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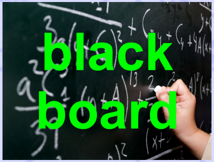
Spring 2023

II. Phenomena

2. Hadronic Form Factors

Or: We Thought the Matter was Closed. . .

References: [HM 8.2 (th); HG 6.5/6; Tho 7.5; Ann. Rev. Nucl. Part. Sci. **54** (2004) 217]



and optional additional details in script.

(a) Recap: Currents & Form Factors of Spin-$\frac{1}{2}$ Target

Most general current for spin-$\frac{1}{2}$ target:

$$J^{\mu}_{S,S'} = -\mathrm{i}\underbrace{e\,F_1(q^2)\,\bar{u}_{S'}(p')\gamma^{\mu}u_S(p)}_{\text{Dirac: modify point-form}} \qquad \text{(I.7.5C)}$$

$$+\underbrace{\frac{e}{2M}F_2(q^2)\,q_\nu\,\bar{u}_{S'}(p')\mathrm{i}\sigma^{\mu\nu}u_S(p)}_{\text{Pauli: anomalous mag. term}}$$

$F_1(0) = Z$ charge; $F_2(0) = \kappa$ anom. mag. mom.

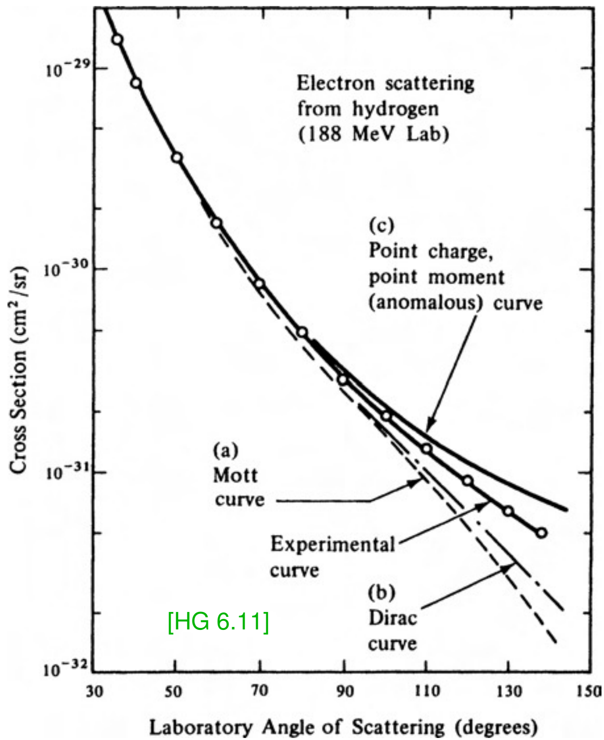
Sachs FFs:

$$G_E = F_1 - \tau F_2,\; G_M = F_1 + F_2; \qquad \tau = -\frac{q^2}{4M^2}$$

Rosenbluth formula/Sachs cross section:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) \Big/ \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott= } e \text{ on point spin-0}}\Bigg|_{\text{lab}} \qquad \text{(I.7.5)}$$

$$= \left[\frac{G_E^2 + \tau\,G_M^2}{1+\tau} + 2\tau\,G_M^2\,\overbrace{\tan^2\frac{\theta}{2}}^{\text{spin-flip}}\right]$$



(b) FF Interpretation in the Breit/Brick-Wall Frame

“Electric” and “magnetic” are frame-dependent decompositions. $\Longrightarrow$ Careful!

One Can Show: The Sachs Form Factors $G_E(q^2)$ and $G_M(q^2)$ are indeed the form factors of electric charge and magnetic current inside the target *in one particular frame*:

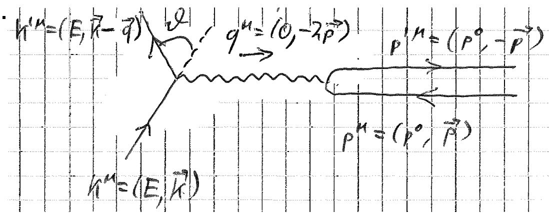
Breit/Brick-Wall Frame

$E_\mathrm{B} = E'_\mathrm{B} \Longrightarrow q^0 := k'^0_\mathrm{B} - k^0_\mathrm{B} = 0$ No energy transfer.

$\vec{p}_\mathrm{B} = -\vec{p}'_\mathrm{B}$ Nucleon recoils like from brick wall.

$$\Longrightarrow \vec{q}_\mathrm{B} = -2\vec{p}_\mathrm{B},\ \sphericalangle(\vec{k}_B,\vec{q}_B) \equiv \sphericalangle(\vec{k}_B,\vec{p}_B)$$

$$\Longrightarrow t = (k'-k)^2 = -2k\cdot k' = -2E_\mathrm{B}^2(1-\cos\theta_\mathrm{B})$$
$$= -2\vec{k}_\mathrm{B}\cdot\vec{q}_\mathrm{B} = +4E_\mathrm{B}\,|\vec{p}_\mathrm{B}|\cos\sphericalangle(\vec{k}_B,\vec{p}_B)$$



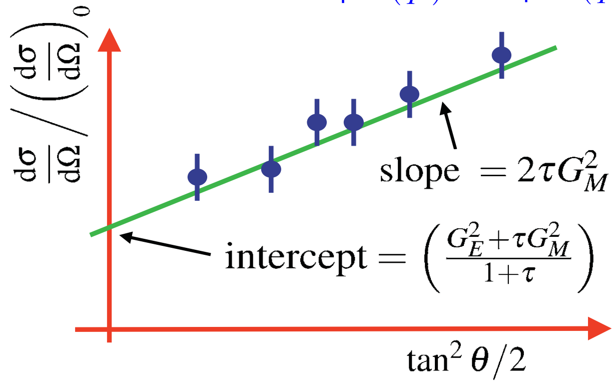
θ_B small $\Longrightarrow |\vec{p}_\mathrm{B}| = \frac{1}{2}|\vec{q}_\mathrm{B}|$ small: grazing shot

θ_B large $\Longrightarrow |\vec{p}_\mathrm{B}| = \frac{1}{2}|\vec{q}_\mathrm{B}|$ large: head-on collision

Optional additional details in script.

