PHYS 6610: Graduate Nuclear and Particle Physics I

THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON DC

H. W. Grießhammer

Institute for Nuclear Studies
The George Washington University

Spring 2021

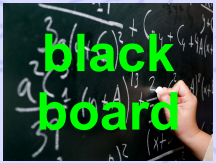
GW INS
Institute for Nuclear Studies
THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

II. Phenomena

3. Resonance Region and Isospin

Or: Hints of Hadron Substructure

References: [PRSZR 2.4, 6.2, 7.1/4; HG 6.8, 14.2, 8.4-7; Per 3.12; HM 2.6/7; PDG 49]

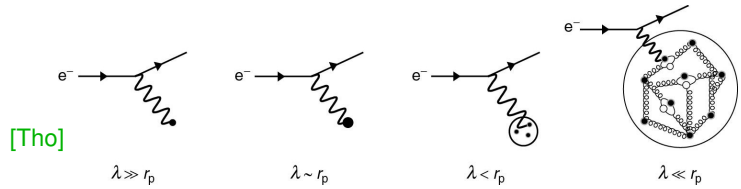


(a) The Resonance Region

$p(e,e')X$ **inelastic inclusive:** $\dfrac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}$ with $\dfrac{E'}{E} \leq \dfrac{1}{1+\frac{E}{M}(1-\cos\theta)}$ unconstrained.

Virtual probe with wave length $\lambda \sim \dfrac{1}{\sqrt{Q^2}}$. $\Longrightarrow$

Dissipate energy & momentum into small volume $\sim \lambda^3$.



[Tho]

$Q^2 \lesssim (0.3\mathrm{GeV})^2 \lesssim r_N^{-2}$: **Response of whole nucleon:**

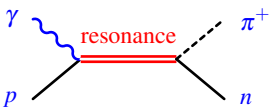
(a) Excite resonances $\Longrightarrow$ **elastic** process with bump:

resonance as global excitation of nucleon.

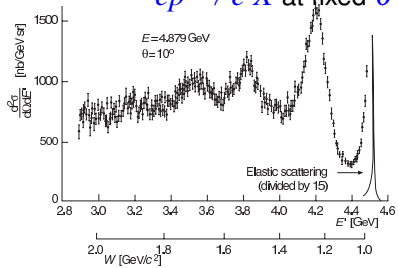
(b) Knock out constituents (virtual particle cloud)

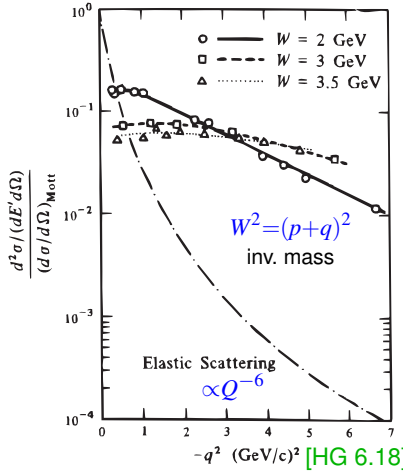
$\Longrightarrow$ **inelastic** process, also via resonance.

e.g.



$ep \to e'X$ at fixed θ



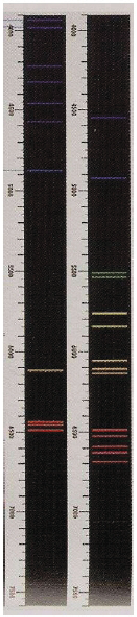


[HG 6.18]

