The George Washington University Combinatorics Seminar

Wednesday, September 17, 2008, 4:45 - 5:45 p.m.

Monroe Hall, Room 267
2115 G Street, N.W., Washington, D.C.

Infinite log-concavity

by

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Let $(a_k) = a_0, a_1, a_2, \ldots$ be a sequence of real numbers. For convenience, we let $a_k = 0$ if $k < 0$. The sequence is *log-concave* if $a_k^2 \geq a_{k-1}a_{k+1}$ for all k. Log-concave sequences frequently appear in algebra, combinatorics, and geometry. Define the $\mathcal{L}$-*operator* on sequences by $\mathcal{L}(a_k) = (b_k)$ where

$$b_k = a_k^2 - a_{k-1}a_{k+1}.$$

So (a_k) is log-concave if and only if $\mathcal{L}(a_k)$ is a nonnegative sequence. Define (a_k) to be *infinitely log-concave* if $\mathcal{L}^i(a_k)$ is nonnegative for all $i \geq 1$.

Boros and Moll conjectured that the sequence of binomial coefficients

$$\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots$$

is infinitely log-concave for any $n \geq 0$. We prove that this conjecture is true for all $n \leq 1450$ by using a computer and a stronger version of log-concavity. We also discuss infinite log-concavity of the columns of Pascal's triangle, q-analogues, symmetric functions, real-rooted polynomials, and Toeplitz matrices. There will be many conjectures sprinkled throughout the lecture.