PHYS 6610: Graduate Nuclear and Particle Physics I

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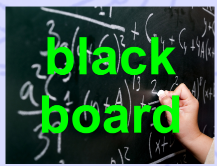
I. Tools

7. Scattering and Decay of Particles

Or: How Long to Count

References: [HH; HG 10.1-2, 5.7/12; PRSZR 4; HM 4.3, 2.10, 4.4; PDG 48, 48.5, 49]

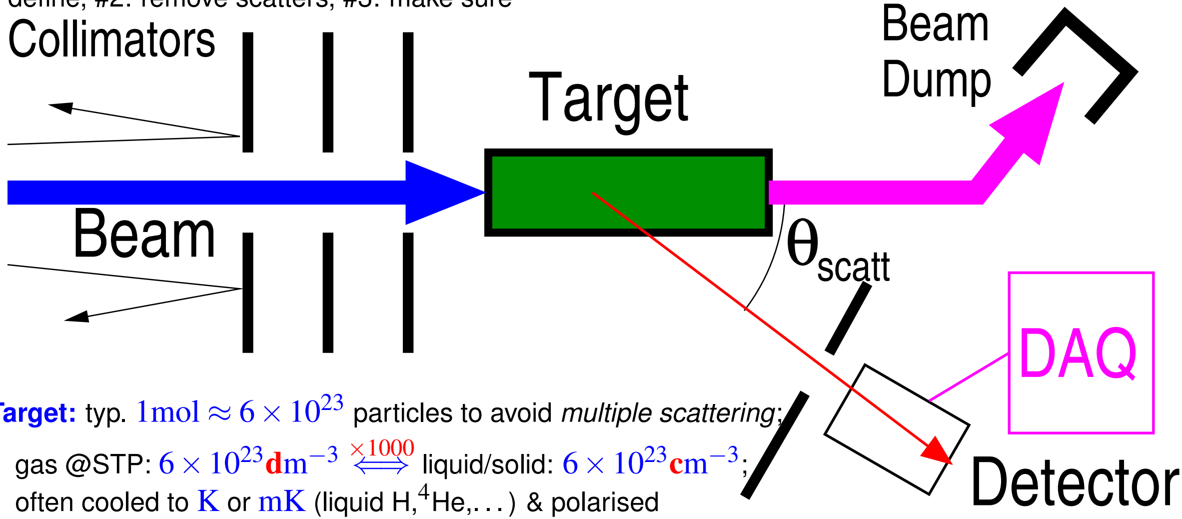


Garbage-In – Garbage-Out

What An Experiment Really Is (Ideally)

Beam Cleanup: remove charged undesireds by $\vec{B}$
Collimators: make sure *all* beam hits target
eliminate "beam halo" (cotravelling undesireds)
#1: define, #2: remove scatters, #3: make sure

Charged-Beam Dump: use Cu; after bend to reduce backscatter; measure charged-beam flux by Faraday cup; often most radioactive piece during run



Target: typ. $1\text{mol} \approx 6 \times 10^{23}$ particles to avoid *multiple scattering*;
gas @STP: $6 \times 10^{23}\mathbf{d}\text{m}^{-3} \overset{\times 1000}{\Longleftrightarrow}$ liquid/solid: $6 \times 10^{23}\mathbf{c}\text{m}^{-3}$;
often cooled to K or mK (liquid H,^{4}He,...) & polarised