(c) Rosenbluth Separation

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) \Big/ \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}}\Bigg|_{\text{lab}} = \Bigg[\underbrace{\frac{G_E^2 + \tau\, G_M^2}{1+\tau}}_{\text{intercept } A(q^2)} + \underbrace{2\tau\, G_M^2}_{\text{slope } B(q^2)} \quad \tan^2\frac{\theta}{2} \Bigg]$$



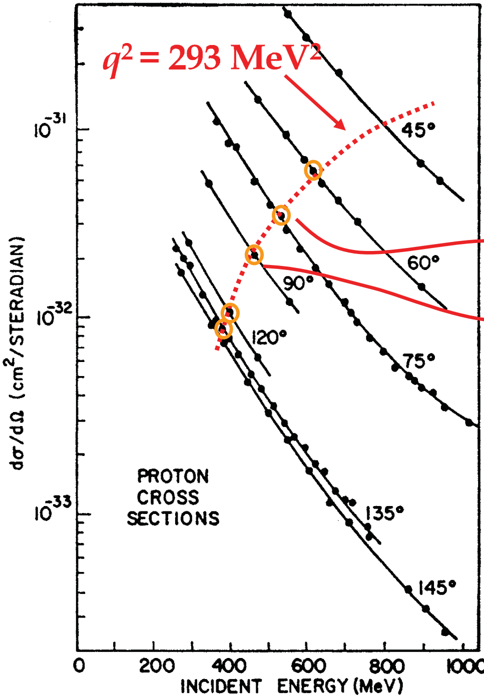
For $q^2 \to 0$: $\tau = -\frac{q^2}{4M^2} \to 0 \implies \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) \Big/ \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} \to G_E^2(q^2) \to 1 - \frac{q^2}{3!}\langle r_E^2\rangle$

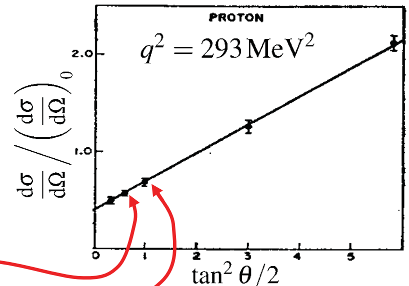
For $q^2 \to -\infty$: $\tau = -\frac{q^2}{4M^2} \to +\infty \implies \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) \Big/ \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} \to \left(1 + 2\tau\tan^2\frac{\theta}{2}\right) G_M^2(q^2)$

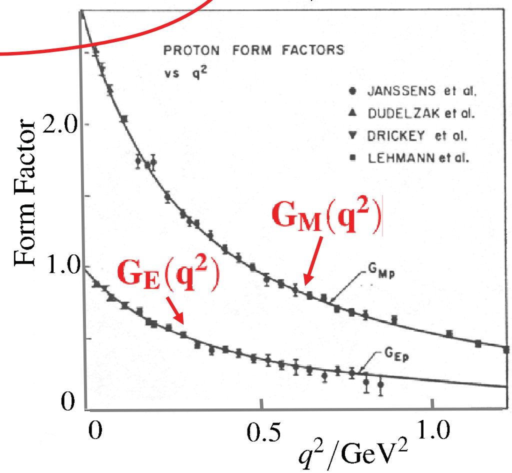
$\implies$ Each limit has 1 FF which is difficult to measure, and 1 easy one.

- **Electron beam energies chosen to give certain values of** q^2
- **Cross sections measured to 2-3 %** $q^2 = -2EE'(1 - \cos\theta_{\text{lab}}) \leq 0$

E.B.Hughes et al., Phys. Rev. 139 (1965) B458







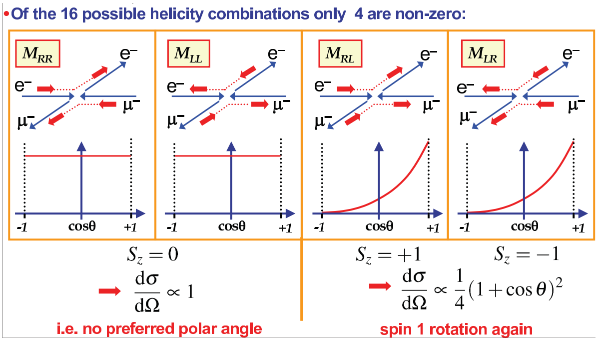
Form Factors at "Any" Q^2 from Polarisation Transfer

MAMI, JLab, SLAC,...

unpolarised beam & target
outgoing spins undetected $\left(\frac{d\sigma}{d\Omega}\right) \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}\Bigg|_{\text{lab}} = \left[\frac{G_E^2 + \tau\, G_M^2}{1+\tau} + 2\tau\, G_M^2 \tan^2\frac{\theta}{2}\right]$

$\Longrightarrow$ Each limit $Q^2 \to 0, \infty$ has 1 FF which is difficult to measure, and 1 easy one: How to do better?

Polarisation-Transfer Method: Use helicity conservation to separate electric and magnetic.



mostly $\propto P^{(\gamma)}_{\text{trans}}\, G_M$ [cf. Tho 8.6] $\propto P^{(\gamma)}_{\text{long}}\, G_E$

Amplitudes have different spin-transfer $\vec{e} \to \vec{p}$:

$\Longrightarrow$ Scatter polarised e^- with definite helicity,
measure recoil p's polarisation (not easy).

longitudinal ("Coulomb") photon: $J_z = 0$

transverse ("real") photon: $J_z = \pm 1 =$ right / left

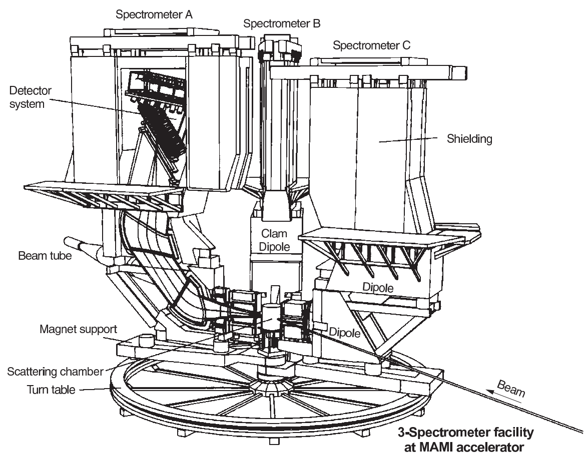
γ-polarisations $P^{(\gamma)}_{\text{long/trans}}$ by e-spin, kinematics.

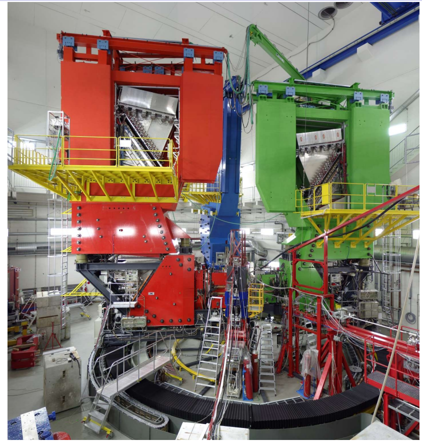
$\Longrightarrow$ Spin-dep. measurement uses QM interference of amplitudes: $\frac{G_E(Q^2)}{G_M(Q^2)} = -\frac{E+E'}{2M}\,\frac{P^{(\gamma)}_{\text{trans}} \tan\frac{\theta}{2}}{P^{(\gamma)}_{\text{long}}}$

No absolute cross section, no absolute beam & recoil polarimetry. $\Longrightarrow$ Many systematics cancel.

So accurate that discrepancies to Rosenbluth led to theory update (2γ exchange) [Afanasev/...2008].

(d) Experiments: Magnetic Spectrometers SLAC, MAMI, Jlab,. . .