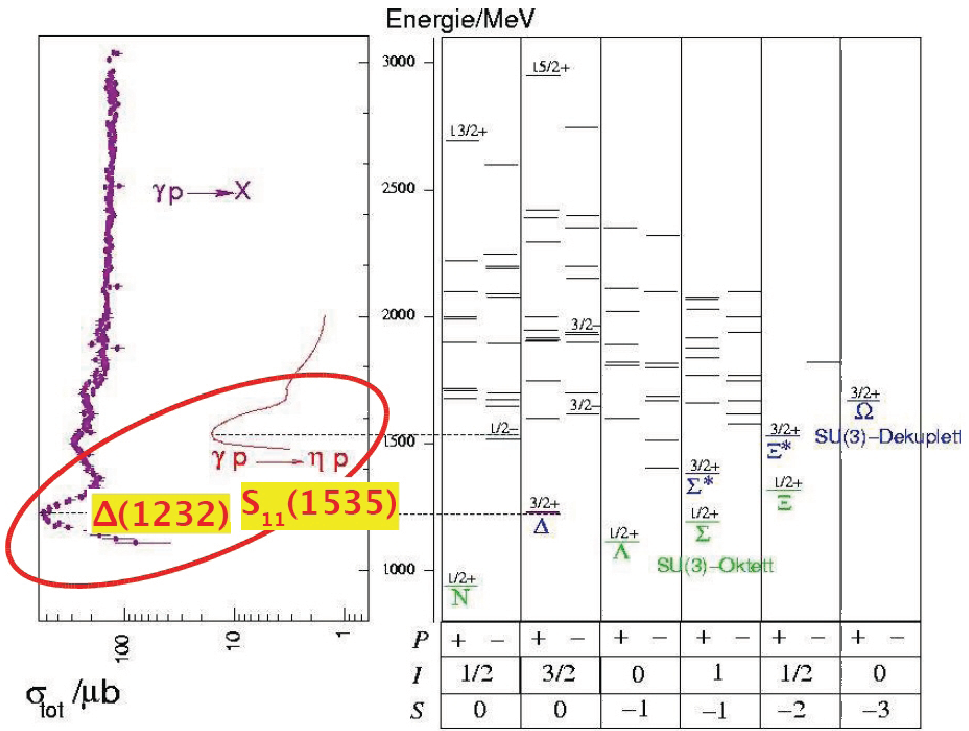
Importance of elastic decreases.

(b) Nucleon Resonances

[PRSZR 7.1]

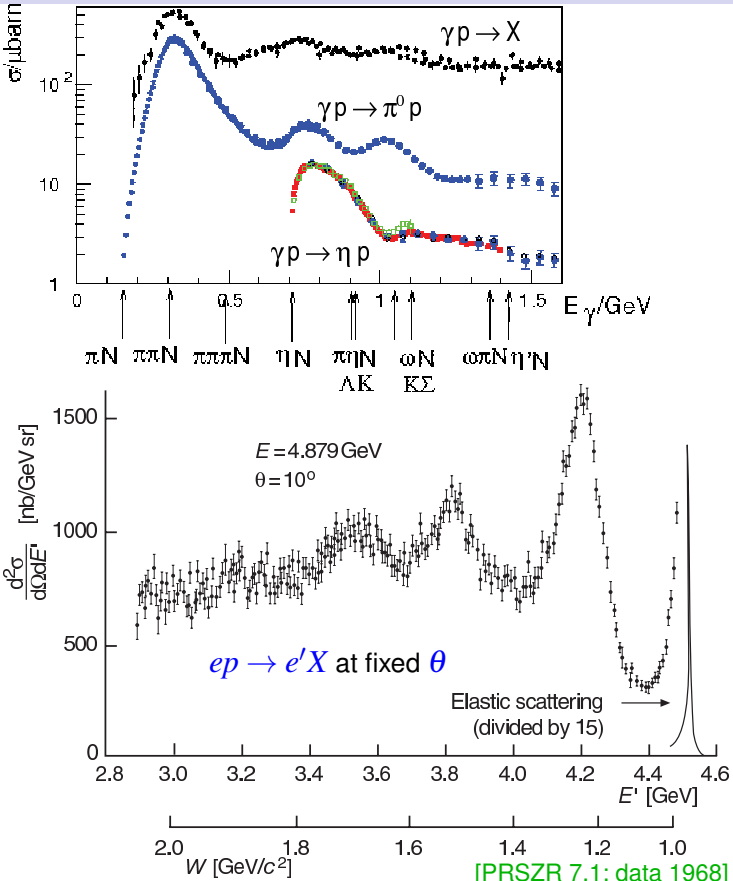


atom



nucleon

More Resonances: Different Probes – Same Pattern



[PRSZR 7.1; data 1968]

$W^2 = (p+q)^2$: inv. mass^2 of fragments

First Resonance Region:

One narrow, isolated bump around

$Q \approx [0.3 \ldots 0.6]\text{GeV} \Leftrightarrow W \approx 1.2\text{GeV}$

also in $\pi^+ p, \pi^0 p, \pi^- p, \pi^- n$ (etc.)

