# PHYS 6610: Graduate Nuclear and Particle Physics I

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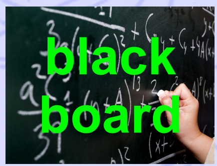
# III. Descriptions

# 1. Non-Abelian Gauge Theories

***Or: Finally!***

References: [HM 14.1-4, 2.15; HG 12.3; CL 8.1]

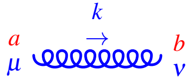


# Particle-Field Feynman Rules of Non-Abelian Gauge Theories

fermion colours $i, j = 1, \ldots, N_c$ for $N_c$ colours

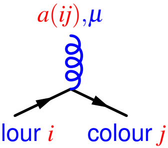
$(ij) \to a(ij)$ form gauge-field colour combinations $a, b, c, d = 1, \ldots, \dim\mathcal{G}(N_c)$ $= N_c^2 - 1$ in $SU(N_c)$.

gauge-field propagator: does not change colour




$$: \frac{-\mathrm{i}\,\delta^{ab}\,g_{\mu\nu}}{k^2 + \mathrm{i}\epsilon}$$

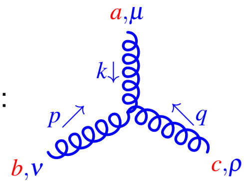
qg vertex:




$$: \mathrm{i}\,g\,\gamma^\mu\,(t^a)^j{}_i$$

flow like for $\gamma$-matrices: left-to-right against fermion arrow

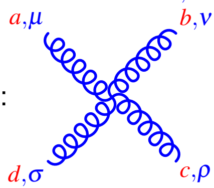
3g self-interaction vertex:




$$: g\,f^{abc}\left[g^{\mu\nu}\,(k-p)^\rho + g^{\nu\rho}\,(p-q)^\mu + g^{\rho\mu}\,(q-k)^\nu\right]$$

$[t^a, t^b] = \mathrm{i}\,f^{abc}\,t_c$ structure constants of $\mathcal{G}$

4g self-interaction: vertex




$$: -\mathrm{i}g^2\left[f^{abe}f^{cde}\left(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right) + f^{ace}f^{bde}\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\rho\nu}\right) + f^{ade}f^{bce}\left(g^{\mu\nu}g^{\sigma\rho} - g^{\mu\rho}g^{\sigma\nu}\right)\right]$$

**Even pure Yang-Mills Theory (no fermions) is self-interacting! $\Longrightarrow$ Chance of confinement?**

**All use same coupling strength $g$: delicate balance of terms to ensure gauge invariance.**

A few more interactions with "ghost" fields only cure a technicality (gauge invariance in loop diagrams).

# $SU(3)$: Gell-Mann Matrices $\lambda^a$ And All That...

$SU_c(3)$ generated by Hermitean, traceless $3\times 3$ matrices $t^a = \frac{\lambda^a}{2}$, ortho-normalised: $2\mathrm{tr}[\frac{\lambda^a}{2}\frac{\lambda^b}{2}] = \delta^{ab}$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},\ \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},\ \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix},\ \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

**Structure Constants** $[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}] = \mathrm{i}\, f^{abc}\, \frac{\lambda_c}{2}$: totally antisymmetric in all indices.

$f^{123} = 1 \qquad f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{2} \qquad f^{458} = f^{678} = \frac{\sqrt{3}}{2}$ rest $0$.

**Explicit form *never* necessary in actual calculations! Only matrix dimension $N_c (= 3)$,**

**Norm $2\mathrm{tr}[t^a t^b] = \delta^{ab}$, $SU(N_c)$ Casimir operators: $t^a t_a = \frac{N_c^2 - 1}{2N_c}\mathbb{1}$, $f^{acd} f_{bcd} = N_c\, \delta^a_b$.**

**[Other groups than $SU(N_c)$ have other numbers for the Casimir operators.]**

sion for the covariant derivatives $D_\mu$, Yang had such difficulty in finding the expression for the covariant field strengths, since these are the simple functions $F_{\mu\nu} = [D_\mu, D_\nu]$ of the $D_\mu$'s. However, the expression of the field strength as a commutator was not common in electromagnetic theory at the time because the direct expression as a curl was so simple, so the commutator expression was not at Yang's disposal. The foresight that warned Yang and Mills that the gauge-field mass problem would not have a simple solution is also interesting. With hindsight, we now know that for the weak interactions this problem is solved only by spontaneous symmetry breaking and that for the strong interactions it remains a deep problem that is intimately connected with asymptotic freedom and confinement.

The question of the gauge-field mass problem was raised by Pauli when Yang was invited to present the Yang-Mills results at the Princeton Institute in February 1954. As Yang [3] relates:

> Pauli was spending the year in Princeton, and was deeply interested in symmetries and interactions. . . . Soon after my seminar began, when I had written on the blackboard,
>
> $$(\partial_\mu - ieB_\mu)\psi,$$
>
> Pauli asked, "What is the mass of this field $B_\mu$?" I said we did not know. Then I resumed my presentation but soon Pauli asked the same question again. I said something to the effect that it was a very complicated problem, we had worked on it and had come to no definite conclusions. I still remember his repartee: "That is not sufficient excuse". I was so taken aback that I decided, after a few moments' hesitation, to sit down. There was general embarrassment. Finally Oppenheimer said, "We should let Frank proceed". I then resumed and Pauli did not ask any more questions during the seminar.

Thus Pauli also was aware of the non-triviality of the mass problem. But some other interesting information is revealed by the following sequel to the episode:

> I do not remember what happened at the end of the seminar. But next day I found the following message:
>
> February 24, Dear Yang, I regret that you made it almost impossible for me to talk to you after the seminar. All good wishes. Sincerely yours, W. Pauli.

# Where the Name "Quark" Comes From

James Joyce: Finnegans Wake (1939)

*— Three quarks for Muster Mark!*
*Sure he hasn't got much of a bark*
*And sure any he has it's all beside the mark.*
*But O, Wreneagle Almighty, wouldn't un be a sky of a lark*
*To see that old buzzard whooping about for uns shirt in the dark*
*And he hunting round for uns speckled trousers around by Palmer-stown Park?*
*Hohohoho, moulty Mark!*
*You're the rummest old rooster ever flopped out of a Noah's ark*
*And you think you're cock of the wark.*
*Fowls, up! Tristy's the spry young spark*
*That'll tread her and wed her and bed her and red her*
*Without ever winking the tail of a feather*
*And that's how that chap's going to make his money and mark!*

Overhoved, shrillgleescreaming. That song sang seaswans. The winging ones. Seahawk, seagull, curlew and plover, kestrel and capercallzie. All the birds of the sea they trolled out rightbold when they smacked the big kuss of Trustan with Usolde.

And there they were too, when it was dark, whilest the wild-caps was circling, as slow their ship, the winds aslight, upborne the fates, the wardorse moved, by courtesy of Mr Deaubaleau Downbellow Kaempersally, listening in, as hard as they could, in Dubbeldorp, the donker, by the tourneyold of the wattarfalls, with their vuoxens and they kemin in so hattajocky (only a

3 quark flavours form meson/baryon octets (8), 3 colours $\Longrightarrow$ 383

**Quark (n., German) a sour curd or cottage cheese.**

# Next: 2. Perturbative QCD

***Familiarise yourself with: [PRSZR 8.1-3, 14; HM 2.15, 10.3-9, 11.4/6-7; Tho 10.7/8; Ryd 3, end of 9.6; HG 12.3; PS 16.7; Per 6.5; lots more. . . ]***