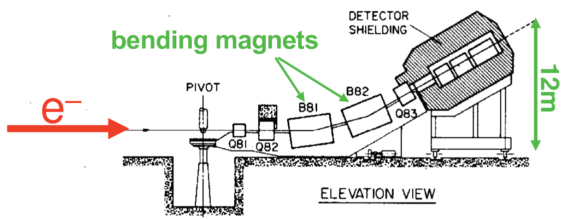


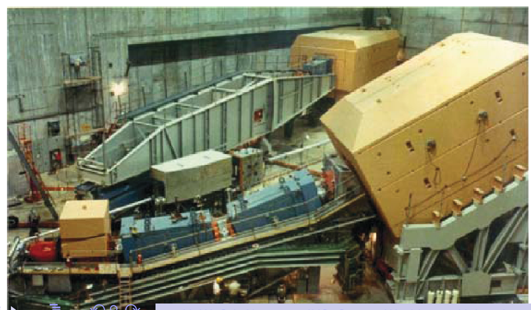


[PRSZR] **Fig. 5.4.** Experimental set-up for the measurement of electron scattering off pro- MAMI-A1 (URL) Spectrometers

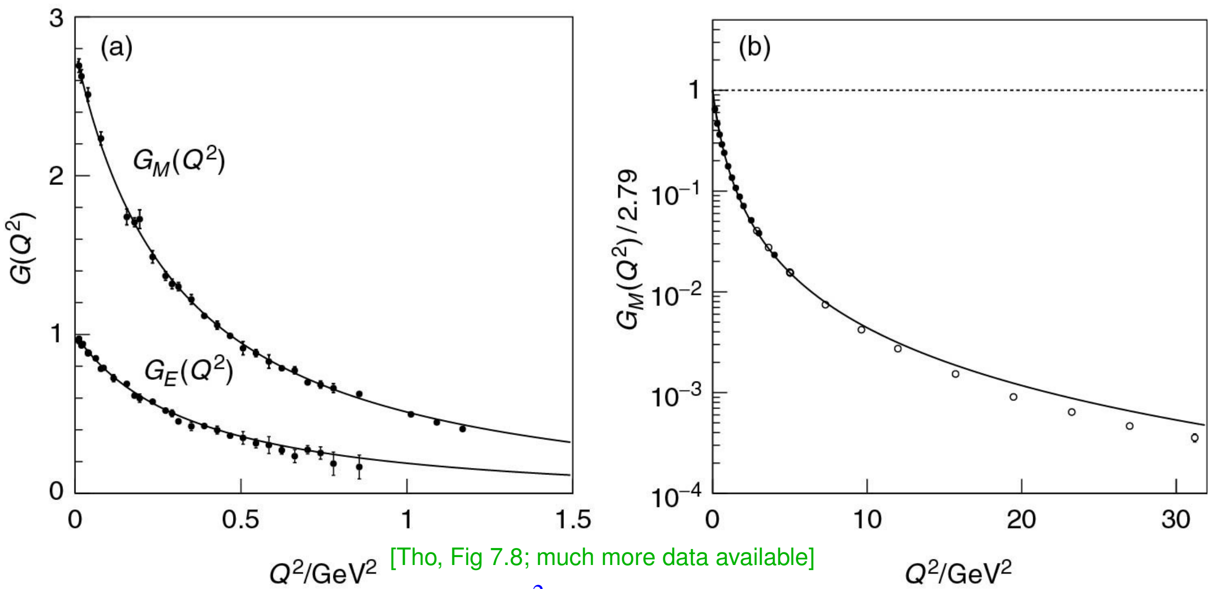
★Use electron beam from SLAC LINAC: $5 < E_{\text{beam}} < 20$ GeV

• Detect scattered electrons using the "8 GeV Spectrometer"





(e) Proton Form Factors: Why So Simple?



[Tho, Fig 7.8; much more data available]

Exp. at low Q^2: **dipole** $G_E \approx \left(1+\frac{Q^2}{a^2}\right)^{-2} \approx \frac{G_M}{\mu^p = 2.79\dots}$ with $a = 4.27\ \mathrm{fm}^{-1} = 0.84\ \mathrm{GeV}$

$\Longrightarrow \rho(r) = \rho_0\, \mathrm{e}^{-ra}$ exponential (in Breit frame) $\quad \langle r^2_{Ep}\rangle = -3!\left.\frac{\mathrm{d}G_E}{\mathrm{d}Q^2}\right|_{Q=0} = \frac{12}{a^2} \approx (0.82\ \mathrm{fm})^2$

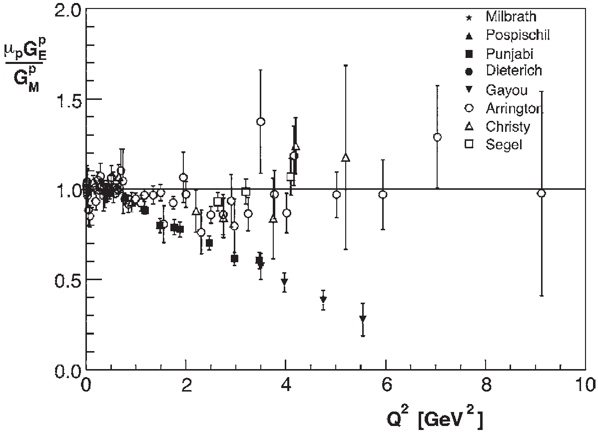
high-accuracy data at $Q^2 \to 0$: $\langle r^2_{Ep}\rangle = ([0.8775 \pm 0.0051]\ \mathrm{fm})^2$ [PDG 2012 but see in a moment]

$Q^2 \to 0 \quad \langle r^2_{Mp}\rangle = ([0.851 \pm 0.026]\ \mathrm{fm})^2$ [PDG 2022]

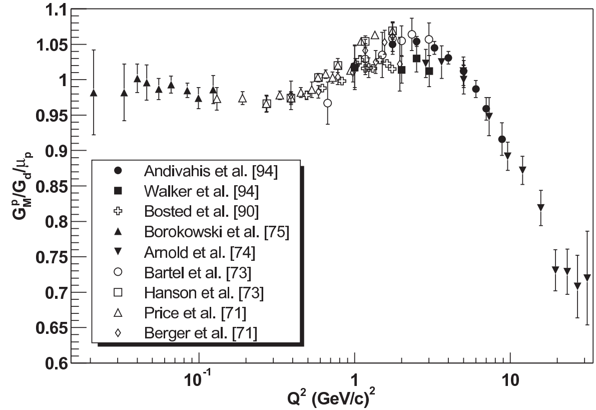
There Is Some Deviation from Simple 1-Dipole Form at High Q^2

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \left[\frac{G_E^2 + \tau\, G_M^2}{1+\tau} + 2\tau\, G_M^2 \tan^2\frac{\theta}{2}\right] \times \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}},\; \tau = \frac{Q^2}{4M^2}$$

Ratio electric-to-magnetic proton FF



Magnetic proton FF: deviation from dipole



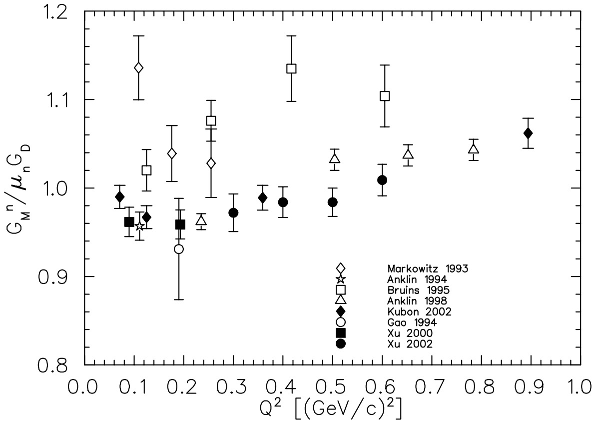
Dipole $G_M(Q^2) \propto \dfrac{1}{\left(1+\frac{Q^2}{a^2}\right)^2}$ largely ok $\Longrightarrow$

