Abstracts of talks delivered at the Third Conference on Knot Theory and its Ramifications:

KNOTS IN WASHINGTON

held on October 18-20, 1996 at the George Washington University.

Friday, October 18

1:00 - 2:00 **Doug Bullock** (Boise State Univ.),
Skein Quantizations

Abstract.

That fact That the Kauffman bracket skein module of a surface is a quantum deformation has been bandied about the knot theory community for a decade or more. In this talk we will see, mostly be example, what the words "quantum" and "deformation" actually mean, what the undeformed object looks like, why it should be of interest, and how the skein module unifies various points of view. In the process, we will touch lightly upon topology, representation theory, non-commutative algebra, and mathematical physics.

2:30 - 3:00 **Michael McDaniel** (GWU),
An Introduction to Cabling of Chord Diagrams
(joint work with Yongwu Rong)

Abstract.

This talk outlines some interesting results about the subspaces of Vassiliev Invariants generated by the Homflypt and Kauffman weight systems of connected cablings of knots. Using certain eigenvectors of cabling, a deframing projector and careful attention to framing, it can be seen that dimensional bounds on these subspaces are quadratic in m (their order).

3:10 - 3:40 **Mark Kidwell** (Naval Academy and GWU),
Fibered Knots from Signed Graphs

Abstract.

We recall how to span a surface in a link by the checkerboard method and by Seifert's algorithm. When these two methods coincide, we use graph theory to study the push-off maps from the surface into its complement. We give a graphical criterion for the link to be fibered and use Eulerian circuits to construct graphs that meet the criterion.

3:50 - 4:20 **Brian Mangum** (Columbia Univ.),
Three-dimensional Representations of Punctured Torus Bundles

Abstract.

We construct a curve of irreducible $SL_3(C)$ representations of the fundamental group of any orientable once punctured torus bundle over the circle. Moreover, infinitely many of these representations are conjugate to $SU(3)$ representations. The construction employs classical matrix group representations of the four-strand braid group.

4:30 - 5.30 **William M. Goldman** (UMD), The topology of relative character varieties of surfaces.

Abstract.

In this talk I will describe the geometry and topology of cubic surfaces arising as sets of R-points of relative character varieties of surfaces of low genus. In particular I will discuss some experimental results begun in an REU activity with Robert Benedetto in 1993 on the quadruply punctured sphere. A compact component of the relative character variety which corresponds to $SL(2,R)$-representations (not $SU(2)$-representations) will be demonstrated, and relations between the relative character varieties of quadruply punctured sphere and punctured torus will be given.

Saturday, October 19

10:30 - 11:00 **Yongwu Rong** (GWU),
Derivatives of Link Polynomials
(joint work with W. B. R. Lickorish)

Abstract.

In this talk, we study the higher order link polynomials – a class of link invariants related to both Homfly polynomial and Vassiliev invariants. We show that a dth partial derivative of the Homfly polynomial is a dth order Homfly polynomial. For $d = 1$, we show that these derivatives span all the first order Homfly polynomials. We also make similar constructions for other link polynomials. In the case of the Conway polynomial, we note that its derivative does not span the space of first order Conway polynomials. Questions related to Vassiliev invariants as well as computational complexities of link polynomials will be raised.

11:15 - 12:15 **Louis H. Kauffman** (Univ. Illinois at Chicago),
Virtual Knot Theory

Abstract.

Knot and link diagrams are usually drawn on the plane or on the two-sphere. Diagrams, taken up to equivalence by local Reidemeister moves, drawn on a surface F_g of genus g, classify links in $F_g \times I$, where I is the unit interval. Such a diagram, when projected back into the plane will have *virtual* crossings that are not regarded as crossings in F_g. A *virtual link diagram* is a link diagram in the plane in which a subset of the crossings are designated as virtual. Such a diagram can be regarded as representing a classical diagram on some F_g, but we define a theory of virtual knots and links (with an appropriate generalization of the Reidemeister moves) that does not make reference to any specific genus g. Quantum link invariants,classical link invariants and Vassiliev invariants all generalize to the category of virtual links. There are many surprising phenomena. We give an example of a non-trivial virtual knot with trivial Jones polynomial. (This is a real phenomenon in $F_g \times I$, for $g = 1$).

The generalization of Vassiliev invariants is non-trivial with the weight systems complicated by extra combinatorics of virtual vertices.

