PHYS 6610: Graduate Nuclear and Particle Physics I

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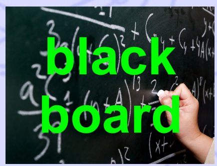
I. Tools

8. Theory of Electron Scattering

Or: Our Analysis Tool

References: [HM 4, 6.1/3-6/9/11/13, 8]



Preview: Electron Scattering on Hadronic Systems

see II.1-3,5

[Martin fig. 2.4]

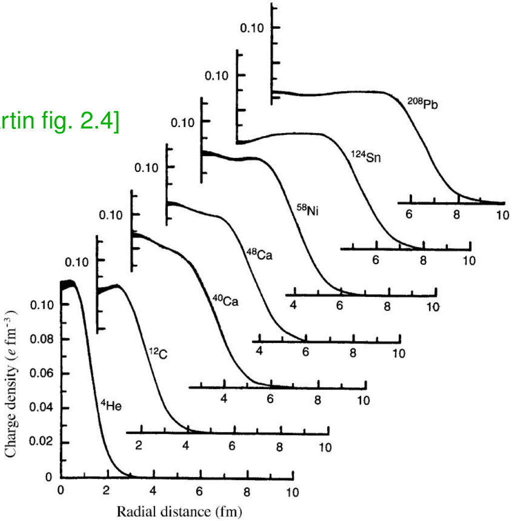
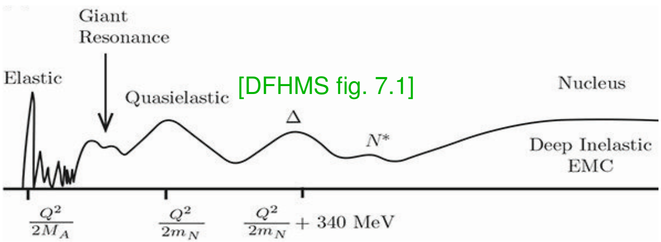


Figure 2.4 Radial charge distributions ρ_{ch} of various nuclei, in units of e fm^{-3}; the thickness of the curves near $r = 0$ is a measure of the uncertaintity in ρ_{ch} (adapted from Fr83)



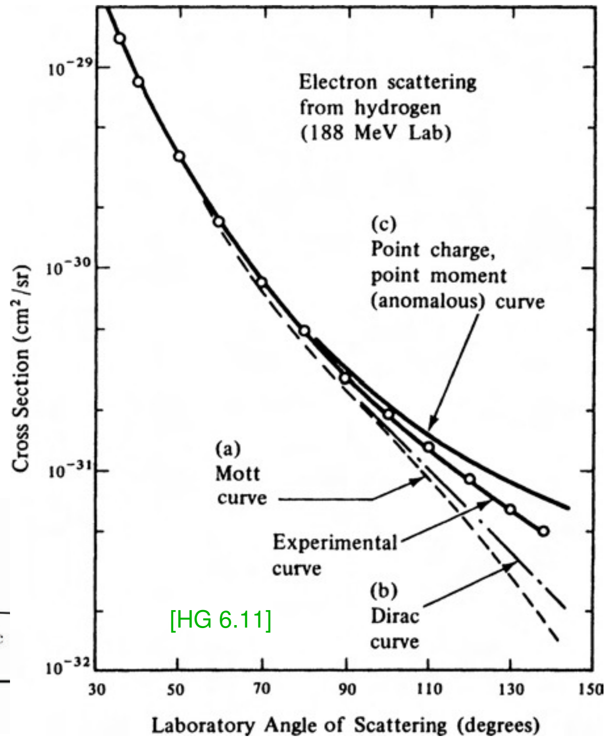
[DFHMS fig. 7.1]

nucleus levels excited

quasi-elast. on bound N in nucleus

inelast. on nucleon

on quarks



[HG 6.11]

Helicity Conservation: "Massless" Spin-$\frac{1}{2}$ on Spin-0 Target

Coulomb interaction of spin-$\frac{1}{2}$ projectile ($m_e \ll E$) on infinitely-heavy, extended spin-0 target:

$$\boxed{\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\text{lab}} = \underbrace{\left(\frac{Z\alpha}{2E\,\sin^2\frac{\theta}{2}}\right)^2}_{\text{Rutherford}} \times \underbrace{\left|F(\vec{q}^{\,2})\right|^2}_{\substack{\text{charge}\\\text{form factor}}} \times \underbrace{\cos^2\frac{\theta}{2}}_{\substack{\text{helicity}\\\text{conservation}}}}$$