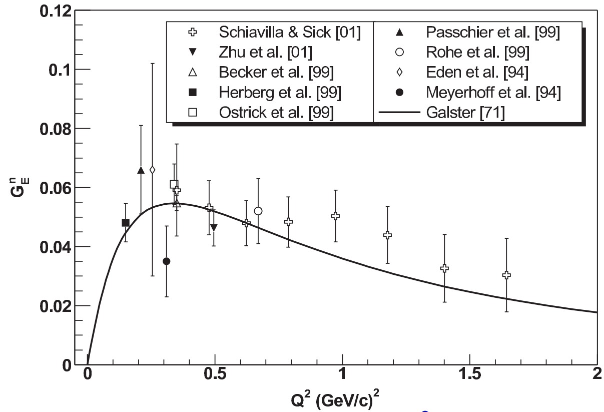
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} (Q^2 \to \infty, \text{ i.e. } \tau = \frac{Q^2}{4M^2} \to \infty) \propto \tau\, |G_M(Q^2)| \propto \frac{\tan^2\frac{\theta}{2}}{Q^6} \times \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}$$

$\Longrightarrow Q^2 \to \infty$ dominated by G_M.

(f) Neutron Form Factors: Why So Similar to Proton?

No neutron targets. $\Longrightarrow d(e,e')$ & subtract binding effects; or at $Q^2 \to 0$: scatter n off atomic e^- cloud.





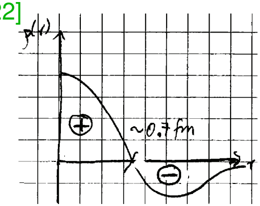
Low Q^2: nearly same dipole as proton for $\dfrac{G_M^n}{\mu^n = -1.91\dots} \approx \left(1 + \dfrac{Q^2}{(0.84\ \mathrm{GeV})^2}\right)^{-2}$

high-accuracy data: $\langle r^2_{Mn}\rangle = ([0.864 \pm 0.009]\ \mathrm{fm})^2 \approx \langle r^2_{Ep}\rangle \approx \langle r^2_{Mp}\rangle$ [PDG 2022]

$\langle r^2_{En}\rangle = -[0.1155 \pm 0.0017]\ \mathrm{fm}^2 < 0$!!

This is allowed: $\displaystyle\int \frac{\mathrm{d}^3 r}{(2\pi)^3}\, r^2\, \big[|\rho_+(r)| - |\rho_-(r)|\big]$

$= \langle r_+^2\rangle - \langle r_-^2\rangle \gtrless 0$!



$\Longrightarrow$ On average, *negative-charged* neutron constituents farther from centre than *positive-charged* ones.

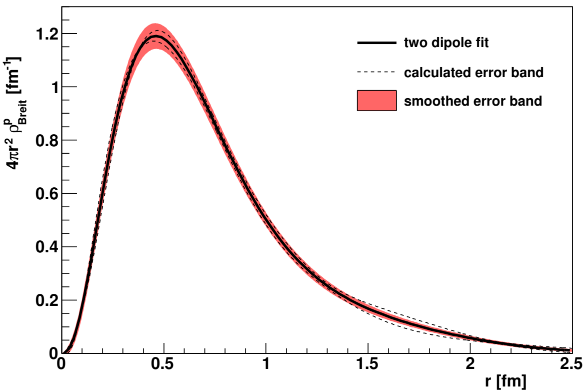
(g) Reconstructed Charge-Densities in Breit Frame

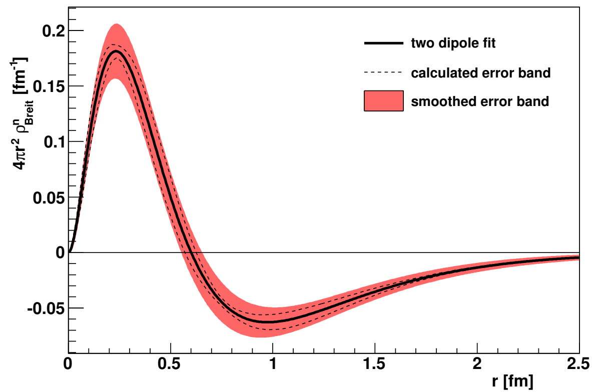
Nucleon Form Factors have surprisingly simple form:

Excellent fit with two dipoles: $G_E^N(Q^2) \simeq \frac{b}{\left(1+\frac{Q^2}{a_1}\right)^2} + \frac{1-b}{\left(1+\frac{Q^2}{a_2}\right)^2}$

$\Longrightarrow 4\pi r^2 \rho(r) \propto r^2 \left[b\, \mathrm{e}^{-ra_1} + (1-b)\, \mathrm{e}^{-ra_2}\right]$ is a pretty good representation.

proton **neutron**

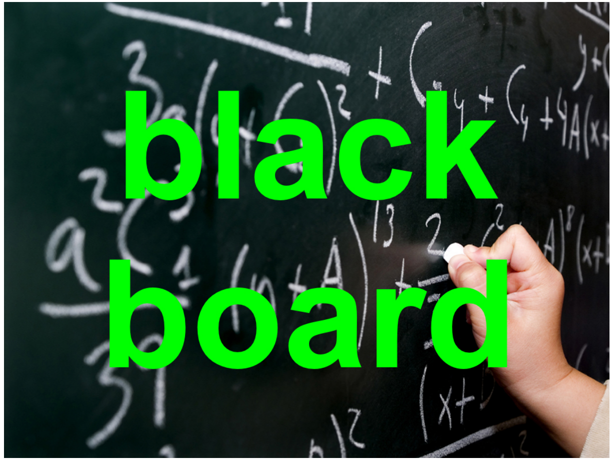




[Crawford et al. PR**C82** (2010) 045211 and 2010 LRP]

(h) Not Even A Model: Meson-Cloud Argument

[PRSZ 6.3]
[HG 6.6]



QFT: Every particle has a virtual cloud. $\Longrightarrow$ **Even point-particle has** $F(Q^2) \neq 1$.

RMS of hadron FFs set by $\dfrac{1}{2 \times \text{mass of lightest constituent of cloud – typically } m_\pi}$

$$\Longrightarrow |\langle r^2 \rangle|_{\text{hadron}} \simeq (0.7\text{fm})^2$$

But need virtual particles produced at good rates! HW!