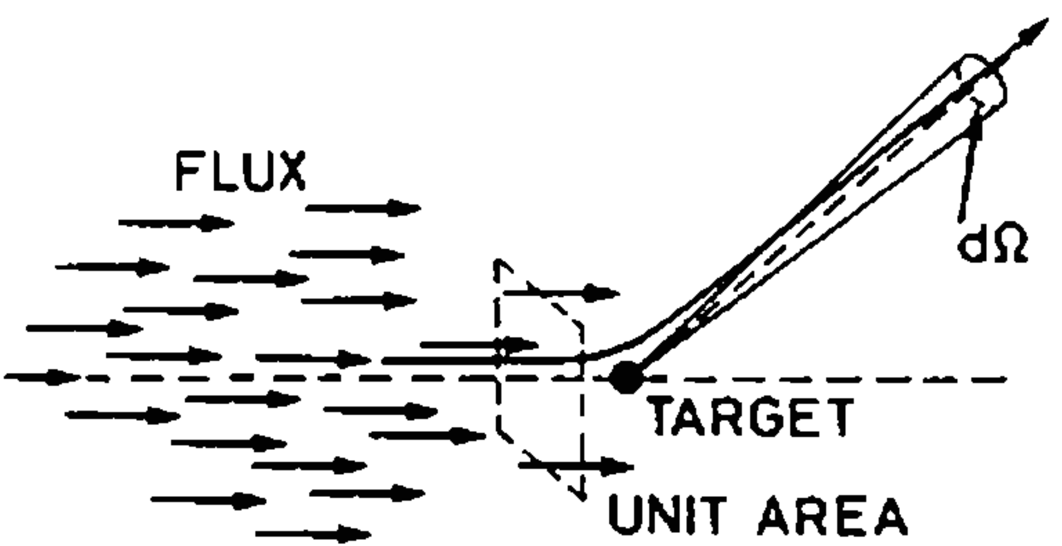
If you are a beam, everything looks like a target:
Nature cannot separate between **signal (good)** and **noise (bad)**:
contaminations: scatter from wrong reaction, atomic e^-,
container, impurities/stabilising compounds (e.g. $NaPO_3$ for P),
collimators, beam dump; environment: concrete, cosmics,...

Detector:
collimator often defines angle

Data Acquisition:
hardware/software filters,
event recording,...

$\Longrightarrow$ **student** $\Longrightarrow$ **paper**

(c) Scattering for Theorists



target has length d

typical target density for liquid/solid: $\frac{1 \text{ particle}}{\text{Ångstrom}} \approx 1 \times 10^{30}\text{m}^{-3}$

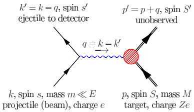
for gas: $\frac{6 \times 10^{23} \text{ particles}}{22\,\text{litres} \widehat{=} 1\text{mol}} \times \frac{\text{pressure}}{1\text{bar}} \approx \frac{1}{4} \times 10^{26}\text{m}^{-3} \times \frac{\text{pressure}}{[\text{bar}]}$

Summary: Electron Scattering Cross Section Handout (link here)

Summary Electron Scattering Cross Sections

cf. [HM 8]

Lowest-order Feynman graph



$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{1}{64\pi^2 s}\,|\overline{\mathcal{M}}|^2$ **elastic, unpolarised, cm-frame**

$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \frac{1}{64\pi^2 M^2}\left(\frac{E'}{E}\right)^2 |\overline{\mathcal{M}}|^2$ **elastic, unpolarised, lab-frame**

$|\overline{\mathcal{M}}|^2 = (e^2)^2\, L_{\mu\nu}\, \frac{1}{q^4}\, W^{\mu\nu}$ **matrix element**: avg. initial, sum final spins

$e^2\, L^{\mu\nu} = \frac{1}{2s+1}\sum_{s,s'} j^\mu_{s,s'}(k,k')\, j^{\nu\dagger}_{s,s'}(k,k')$ **lepton tensor**: scatter off virtual γ

$j^\mu_{s,s'}(k,k') = \langle k', s'|j^\mu|k,s\rangle$ **lepton current** (electromagnetic)

$j^\mu_{s,s'} = -\mathrm{i}e\, \bar{l}_{s'}(k')\,\gamma^\mu\, l_s(k)$ for electron

$L^{\mu\nu} = 2[k^\mu k'^\nu + k'^\mu k^\nu - g^{\mu\nu}\, k\cdot k']$ for electron ($m = 0, s = \frac{1}{2}$); cf. (I.7.4W)

$e^2\, W^{\mu\nu} = \frac{1}{2S+1}\sum_{S,S'} J^\mu_{S,S'}(p,p')\, J^{\nu\dagger}_{S,S'}(p,p')$ **hadron tensor**: scatter off virt. γ

$J^\mu_{S,S'}(p,p') = \langle p', S'|J^\mu|p,S\rangle$ **hadron current** (electromagnetic)

$q_\mu j^\mu = 0 = q_\mu J^\mu \implies q_\mu L^{\mu\nu} = 0 = q_\mu W^{\mu\nu}$ **elmag. current conservation**