The $\Delta(1232)$: $\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$

Width $\Gamma \approx 100\text{MeV}$ (FWHM)

$$\Longrightarrow \tau = \frac{1}{\Gamma} \approx \frac{2}{3} \times 10^{-23}\text{s}$$

Second Resonance Region:

$Q \sim [0.5 \ldots 1]\text{GeV}$ ($W \sim [1.5 \ldots 2]\text{GeV}$)

Two isolated slightly broader bumps

in $\pi^- p$, $\pi^+ n$, γp

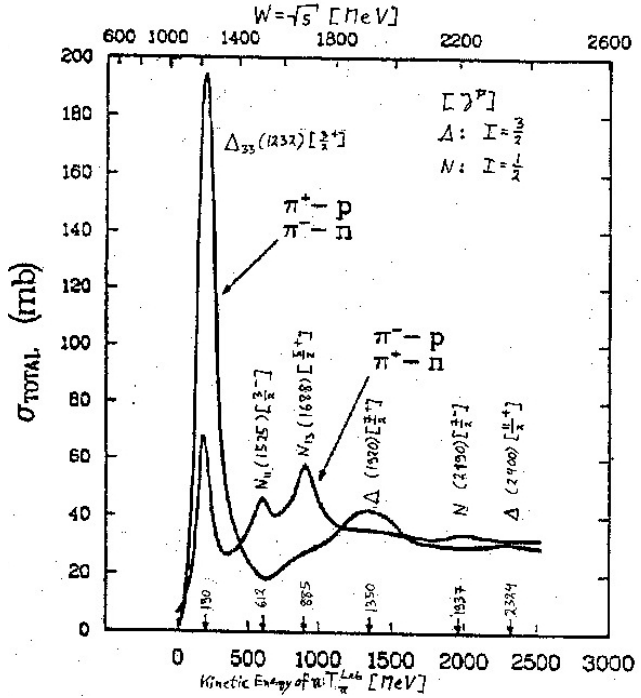
but *nothing* in $\pi^+ p$, $\pi^- n$

(see next slide)

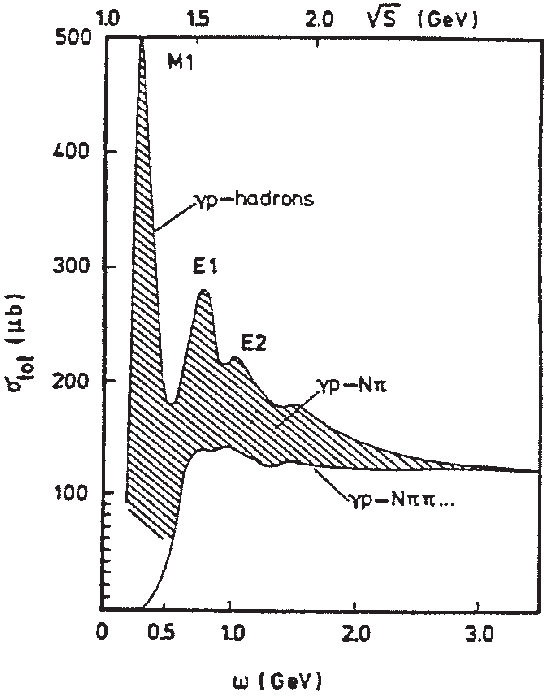
Resonances in $\pi N \to \pi N$ Scattering & $\gamma N \to X$ Photo-Absorption

Not all bumps in all channels, but some pattern!

$\pi N \to \pi N$ **elastic**



$\gamma p \to X$ **photo-absorption (inelastic)**



[Weise: "Quarks, Hadrons, . . .", Les Houches Lecture 1996]

Some Baryon Resonances Identified by the Particle Data Group

Table 1. The status of the N resonances. Only those with an overall status of *** or **** are included in the main Baryon Summary Table.