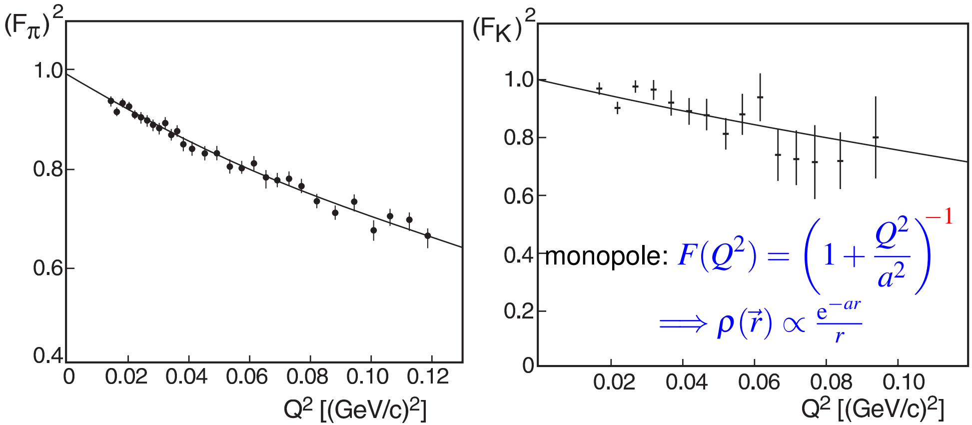
Application: Meson Form Factors Still Quite Simple

Expect $|\langle r^2\rangle| \approx (0.7\text{ fm})^2$ of all hadrons still set by pion cloud.

Pion, Kaon: spin $0 \implies$ only electric $F(q^2)$, no magnetic FF.

Unstable Particle $\implies$ Experiment in "inverse kinematics": (cf. neutron)

scatter secondary beam on electron cloud of atoms, detect *recoil electron* (not meson)



monopole: $F(Q^2) = \left(1 + \frac{Q^2}{a^2}\right)^{-1}$

$\implies \rho(\vec{r}) \propto \frac{e^{-ar}}{r}$

Fig. 6.4. Pion and kaon form factors as functions of Q^2 (from [Am84] and [Am86]). The solid lines correspond to a monopole form factor, $(1 + Q^2/a^2\hbar^2)^{-1}$. [PRSZR]

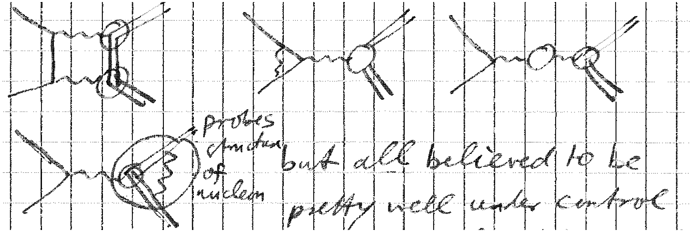
$$\langle r_\pi^2\rangle = \frac{6}{a_\pi^2} = ([0.67 \pm 0.02]\text{ fm})^2 \qquad \langle r_K^2\rangle = \frac{6}{a_\pi^2} = ([0.58 \pm 0.04]\text{ fm})^2$$ (s-quark!)

(i) List of Accomplishments [PRSZR 6.3, HG 6.6]

– Measurements so accurate that one has to go beyond **One-Photon Approximation**:

Example contributions at $\mathcal{O}(\alpha^3)$:

[Afanasev, Koshchii, Solyanik]



– Nucleons have common **dipole** form: $G_E^p \approx \frac{G_M^p}{\mu^p} \approx \frac{G_M^n}{\mu^n} \approx \left(1+\frac{Q^2}{(0.84\text{ GeV})^2}\right)^{-2}$

$$\Longrightarrow \rho(r) = \rho_0\, \mathrm{e}^{-ra},\ a = 4.27\text{ fm}^{-1}.$$

– $\langle r^2_{Ep}\rangle \approx \langle r^2_{Mp}\rangle \approx \langle r^2_{Mn}\rangle \approx (0.8\text{ fm})^2 \approx \frac{1}{(2m_\pi)^2}$.

– Distribution of charges inside hadrons similar, but different to that of currents.

– **Proton:** positive charges more on surface; mag. currents less spread.

– **Neutron:** $\langle r^2_{En}\rangle \lesssim 0$: charges about equally distributed, but negative charges more on surface.

	$\langle r^2_E\rangle$	$\langle r^2_M\rangle$
proton	$([0.8775 \pm 0.0051]\text{ fm})^2$	$([0.851 \pm 0.026]\text{ fm})^2$
neutron	$-\ [0.1155 \pm 0.0017]\text{ fm}^2$	$([0.864 \pm 0.009]\text{ fm})^2$

– **Mesons: Monopole** FFs with small dependence on constituent quark content.

(j) …Then Someone Had To Do An Experiment

Downie
Briscoe

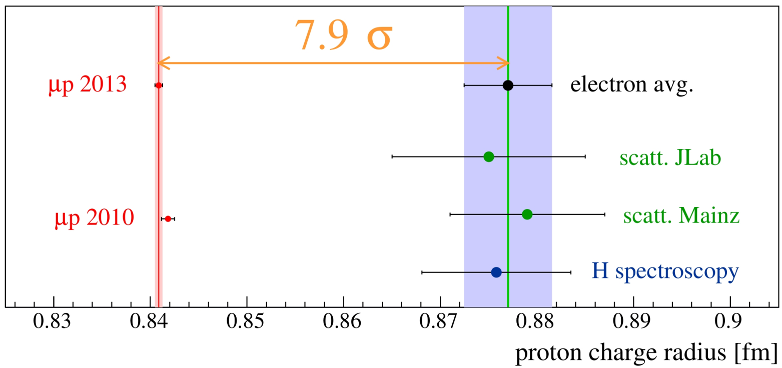
New Method in 2000: Hyperfine Splitting $\propto \vec{\sigma}_e \cdot \vec{\sigma}_p \left[\delta^{(3)}(\vec{r}) + \langle r_p^2 \rangle \vec{\nabla}^2 \delta^{(3)}(\vec{r})\right] + \dots$ [Hänsch et al.]

$\Longrightarrow$ **Atomic Precision Spectroscopy**: $\langle r_p^2 \rangle$ from Hydrogen-atom *near-identical*, compatible error bars.

$\Longrightarrow$ **Until 2010:** static properties of proton very well known.

Idea: Muonic Hydrogen μH: $m_\mu \approx 200 m_e \Longrightarrow$ Bohr-radius of μH is $a_B(\mu) \approx \dfrac{a_B}{m_\mu / m_e \approx 200}$:

$\Longrightarrow$ μ closer to proton $\Longrightarrow$ Better signal. Indeed, much smaller error bars.



μH result is 7 standard-deviations off accepted value!!

[PDG 2014-2019] **"The μp and ep results for the charge radius are much too different to average them. The disagreement is not yet understood."**

How do we Resolve the Radius Puzzle

Beyond-The-Standard Model: Break **Lepton Universality**:

An interaction which is seen by μ but not by e??

- New data needed to test that the e and μ are really different, and the implications of novel BSM and hadronic physics
  - **BSM:** scattering modified for Q^2 up to m^2_{BSM} (typically expected to be MeV to 10s of MeV), enhanced parity violation
  - **Hadronic:** enhanced 2γ exchange effects [Afanasev, Koshchii, Solyanik]

MUSE tests these

- Experiments include:
  - Redoing atomic hydrogen
  - Light muonic atoms for radius comparison in heavier systems **CREMA**
  - Redoing electron scattering at lower Q^2 **Jlab & Mainz**
  - **Muon scattering!**

And, of course: check & recheck *Theory* of previous analyses!!

