$ satisfying $\rho = r\varphi$. Furthermore, I show how to find the unique ρ that minimizes the rank and cardinality of a matroid.

Second, I find the necessary and sufficient condition to have $R(X)$ be an interval in $\mathcal{Z}(M)$. In particular, M is a fundamental transversal matroid if and only if any interval can arise as $R(X)$ with an appropriate choice of X.