		Status			Status as seen in —						
Particle	J^P	overall	πN	γN	$N\eta$	$N\sigma$	$N\omega$	ΛK	ΣK	$N\rho$	$\Delta\pi$
N	$1/2^+$	****									
$N(1440)$	$1/2^+$	****	****	****		***				*	***
$N(1520)$	$3/2^-$	****	****	****	***					***	***
$N(1535)$	$1/2^-$	****	****	****	****					**	*
$N(1650)$	$1/2^-$	****	****	***	***			***	**	**	***
$N(1675)$	$5/2^-$	****	****	***	*			*		*	***
$N(1680)$	$5/2^+$	****	****	****	*	**				***	***
$N(1685)$	$?^?$	*									
$N(1700)$	$3/2^-$	***	***	**	*			*	*	*	***
$N(1710)$	$1/2^+$	***	***	***	***		**	***	**	*	**
$N(1720)$	$3/2^+$	****	****	***	***			**	**	**	*
$N(1860)$	$5/2^+$	**	**							*	*
$N(1875)$	$3/2^-$	***	*	***			**	***	**		***
$N(1880)$	$1/2^+$	**	*	*		**		*			
$N(1895)$	$1/2^-$	**	*	**	**			**	*		
$N(1900)$	$3/2^+$	***	**	***	**		**	***	**	*	**
$N(1990)$	$7/2^+$	**	**	**					*		
$N(2000)$	$5/2^+$	**	*	**	**			**	*	**	
$N(2040)$	$3/2^+$	*									
$N(2060)$	$5/2^-$	**	**	**	*				**		
$N(2100)$	$1/2^+$	*									
$N(2150)$	$3/2^-$	**	**	**				**			**
$N(2190)$	$7/2^-$	****	****	***			*	**		*	
$N(2220)$	$9/2^+$	****	****								
$N(2250)$	$9/2^-$	****	****								
$N(2600)$	$11/2^-$	***	***								
$N(2700)$	$13/2^+$	**	**								

**** Existence is certain, and properties are at least fairly well explored.
*** Existence is very likely but further confirmation of quantum numbers and branching fractions is required.
** Evidence of existence is only fair.
* Evidence of existence is poor.

Table 2. The status of the Δ resonances. Only those with an overall status of *** or **** are included in the main Baryon Summary Table.

		Status			Status as seen in —						
Particle	J^P	overall	πN	γN	$N\eta$	$N\sigma$	$N\omega$	ΛK	ΣK	$N\rho$	$\Delta\pi$
$\Delta(1232)$	$3/2^+$	****	****	****	F						
$\Delta(1600)$	$3/2^+$	***	***	***	o					*	***
$\Delta(1620)$	$1/2^-$	****	****	***	r					***	***
$\Delta(1700)$	$3/2^-$	****	****	****	b					**	***
$\Delta(1750)$	$1/2^+$	*	*		i						
$\Delta(1900)$	$1/2^-$	**	**	**			d		**	**	**
$\Delta(1905)$	$5/2^+$	****	****	****			d		***	**	**
$\Delta(1910)$	$1/2^+$	****	****	**			e		*	*	**
$\Delta(1920)$	$3/2^+$	***	***	**			n		***		**
$\Delta(1930)$	$5/2^-$	***	***								
$\Delta(1940)$	$3/2^-$	**	*	**	F				(seen	in	$\Delta\eta$)
$\Delta(1950)$	$7/2^+$	****	****	****	o				***	*	***
$\Delta(2000)$	$5/2^+$	**			r						**
$\Delta(2150)$	$1/2^-$	*	*		b						
$\Delta(2200)$	$7/2^-$	*	*		i						
$\Delta(2300)$	$9/2^+$	**	**				d				
$\Delta(2350)$	$5/2^-$	*	*				d				
$\Delta(2390)$	$7/2^+$	*	*				e				
$\Delta(2400)$	$9/2^-$	**	**				n				
$\Delta(2420)$	$11/2^+$	****	****	*							
$\Delta(2750)$	$13/2^-$	**	**								
$\Delta(2950)$	$15/2^+$	**	**								

**** Existence is certain, and properties are at least fairly well explored.
*** Existence is very likely but further confirmation of quantum numbers and branching fractions is required.
** Evidence of existence is only fair.
* Evidence of existence is poor.