Sagan: Extraordinary claims need extraordinary evidence!

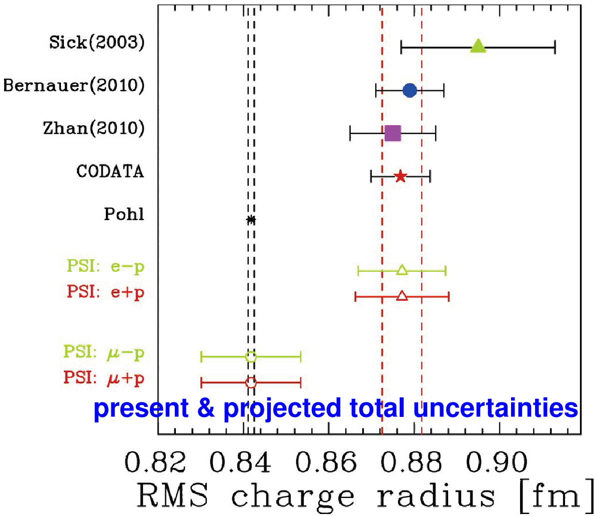
The *MU*on Proton *S*cattering *E*xperiment MUSE at PSI

Link to proposal here. **GW: Downie (spokesperson), Briscoe, Afanasev, Lavrukhin, Ratvasky,...**

Idea: Use PSI mixed meson/muon/electron beam at $E_{\text{beam}} = 115, 153, 210\,\text{MeV}$

to **simultaneously** measure ep and μp and πp scattering.

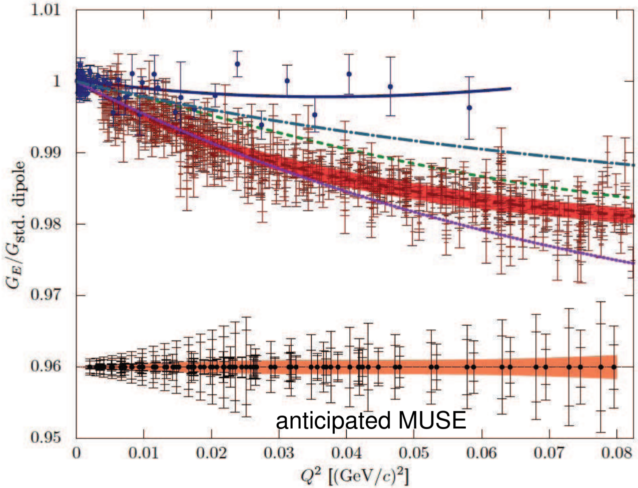
$\Longrightarrow$ **Simultaneous determination of proton radius from $e^- p$ and $e^+ p$ and $\mu^- p$ and $\mu^+ p$.**

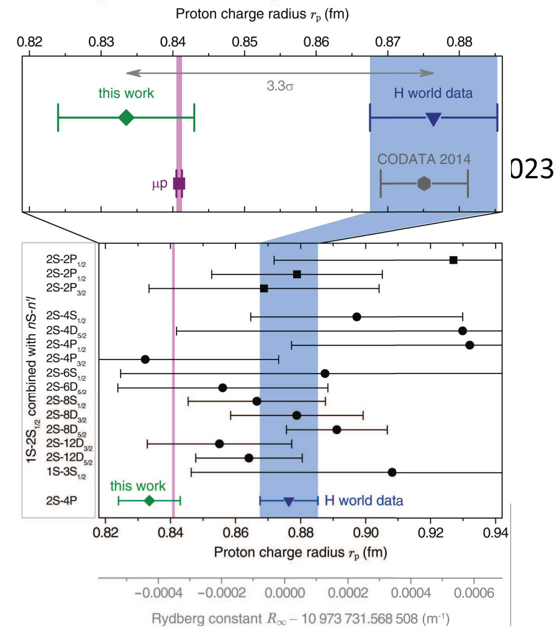


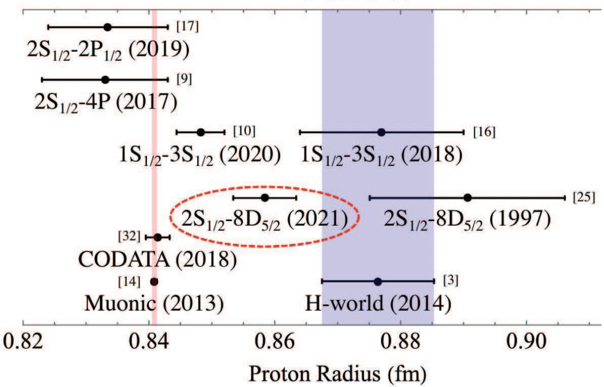
Goals: – Test theory understanding of two-photon effects. – Test Lepton Universality.

Funding by NSF: US$2.5M

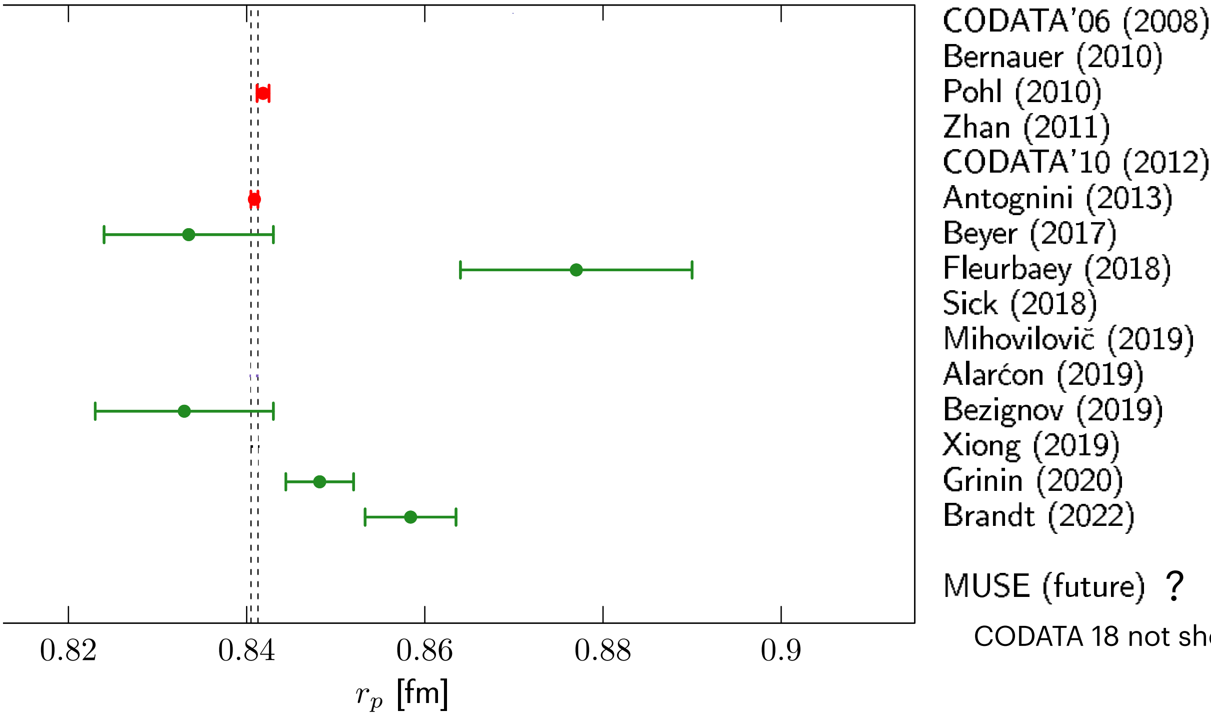
From Data to Values: e Scattering vs. Atomic Spectra