Mandelstam: $s = (k+p)^2$; $t = (k'-k)^2$; $u = (p'-k)^2$

$k = (E, \vec{k})$, $k' = (E', \vec{k}')$, $q^2 = -Q^2 < 0$

$M\nu = p\cdot q = M(E'-E)_{\text{lab}}$

scatt. angle θ: $\vec{k}\cdot\vec{k}' = |\vec{k}|\cdot|\vec{k}'|\,\cos\theta$

for elastic in lab frame $\frac{E'}{E} = \frac{1}{1 + \frac{E}{M}(1-\cos\theta)}$

Relativistic Rutherford: Coulomb of massless, spin-0 on infinitely heavy, spin-0 point-target ($s = 0; M = \infty, S = 0$)

$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2$ $J^\mu = -\mathrm{i}Ze\delta^{\mu 0}$, i.e. point charge at rest (I.7.1) (I.7.1C)

$e^\pm$ **Coulomb scattering on infinitely heavy, composite spin-0 target:** ($s = \frac{1}{2}; M = \infty, S = 0$)

helicity forbids back-scattering; isotropic charge density $\rho(r)$

$\left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2} \times |F(\vec{q}^2)|^2$ $\overset{\text{Fourier}}{\Longrightarrow}$ form factor $F(\vec{q}^2) := \frac{4\pi}{Ze}\int_0^\infty dr\, \frac{r}{q}\, \sin(qr)\,\rho(r)$ (I.7.2)

normalisation: $F(0) = 1$; charge radius: $\langle r^2\rangle = -6\left.\frac{dF(\vec{q}^2)}{d\vec{q}^2}\right|_{\vec{q}^2=0}$

$e^\pm$ **full electromagnetic scattering on massive composite spin-0 target:** ($s = \frac{1}{2}; M$ finite, $S = 0$)

$\underbrace{\left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}\,\frac{E'}{E}}_{\textbf{Mott: } \text{no structure}} \times |F(q^2)|^2$

adds target recoil; $\vec{q}^2 \to q^2$: 4-momentum transfer (I.7.3)

$J^\mu = -\mathrm{i}Ze\, F(q^2)\,(p^\mu + p'^\mu)$ (most general for $S = 0$) (I.7.3C)

$\Longrightarrow W^{\mu\nu} = Z^2 (p+p')^\mu\,(p+p')^\nu\, |F(q^2)|^2$ (I.7.3W)

$e^\pm\mu^\pm \to e^\pm\mu^\pm$ **scattering on massive spin-$\frac{1}{2}$ target without structure:** ($s = \frac{1}{2}; M, S = \frac{1}{2}$)

$\left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}\,\frac{E'}{E} \times \left[1 - \frac{q^2}{2M^2}\tan^2\frac{\theta}{2}\right]$ back-scattering by helicity transfer (spin-spin/mag. moment interaction) (I.7.4)

Massive $S = \frac{1}{2}$ hadr. tensor, no structure: $W^{\mu\nu} = 2Z^2\left[p^\mu p'^\nu + p'^\mu p^\nu - g^{\mu\nu}(p\cdot p' - M^2)\right]$ (I.7.4W)

$e^\pm$ **on composite, massive spin-$\frac{1}{2}$ target:** form factors $F_1(q^2)$: Dirac; $F_2(q^2)$: Pauli ($s = \frac{1}{2}; M, S = \frac{1}{2}$)

$\left(\frac{\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}\,\frac{E'}{E} \times \left[\left[F_1^2(q^2) + \tau F_2^2(q^2)\right] + 2\tau\,\left[F_1(q^2) + F_2(q^2)\right]^2\tan^2\frac{\theta}{2}\right]$ $\tau := -\frac{q^2}{4M^2}$ (I.7.5)

Variant: **Rosenbluth/Sachs formula** uses **Sachs form factors** $G_E = F_1 - \tau F_2$, $G_M = F_1 + F_2$

$= [\ldots] \times \left[\frac{G_E^2(q^2) + \tau\, G_M^2(q^2)}{1+\tau} + 2\tau\, G_M^2(q^2)\,\tan^2\frac{\theta}{2}\right]$ (I.7.5)

$J^\mu = -\mathrm{i}\underbrace{e\,F_1(q^2)\,\bar{u}(p')\gamma^\mu u(p)}_{\substack{\text{modify point form}\\ F_1(0) = Z = G_E(0)}} + \frac{e}{2M}\underbrace{F_2(q^2)\,q_\nu\,\bar{u}(p')\mathrm{i}\sigma^{\mu\nu}u(p)}_{\substack{\text{anomalous mag. term, } F_2(0) = \kappa = G_M(0) - Z;\\ \kappa \text{ anomalous mag. moment, } \mu = Z + \kappa \text{ mag. moment}}}$ (most general for $S = \frac{1}{2}$) (I.7.5C)

$e^\pm$ ***inelastic*, inclusive scattering:** E' independent variable, p' not detected ($s = \frac{1}{2}; M, S =$ any [sic!])