What is the guiding principle/underlying symmetry?

(c) Isospin in Nuclear Physics

Heisenberg 1932

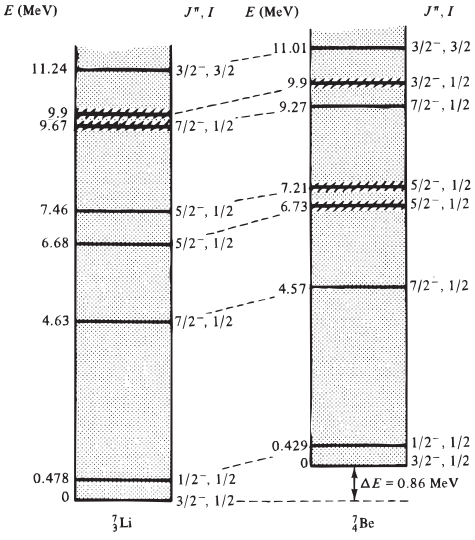


Figure 8.6: Level structure in the two isobars ^{7}Li and ^{7}Be. These two nuclides contain the same number of nucleons; apart from electromagnetic effect, their level schemes should be identical. J^x denotes spin and parity of a level, I its isospin. Parity will be discussed in Chapter 9. [For reference see F. Ajzenberg-Selove, *Nucl. Phys.* **A490**, 1 (1988).]

isodoublet $I = \frac{1}{2}$ [HG]

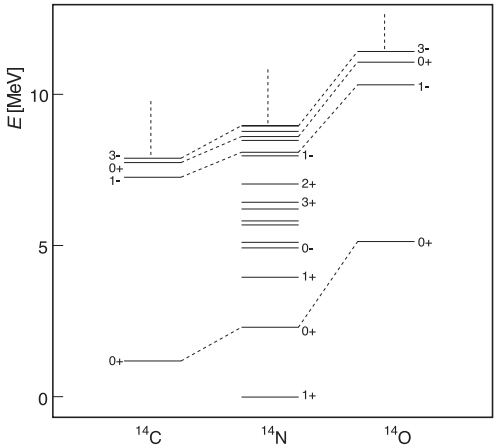


Fig. 2.6. Low-lying energy levels of the three most stable $A = 14$ isobars. Angular momentum J and parity P are shown for the most important levels. The analogous states of the three nuclei are joined by dashed lines. The zero of the energy scale is set to the ground state of $^{14}_{7}N_7$.

[PRSZR]

isotriplet $I = 1$

(d) Isospin in Particle Physics

nucleon — EV to I_3

iso-doublet

$$I(J^P) = \tfrac{1}{2}(\tfrac{1}{2}^+)\begin{pmatrix}|p\rangle \\ |n\rangle\end{pmatrix} \quad \begin{matrix}+\frac{1}{2}\\ -\frac{1}{2}\end{matrix}$$

$$\frac{M_p - M_n}{M_p + M_n} \approx 0.7 \times 10^{-3}$$

pion — EV to I_3

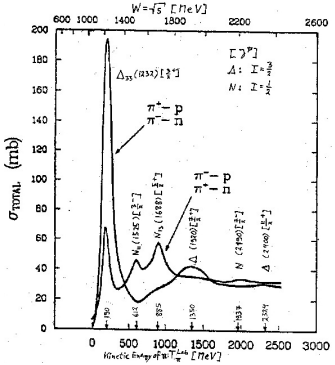
iso-triplet

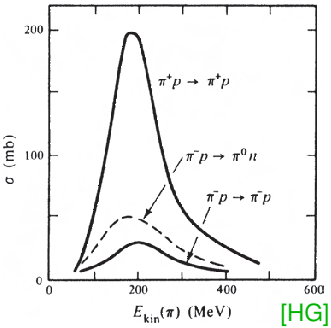
$$I(J^P) = 1(0^-)\begin{pmatrix}|\pi^+\rangle \\ |\pi^0\rangle \\ |\pi^-\rangle\end{pmatrix} \quad \begin{matrix}+1\\ 0\\ -1\end{matrix}$$

$$\frac{m_{\pi^\pm} - m_{\pi^0}}{m_{\pi^\pm} + m_{\pi^0}} \approx 1.7 \times 10^{-2}$$