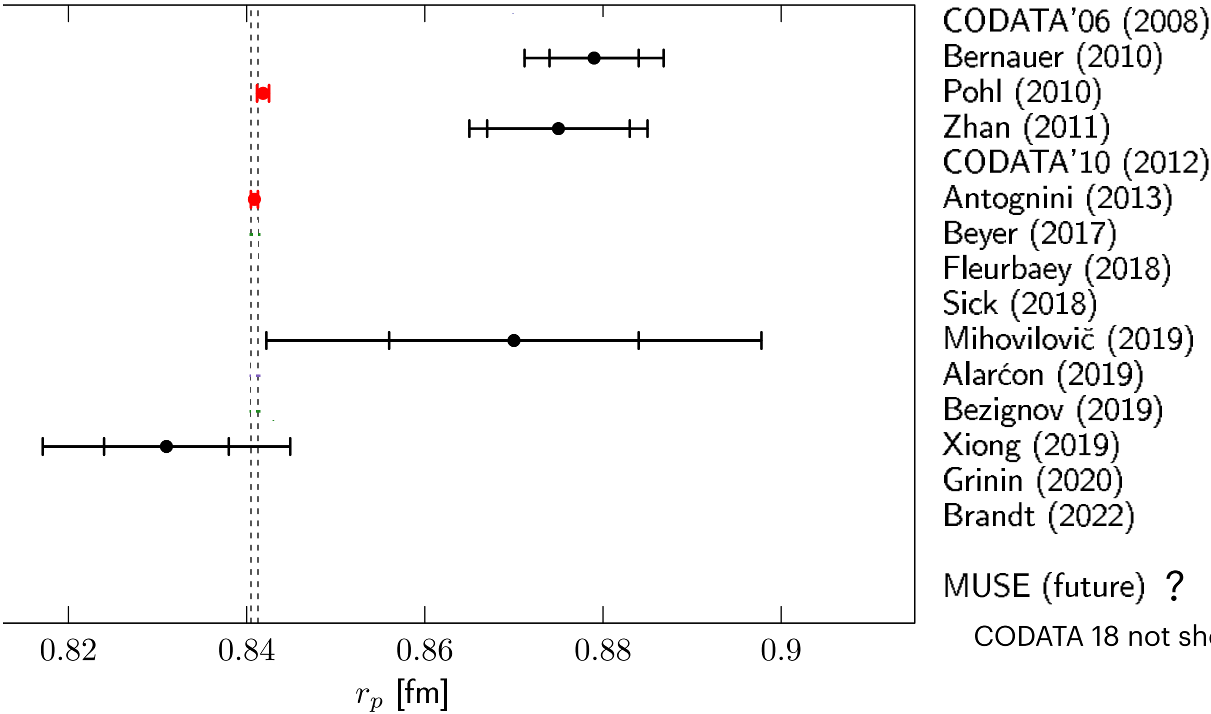




H Spectroscopy: are some results wrong? Why?

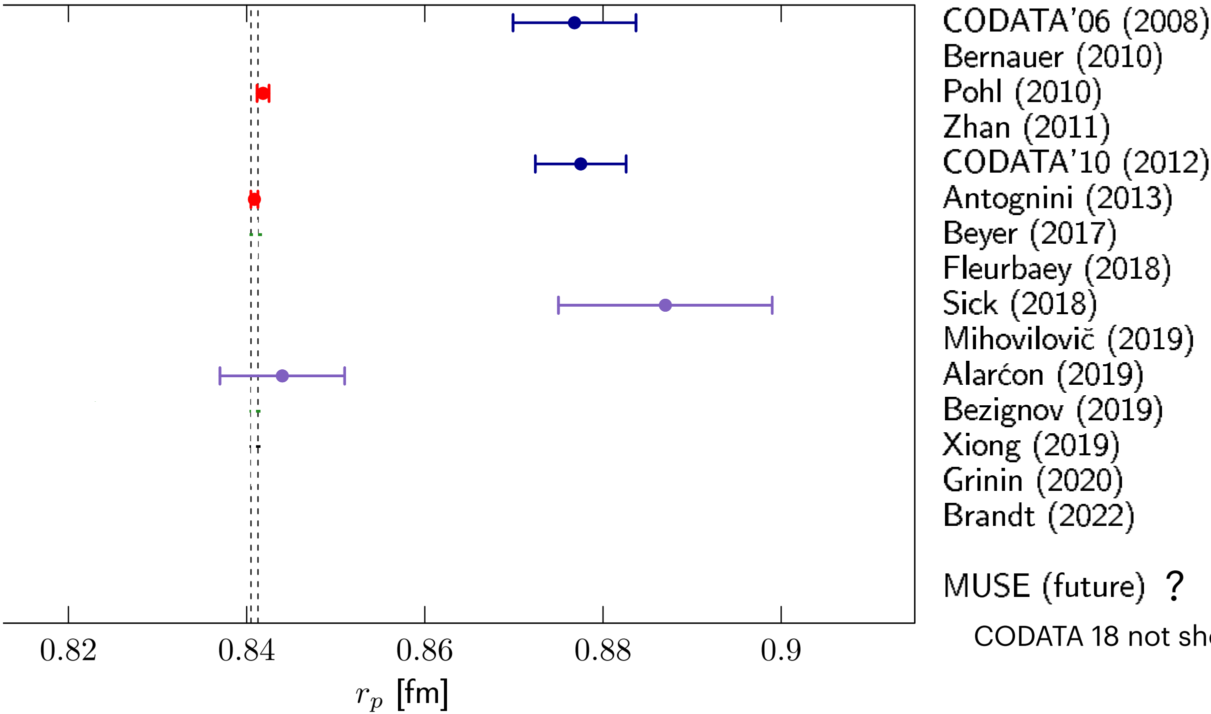


ep scattering: are some results wrong? Why?