$\left.\frac{d\sigma}{d\Omega\, dE'}\right|_{lab} = \left(\frac{2\alpha}{q^2}\right)^2 E'^2\,\cos^2\frac{\theta}{2} \times \left[\frac{F_2(q^2,x)}{\nu} + \frac{2\,F_1(q^2,x)}{M}\tan^2\frac{\theta}{2}\right]$ (I.7.6)

inelasticity measure: **Bjorken-**$x := -\frac{q^2}{2p\cdot q} = \frac{Q^2}{2M\nu} \in [0;1]$; elastic: $x = 1$, i.e. $F_{1,2}(q^2,x) \propto \delta(\nu + \frac{q^2}{2M})$

Most general elmag. hadronic ME (symmetric $\mu \leftrightarrow \nu$, charge conservation; any spin [sic!]):

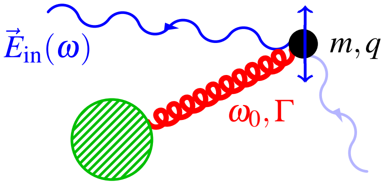
$$W^{\mu\nu} = \frac{F_1(q^2,x)}{M}\left[\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right] + \frac{F_2(q^2,x)}{M^2\nu}\left[p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right]\left[p^\nu - \frac{p\cdot q}{q^2}\,q^\nu\right] \quad \text{(I.7.6W)}$$

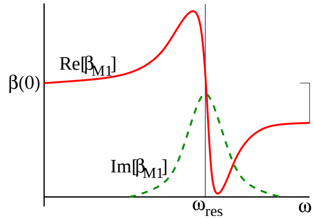
Structure functions F_1, F_2 *are not the Dirac, Pauli FFs of Eq.* (I.7.5)!

(f) Resonances in Quantum Mechanics

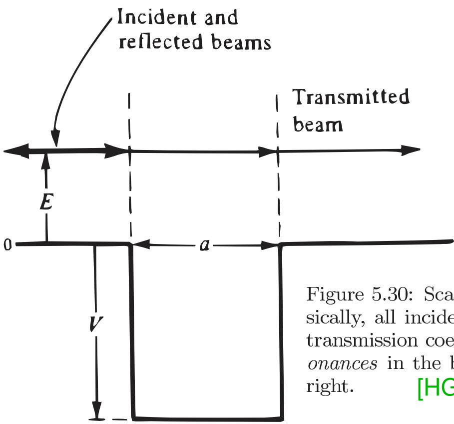
Classical Mechanics: resonance frequencies reveal properties of materials.

Electrodynamics: Lorentz-Drude model, resonance fluorescence





Quantum Mechanics: interference $\Longrightarrow$ resonance even when no bound states.



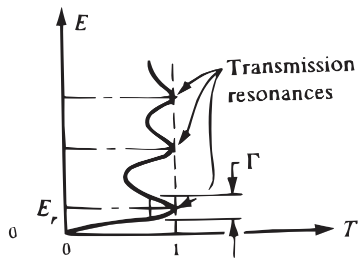


Figure 5.30: Scattering of a particle with energy E from a one-dimensional potential well. Classically, all incident particles will be transmitted. Quantum mechanically, at small energies, the transmission coefficient T is unity only at certain energies. The appearance of *transmission resonances* in the behavior of the transmission as a function of particle energy E is shown at the right. [HG]

Describe Resonance as Creation & Decay of Unstable Particle

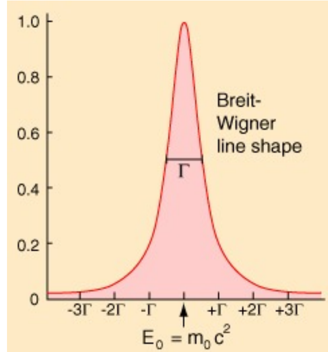
$$\sigma(1+2 \to BC\ldots) \propto |\mathcal{M}(1+2 \to A^* \to BC\ldots)|^2$$

***IF[!!]* Modelled as Nonrelativistic Breit-Wigner:**

Collision with total cm-energy E_{cm}, relative momentum $\vec{k}_{\text{cm}}$, spins S_1, S_2.