$\Delta(1232)$ **iso-quadruplet** — EV to I_3

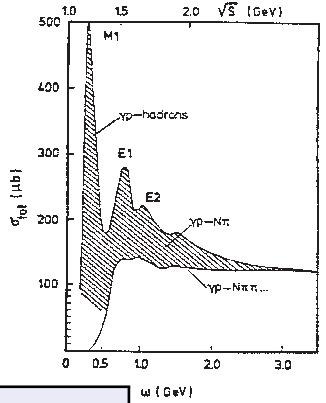
$$I(J^P) = \tfrac{3}{2}(\tfrac{3}{2}^+)\begin{pmatrix}|\Delta^{++}\rangle \\ |\Delta^{+}\rangle \\ |\Delta^{0}\rangle \\ |\Delta^{-}\rangle\end{pmatrix} \quad \begin{matrix}+\frac{3}{2}\\ +\frac{1}{2}\\ -\frac{1}{2}\\ -\frac{3}{2}\end{matrix}$$





[HG]

Figure 14.5: Cross sections for the low-energy elastic and charge-exchange pion–proton reactions.



Relation between charge, baryon number, isospin:
Gell-Mann–Nishijima without strangeness

$$Q = I_3 + \frac{B}{2}$$

Postulate: **Strong Interactions are approximately isospin-independent:** $[\vec{I}, H_{\text{strong}}] \approx 0$.

Mass difference from electromagnetism & small explicit breaking: $\varepsilon(H_{\text{em}} + H_{\text{expl}})$ **with** $\varepsilon \sim 10^{-2}$.

(e) Iso-Multiplets Predict Cross Section Ratios

Postulate: Isospin Multiplets have common origin. $\Longrightarrow$ Combine like spin.

$$\vec{I}_{\rm tot} = \vec{I}_N \otimes 1_\pi + 1_N \otimes \vec{I}_\pi \Longrightarrow |IM\rangle = \sum_{m_N m_\pi} \underbrace{(IM|I_N m_n, I_\pi m_\pi)}_{\text{Clebsch-Gordan}} |I_N m_N\rangle \otimes |I_\pi m_\pi\rangle$$

with $M = m_N + m_\pi$, $|I_N - I_\pi| \leq I \leq I_N + I_\pi$ like spinology.

$\Longrightarrow \langle I'M'|\mathcal{M}_{\rm strong}|IM\rangle = \delta_{II'}\ \delta_{MM'}\ \mathcal{M}_{2I+1}(E)$: **reduced ME is function of E only, labelled by I.**

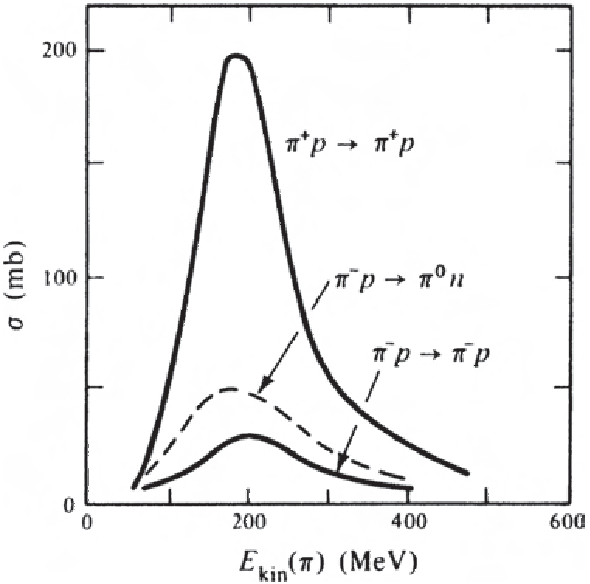
$\frac{1}{2}_{\rm N} \otimes 1_\pi = \frac{1}{2} \oplus \frac{3}{2} \Longrightarrow$ **2 independent functions $\mathcal{M}_2(E)$, $\mathcal{M}_4(E)$ characterise 8 πN processes:**