CODATA 18 not sh

Analyses inconsistent



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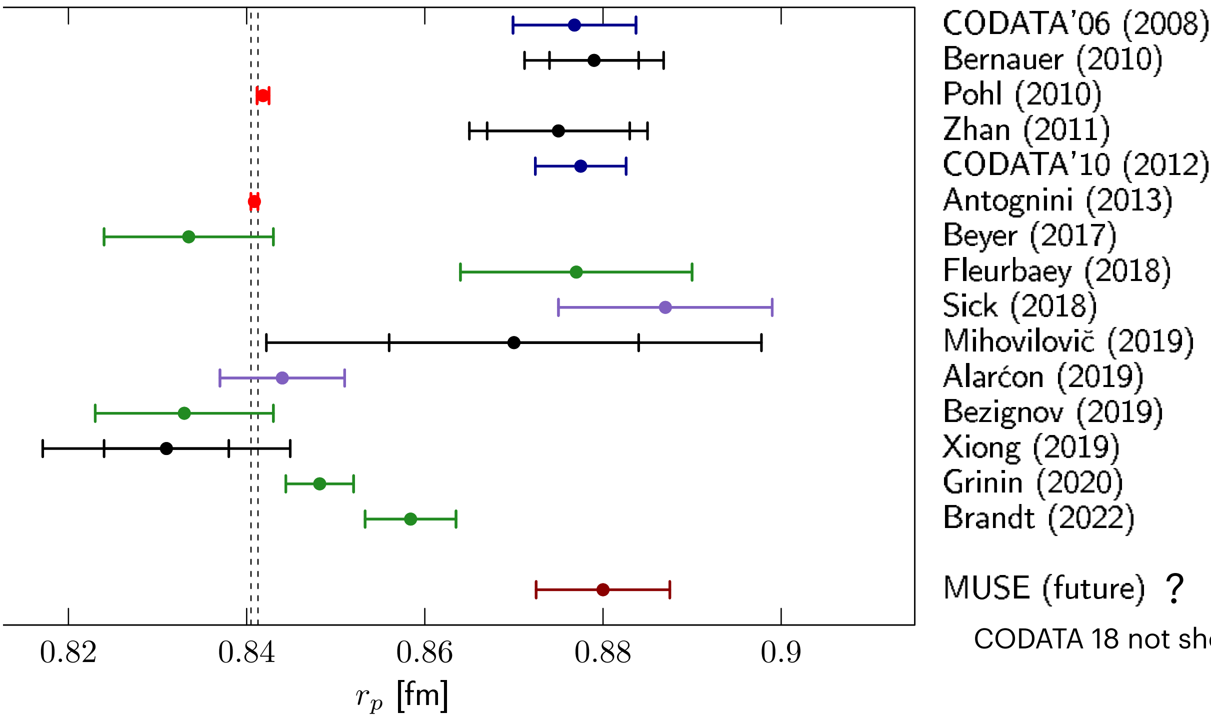
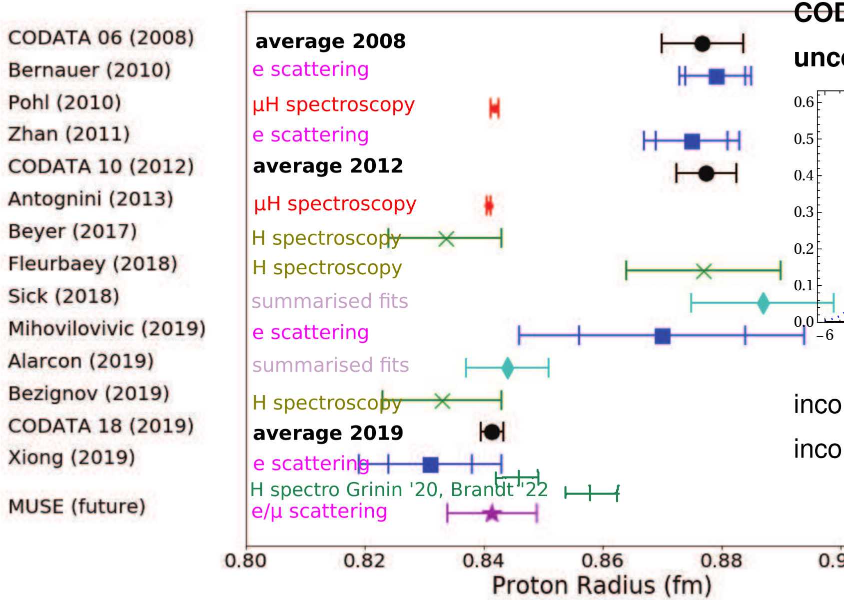
Update 2020/22: Puzzle Solved?



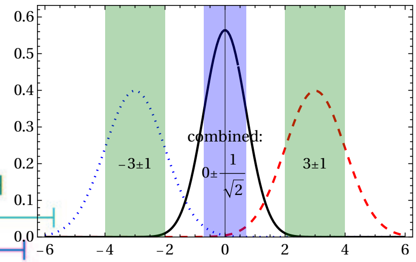
CODATA 18 not sh

Update 2020/22: Puzzle Solved?

[PDG 2020]: *However, reflecting the new electronic measurements, the 2018 CODATA recommended value is* $0.8414(19)\,\mathrm{fm}$*, and the puzzle appears to be resolved.*

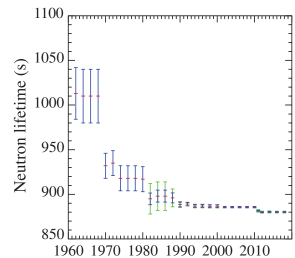


CODATA does not disclose how uncertainties are combined.



inconsistencies PRad/MAMI

inconsistencies H: μ/e, old/new



My (non-expert) Questions: Result of re-analysis of old data?

What's *methodologically* wrong with pre-2012 exp's?

Unconscious Bias?

Update 2021: Hiding A Justification

C. Proton charge radius and Rydberg constant or frequency

The disagreement between the (root-mean-square) charge radius of the proton r_p obtained from Lamb-shift measurements in muonic hydrogen (a muon bound to a proton) and the value obtained from transition frequency measurements in hydrogen and electron-proton elastic scattering data, sometimes referred to as the "proton-radius puzzle," has been partly resolved. Therefore, for this 2018 CODATA adjustment, the TGFC decided that the muonic hydrogen data, some of which were already available in 2010, as well as related muonic deuterium data, should no longer be excluded.

The reduced disagreement in the determinations of the proton charge radius is mainly due to two new hydrogen spectroscopic measurements (Beyer *et al.*, 2017; Bezginov *et al.*, 2019), as they imply a smaller r_p closer to that found from muonic hydrogen data. Figure 1 illustrates the improved agreement for r_p as well as its strong correlation with the determination of the Rydberg constant R_∞. We observe that our 2018 value for r_p has a three-times improved uncertainty compared to that found in the 2014 CODATA evaluation. Moreover, the correlation coefficient between r_p and R_∞ has significantly decreased. The covariance error ellipse is more circular in the 2018 adjustment. Similar observations hold for the determination of the deuteron charge radius r_d. Our 2018 relative standard uncertainties for r_p, r_d, and R_∞ are 2.2×10^{-3}, 3.5×10^{-4}, and 1.9×10^{-12}, respectively.

The tension between the two approaches determining r_p and r_d has not been fully resolved. In fact, to obtain consistency among the many input data that contribute to the determination of R_∞, r_p, and r_d, a multiplicative expansion factor of 1.6 is applied to their uncertainties. Further experiments are needed.

CODATA's explanation for the error assessment in its 2018 value has to wait for 3 years until the next-to-last sentence in a paragraph on p. 6 of a 62-page article published in 2021.

[E. Tiesinga et al (CODATA) J. Phys. Chem. Ref. Data **50** (2021) 033105]

Next: 3. Resonance Region, Isospin

Familiarise yourself with: [PRSZR 2.4, 6.2, 7.1/4; HG 6.8, 14.2, 8.4-7; Per 3.12; HM 2.6/7; PDG 49]