The George Washington University Combinatorics Seminar

Wednesday, October 8, 2008, 4:45 - 5:45 p.m.

Monroe Hall, Room 267
2115 G Street, N.W., Washington, D.C.

Questions (and a few answers) in additive number theory and combinatorial geometry

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If you are like me, you have been wondering how to best approximate the entire nine-dimensional unit sphere by just twenty-five points. It turns out that a good starting point is to find five elements in the cyclic group of order twenty-five which are least dependent on one another. Let me explain.

A finite set X of points on the sphere S^d is a *spherical t-design*, if for every polynomial f of total degree t or less, the average value of f over S^d is equal to the arithmetic average of its values on X.

A finite subset A of an abelian group G is *t-independent* in G, if no multi-subset of A with at most t elements can be divided into two disjoint parts so that the two parts have the same (multi-subset) sum.

Spherical 2-designs and 2-independent sets are not difficult to find; for example, the vertices of a regular simplex form a 2-design of size $d+2$ on S^d, and taking exactly one of each element or its negative (when different) yields a 2-independent set of size $\lfloor (n-1)/2 \rfloor$ in $\mathbb{Z}_n$. (These cardinalities are minimal and maximal, respectively.)

Much less is known for $t \geq 3$, but we can answer the questions in our first paragraph, as follows. Consider $A = \{1, 4, 6, 9, 11\}$ in $G = \mathbb{Z}_{25}$. We find that A is 3-independent in G (of maximum size; also, there is no set of size five which is 4-independent in G). Now take

$$X = \left\{ \frac{1}{\sqrt{5}} \left(z^j(1), z^j(4), z^j(6), z^j(9), z^j(11) \right) : 1 \leq j \leq 25 \right\}$$

on S^9 where

$$z(a) = \cos\left(\frac{2\pi}{25} a\right) + i \cdot \sin\left(\frac{2\pi}{25} a\right).$$

Then X is a 3-design on S^9 (and there is no set of size twenty-five which is a 4-design on S^9).

In this talk we discuss these and other concepts in additive number theory and geometric combinatorics from a new, unified viewpoint, and feature some of the famous and less well known conjectures along the way.