6 direct/elastic reactions: $N\pi^{\pm 0} \to N\pi^{\pm 0}$ & 2 charge-exchange reactions $p\pi^0 \leftrightarrow n\pi^+$, $p\pi^- \leftrightarrow n\pi^0$

Clebsch-Gordan coefficient $(IM|I_N m_n, I_\pi m_\pi)$

N doublet		π triplet		$I = \frac{3}{2}$				$I = \frac{1}{2}$	
$\lvert I_N = \frac{1}{2}; m_N\rangle$		$\lvert I_\pi = 1; m_\pi\rangle$		$M = \frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
p	$+\frac{1}{2}$	π^+	$+1$	1					
n	$-\frac{1}{2}$	π^+	$+1$		$\sqrt{\frac{1}{3}}$			$\sqrt{\frac{2}{3}}$	
p	$+\frac{1}{2}$	π^0	0		$\sqrt{\frac{2}{3}}$			$-\sqrt{\frac{1}{3}}$	
n	$-\frac{1}{2}$	π^0	0			$\sqrt{\frac{2}{3}}$			$\sqrt{\frac{1}{3}}$
p	$+\frac{1}{2}$	π^-	-1			$\sqrt{\frac{1}{3}}$			$-\sqrt{\frac{2}{3}}$
n	$-\frac{1}{2}$	π^-	-1				1		

Compare to Data



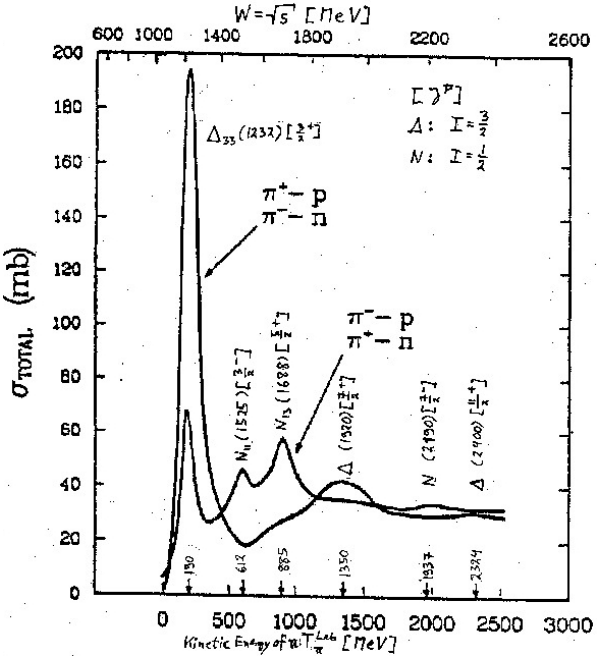


Figure 14.5: Cross sections for the low-energy elastic and charge-exchange pion–proton reactions.

Baryon Resonances, Again: $I(J^P)$ Assignment

Nucleon Resonances: $I = \frac{1}{2}$

Table 1. The status of the N resonances. Only those with an overall status of *** or **** are included in the main Baryon Summary Table.

Particle J^P	Status overall	Status πN	Status γN	Status as seen in — $N\eta$	$N\sigma$	$N\omega$	ΛK	ΣK	$N\rho$	$\Delta\pi$
N $1/2^+$	****									
$N(1440)$ $1/2^+$	****	****	****		***				*	***
$N(1520)$ $3/2^-$	****	****	****	***					***	***
$N(1535)$ $1/2^-$	****	****	****	****					**	*
$N(1650)$ $1/2^-$	****	****	***	***			***	**	**	***
$N(1675)$ $5/2^-$	****	****	***	*			*		*	***
$N(1680)$ $5/2^+$	****	****	****	*	**				***	***
$N(1685)$ $?^?$	*									
$N(1700)$ $3/2^-$	***	***	**	*			*	*	*	***
$N(1710)$ $1/2^+$	***	***	***	***		**	***	**	*	**
$N(1720)$ $3/2^+$	****	****	***	***			**	**	**	*
$N(1860)$ $5/2^+$	**	**							*	*
$N(1875)$ $3/2^-$	***	*	***			**	***	**		***
$N(1880)$ $1/2^+$	**	*	*		**		*			
$N(1895)$ $1/2^-$	**	*	**	**			**	*		
$N(1900)$ $3/2^+$	***	**	***	**		**	***	**	*	**
$N(1990)$ $7/2^+$	**	**	**					*		
$N(2000)$ $5/2^+$	**	*	**	**			**	*	**	
$N(2040)$ $3/2^+$	*									
$N(2060)$ $5/2^-$	**	**	**	*				**		
$N(2100)$ $1/2^+$	*									
$N(2150)$ $3/2^-$	**	**	**				**			**
$N(2190)$ $7/2^-$	****	****	***			*	**		*	
$N(2220)$ $9/2^+$	****	****								
$N(2250)$ $9/2^-$	****	****								
$N(2600)$ $11/2^-$	***	***								
$N(2700)$ $13/2^+$	**	**								

