

Feynman Rules of A Few Popular Interactions and Fields v1.0, Feb 2019

Propagators	nonrelativistic (boson or fermion) field	$----- : i\Delta_F^{\text{nonrel}}(q) = \frac{i}{q_0 - \frac{\vec{q}^2}{2m} + i\epsilon}$
real/complex scalar	$----- : i\Delta_F(q) = \frac{i}{q^2 - m^2 + i\epsilon}$	fermion Dirac $\alpha\beta$
photon ($\partial \cdot A = 0$: Lorenz gauge)	$\nu \text{---} \mu : i\Delta_F^{\mu\nu}(q) = \frac{-i g^{\mu\nu}}{q^2 + i\epsilon}$	$\alpha \xrightarrow{q} \beta : iS_F(q) = \frac{i[\not{q} + m]^\beta_\alpha}{q^2 - m^2 + i\epsilon}$
		massive spin-1
		$\nu \text{---} \mu : i\Delta_F^{\mu\nu}(q) = \frac{-i(g^{\mu\nu} - \frac{q^\mu q^\nu}{m^2})}{q^2 - m^2 + i\epsilon}$

Interactions and Their Lagrangeans

Φ^n Theories: Interactions of Scalar With Itself

$$\begin{aligned}
 &-\frac{\lambda_3}{3!} \Phi^3 \implies \text{---} \text{---} \text{---} : -i \frac{\lambda_3}{3!} \\
 &\left. \begin{aligned}
 \text{real: } &-\frac{\lambda_4}{4!} \Phi^4 \\
 \text{complex: } &-\frac{\lambda_4}{2!^2 = 4} (\Phi^\dagger \Phi)^2
 \end{aligned} \right\} \implies \text{---} \text{---} : -i \frac{\lambda_4}{4! \text{ or } 4} \\
 &\left. \begin{aligned}
 \text{real: } &-\frac{\lambda_6}{6!} \Phi^6 \\
 \text{complex: } &-\frac{\lambda_6}{3!^2} (\Phi^\dagger \Phi)^3
 \end{aligned} \right\} \implies \text{---} \text{---} : -i \frac{\lambda_6}{6! \text{ or } 3!^2}
 \end{aligned}$$

Some self-interactions with derivatives (all p incoming):

$$\begin{aligned}
 \text{via s-wave: } &-4\lambda_s [\Phi^\dagger \Phi] [\Phi^\dagger (\partial^2 \Phi) - (\partial^2 \Phi^\dagger) \Phi] \implies \text{---} \text{---} : -i \lambda_s \sum_{i=1}^4 p_i^2 \\
 \text{via p-wave: } &-6\lambda_p [(\partial_\mu \Phi)^\dagger (\partial^\mu \Phi)]^2 \implies \text{---} \text{---} : -i \lambda_p \sum_{i=1}^3 \sum_{j=i+1}^4 p_i \cdot p_j
 \end{aligned}$$

4-Fermion/Fermi-like Theory:

$$-\lambda_F (\bar{\Psi} \Psi)^2 \implies \text{---} \text{---} : -i \lambda_F$$

Yukawa(-like) Theory: fermion-scalar $f_0 \bar{\Psi} \Phi \Psi + f_5 \bar{\Psi} \gamma_5 \Phi \Psi$ (+ H.c. if Φ complex) \implies

$$\text{---} \text{---} : i f_0 + i f_5 \gamma_5$$

Scalar QED: $-ieA^\mu [(\partial_\mu \Phi^\dagger) \Phi - \Phi^\dagger (\partial_\mu \Phi)] + e^2 A^\mu A_\mu \Phi^\dagger \Phi \implies$

$$\text{---} \text{---} : -ie(p^\mu + p'^\mu), \quad \text{---} \text{---} : ie^2 g^{\mu\nu}$$

(Fermionic) QED:

$$-e \bar{\Psi} \not{A} \Psi \implies \text{---} \text{---} : -ie(\gamma^\mu)^\beta_\alpha$$

πN Toy-Model Pauli bi-spinor $N = \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}$ with spin $\vec{\sigma}$: $-\frac{gA}{2f_\pi} N^\dagger (\vec{\sigma} \cdot (\vec{\partial}\pi)) N \implies$

$$\text{---} \text{---} : \frac{gA}{2f_\pi} (\vec{\sigma})^\beta_\alpha \cdot \vec{q}$$

Kinematics: Mandelstam variables $s = (k + p)^2$; $t = (k' - k)^2$; $u = (p' - k)^2$

Energies of massless particles for elastic $A + B \rightarrow A + B$ in lab frame: $\frac{E' \text{ (out)}}{E \text{ (in)}} = \frac{1}{1 + \frac{E}{M}(1 - \cos \theta)}$

Elastic cross section of unpolarised scattering $A + B \rightarrow A + B$:

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{cm}} = \frac{1}{64\pi^2 s} |\overline{\mathcal{M}}|^2 \quad \text{massless on massive in lab: } \left. \frac{d\sigma}{d\Omega} \right|_{\text{lab}} = \frac{1}{64\pi^2 M^2} \left(\frac{E'}{E}\right)^2 |\overline{\mathcal{M}}|^2$$

$|\overline{\mathcal{M}}|^2$: squared transition amplitude, averaged over initial spins and summed over final ones.