$\Longrightarrow$ Produces **resonance** at E_0, total decay width Γ_{total}, spin J.

$\Longrightarrow A^* \xrightarrow{\text{decays into}} BC\ldots$ (final state fully specified).



$$\sigma(1+2 \to A^* \to BC\ldots) = \underbrace{\frac{\overbrace{2J+1}^{\text{multiplicity of resonance}}}{(2S_1+1)(2S_2+1)} \frac{4\pi}{|\vec{k}_{\text{cm}}|^2}}_{\text{flux factor for in-multiplicities}} \underbrace{\frac{B_{\text{in}}^{1+2\to A^*} B_{\text{out}}^{BC\ldots} \Gamma_{\text{total}}^2/4}{(E_{\text{cm}}-E_0)^2+\Gamma_{\text{total}}^2/4}}_{\text{Lorentzian/Breit-Wigner}}$$

Γ_{total}: decay width into *any* final state: "Full Width at Half-Maximum" FWHM

$\Gamma_{A^*\to BC\ldots} = B_{\text{out}}^{BC\ldots} \times \Gamma_{\text{total}}$: partial decay width into specific final state $BC\ldots$

$$\Gamma_{\text{total}} = \sum_{\text{all finals}} \Gamma_{BC\ldots}, \quad \sum_{\text{all finals}} B^{BC\ldots} = 1$$

Branching Ratios: $B_{\text{out}}^{BC\ldots}$: percentage of resonances decaying into specific final state $BC\ldots$

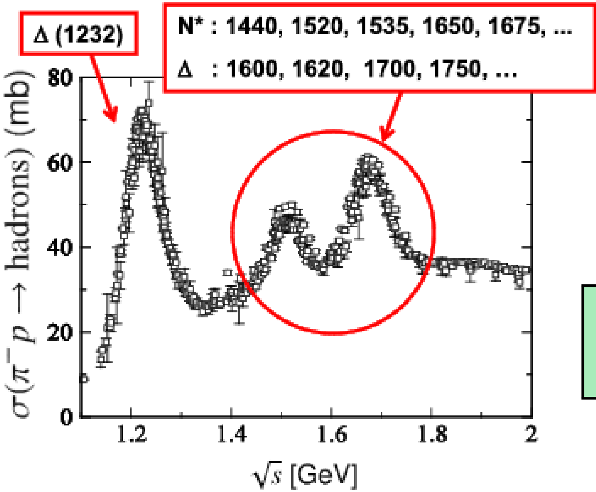
$B_{\text{in}} = B^{1+2}$ by detailed balance: "probability" to produce A^* by colliding $1+2$.

Be Wary of Breit-Wigner Parametrisations in Hadron Physics!

Must account for energy constraints (thresholds) in decay! $\Longrightarrow$ **energy-dependent width $\Gamma_{\mathrm{BW}}(s)$**

Relativistic Breit-Wigner *parametrisation*:
proposed by PDG, often used but not unique

$$\mathcal{M}_{\mathrm{res}} = \frac{\sqrt{s}\,\Gamma_{\mathrm{BW}}^{\mathrm{elastic}}(s)}{s - M_{\mathrm{BW}}^2 + \mathrm{i}\,\sqrt{s}\,\Gamma_{\mathrm{BW}}^{\mathrm{total}}(s)}$$



***BUT* Breit-Wigner parametrisations work only for narrow, well-separated resonances!**

Problems: $\rightarrow$ HW

– $\mathcal{M} = \mathcal{M}_{\mathrm{res}} + \mathcal{M}_{\mathrm{background}}$: split is arbitrary!
Where does background start/end?

– Resonances overlap $\Longrightarrow$ interference!

$\Longrightarrow$ Only positions s_R and residues $\Gamma_{\mathrm{residue}}(s_R)$ of poles in scattering amplitude $\mathcal{M}$ are unique!

$$\sqrt{s_R} \neq M_{\mathrm{BW}} - \mathrm{i}\frac{\Gamma_{\mathrm{BW}}}{2}:$$

Breit-Wigner mass is *not* pole position!

More in PHYS 6710: Nuclear & Particle Physics II

Next: 8. Electron Scattering

Familiarise yourself with: [HM 4, 6.1/3-6/9/11/13, 8]