**** Existence is certain, and properties are at least fairly well explored.
*** Existence is very likely but further confirmation of quantum numbers and branching fractions is required.
** Evidence of existence is only fair.
* Evidence of existence is poor.

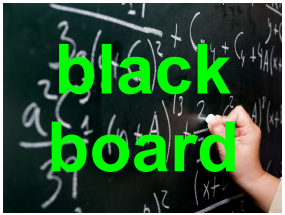
Δ Resonances: $I = \frac{3}{2}$

Table 2. The status of the Δ resonances. Only those with an overall status of *** or **** are included in the main Baryon Summary Table.

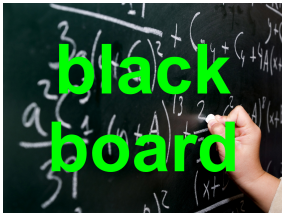
Particle J^P	Status overall	Status πN	Status γN	Status as seen in — $N\eta$	$N\sigma$	$N\omega$	ΛK	ΣK	$N\rho$	$\Delta\pi$
$\Delta(1232)$ $3/2^+$	****	****	****	F						
$\Delta(1600)$ $3/2^+$	***	***	***	o					*	***
$\Delta(1620)$ $1/2^-$	****	****	***	r					***	***
$\Delta(1700)$ $3/2^-$	****	****	****	b					**	***
$\Delta(1750)$ $1/2^+$	*	*		i						
$\Delta(1900)$ $1/2^-$	**	**	**			d		**	**	**
$\Delta(1905)$ $5/2^+$	****	****	****			d		***	**	**
$\Delta(1910)$ $1/2^+$	****	****	**			e		*	*	**
$\Delta(1920)$ $3/2^+$	***	***	**			n		***		**
$\Delta(1930)$ $5/2^-$	***	***								
$\Delta(1940)$ $3/2^-$	**	*	**	F				(seen	in	$\Delta\eta$)
$\Delta(1950)$ $7/2^+$	****	****	****	o				***	*	***
$\Delta(2000)$ $5/2^+$	**			r						**
$\Delta(2150)$ $1/2^-$	*	*		b						
$\Delta(2200)$ $7/2^-$	*	*		i						
$\Delta(2300)$ $9/2^+$	**	**				d				
$\Delta(2350)$ $5/2^-$	*	*				d				
$\Delta(2390)$ $7/2^+$	*	*				e				
$\Delta(2400)$ $9/2^-$	**	**				n				
$\Delta(2420)$ $11/2^+$	****	****	*							
$\Delta(2750)$ $13/2^-$	**	**								
$\Delta(2950)$ $15/2^+$	**	**								

**** Existence is certain, and properties are at least fairly well explored.
*** Existence is very likely but further confirmation of quantum numbers and branching fractions is required.
** Evidence of existence is only fair.
* Evidence of existence is poor.

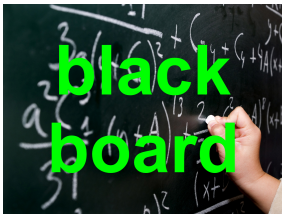
(f) Isospin-Invariant Interactions



(g) Photons and Isospin



(h) The Fundamental Doublet



Next: 4. Deep Inleastic Scattering and Partons

Familiarise yourself with: [HM 9; PRSZR 7.2, 8.1/4-5; HG 6.8-10]