The topic of this talk is new, and it yields a chance to examine knot theory from an unusual angle. I am following an analogy with virtual knots that is well known to graph theorists who study non-planar graphs that are uncolorable in order to understand coloring problems for planar graphs. Here we regard combinatorial knot theory as a theory

of planar knot diagrams and extend to non-planar knot diagrams whose abstract structure parallels that of classical knot theory.

2:00 - 3:00 **Joanna Kania-Bartoszynska** (Boise State Univ.),
Lattice gauge field theory

Abstract.

I will present a combinatorial setting in which the Kauffman bracket skein module can be seen as a quantization of the SL_2C gauge field theory. This is joint work with Doug Bullock and Charlie Frohman.

3:15 - 3:45 **Maxim Sokolov** (GWU),
Turaev-Viro invariants parameterized by non-primitive roots of unity.

Abstract.

Every Turaev-Viro invariant, $TV(M)_q$, is a sum of three non-trivial summand-invariants: $TV_0(M)_q$, $TV_1(M)_q$, and $TV_2(M)_q$. Here q is a $2r$-th root of unity such that q^2 is a primitive root of unity of degree r, and $r \geq 3$. We can prove that

$$TV_N(M)_{-q} = (-1)^N TV_N(M)_q$$

for any 3-manifold M (even non-oriented) and $N \in \{0, 1, 2\}$.

¿From the conditions on q it follows that if q is not a primitive $2r$-th root of unity then $-q$ is. Hence, by our formula, we can express any Turaev-Viro invariant parameterized by a non-primitive q via the summand-invariants parameterized by the primitive root $-q$.

It is worth noting that the above mentioned formula allows us to express the numbers $TV_1(M)_q$ and $(TV_0(M)_q + TV_2(M)_q)$ via Turaev-Viro invariants. Moreover, if $q = exp(\pi i/r)$ and r is odd, then sometimes we can get $TV_0(M)_q$ from the following analog of the well-known Kirby-Melvin formula:

$$TV_0(M)_q TV(M)_{exp(\pi i/3)} = TV(M)_q.$$

4:00 - 4:30 **Jozef H. Przytycki** (GWU),
Kauffman bracket skein algebra of a product of a surface and interval is an integral domain

Abstract.

We consider the Kauffman bracket skein module, $\mathcal{S}_{2,\infty}(M; R, A)$, of an oriented 3-manifold M, that is the quotient of the module of formal linear combinations of unoriented framed links in M with coefficients in a commutative ring with unit, R, by the submodule generated by the classical Kauffman bracket relations $L_+ - AL_0 - A^{-1}L_\infty$ and $L \sqcup O + (A^2 + A^{-2})L$, where A is a fixed invertible element in R. If $M = F \times I$, for an oriented surface F, or $A = -1$ then $\mathcal{S}_{2,\infty}(M; R, A)$ is an algebra. We show that

(i) $\mathcal{S}_{2,\infty}(F \times I; R, A)$ has no zero divisors, provided R has no zero divisors.

(ii) If M is a twisted I bundle over unoriented surface F then $\mathcal{S}_{2,\infty}(M; R, A)$ has no zero divisors, provided R has no zero divisors.

(iii) $\mathcal{S}_{2,\infty}(T^2 \times I; Z, -1)$ is a unique factorization domain.

As a corollary we prove Bullock conjecture for surface groups that the skein algebra of a fundamental group of an oriented compact surface, (with coefficients in C), is isomorphic to the coordinate ring of the $SL(2, C)$ character variety of the group.

4:45 - 5:15 **Adam S. Sikora** (GWU),
Skein algebras and quaternion representations

Abstract.

We are going to give an introduction to a theory of group representations into units of generalized quaternion algebras. In particular, we will present a theorem which will generalize results of Brumfiel and Hilden ('Sl_2 Representations of Finitely Presented Groups', Cont. Math. 187) and the theorem of Culler and Shalen ('Varieties of group representations and splittings of 3-manifolds', Ann. of Math. 117(1983), 109-146) on $Sl_2(C)$-character varieties. Moreover, we are going to show interesting connections between quaternion representations and skein algebras. In particular, we will generalize the Bullock, Przytycki-Sikora Theorem asserting that skein algebra of a manifold is, up to nilpotent elements, isomorphic to the coordinate ring of $Sl_2(C)$-character variety associated with the fundamental group.

Sunday, October 20
Workshops

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