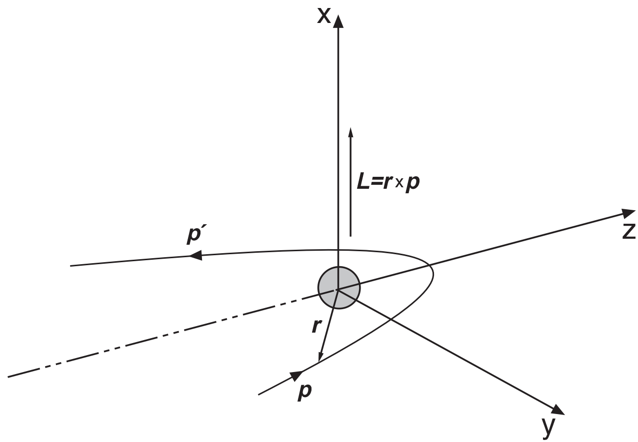
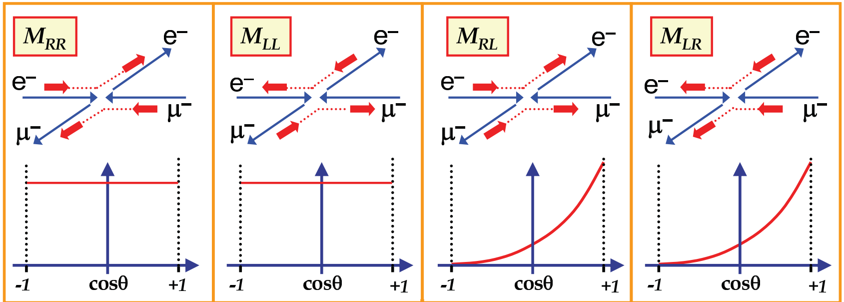


Fig. 5.3. Helicity, $h = \boldsymbol{s}\cdot\boldsymbol{p}/(|\boldsymbol{s}|\cdot|\boldsymbol{p}|)$, is conserved in the $\beta \to 1$ limit. This means that the spin projection on the z-axis would have to change its sign in scattering through 180°. This is impossible if the target is spinless, because of conservation of angular momentum.

[PRSZR]

Helicity Conservation: Massless Spin-$\frac{1}{2}$ on Massive Spin-$\frac{1}{2}$ Target

- Of the 16 possible helicity combinations only 4 are non-zero:



$S_z = 0$ $\qquad\qquad$ $S_z = +1$ $\qquad\qquad$ $S_z = -1$

$\Rightarrow \dfrac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto 1$ $\qquad\qquad$ $\Rightarrow \dfrac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto \dfrac{1}{4}(1+\cos\theta)^2$ [cf. Tho 8.6]

i.e. no preferred polar angle $\qquad\qquad$ **spin 1 rotation again**

HW: Electromagnetic interaction of spin-$\frac{1}{2}$ projectile ($m_e \ll E$) on massive, point-like spin-$\frac{1}{2}$ target:

$W^{\mu\nu} = \frac{1}{2}\mathrm{tr}[\gamma^\mu(\not p + M)\gamma^\nu(\not p' + M)] = 2[p^\mu p'^\nu + p'^\mu p^\nu - g^{\mu\nu}(p\cdot p' - M^2)]$: like $L_{\mu\nu}$ but $M \neq 0$

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\text{lab}} = \overbrace{\underbrace{\left(\frac{Z\alpha}{2E\,\sin^2\frac{\theta}{2}}\right)^2}_{\text{Rutherford}}\;\underbrace{\frac{E'}{E}}_{\text{recoil}}\times\underbrace{\cos^2\frac{\theta}{2}}_{\text{helicity conservation}}}^{\textbf{Mott}\text{ cross section}}\times\left(1-\underbrace{\frac{q^2}{2M^2}\tan^2\frac{\theta}{2}}_{\text{backscattering on target spin}}\right)$$

Reminder: Spinor-Vector Interaction in the Chiral Basis

$\mathcal{L}_{\text{QED}} = \overline{\Psi}\left[\mathrm{i}\left(\partial\!\!\!/ + \mathrm{i}q A\!\!\!/\right) - m\right]\Psi$ with $\Psi := \begin{pmatrix} \varphi_R \\ \varphi_L \end{pmatrix}$:

$$\Longrightarrow \mathcal{L}_{QED} = \left(\varphi_R^\dagger, \varphi_L^\dagger\right) \begin{pmatrix} E + qA_0 + \vec{\sigma}\cdot\vec{p} - q\,\vec{\sigma}\cdot\vec{A} & \mathbf{m} \\ \mathbf{m} & E + qA_0 - \vec{\sigma}\cdot\vec{p} - q\,\vec{\sigma}\cdot\vec{A} \end{pmatrix} \begin{pmatrix} \varphi_R \\ \varphi_L \end{pmatrix}$$

$\begin{pmatrix} \varphi_R \\ \varphi_L \end{pmatrix}$ EFunctions to **Helicity Operators** $h_\pm := \frac{1}{2}\left[1 \pm \gamma^5\right]$, EValues $\begin{matrix} +1 \\ -1 \end{matrix}$ for spin $\begin{matrix} \text{parallel} \\ \text{anti-parallel} \end{matrix}$ to $\vec{p}$.

(1) ***Any* interaction $\gamma^\mu A_\mu$ (spinor-vector/spin-$\frac{1}{2}$-spin1) does not mix φ_{LR}.**
$\Longrightarrow$ **Conserves helicity** (esp. QED and any gauge theory!).

(2) **Mass mixes helicities $\propto \dfrac{m}{E}$.**

(3) $\Longrightarrow$ **Processes $\gamma^\mu A_\mu$ with ultra-relativistic fermions, (approximately) *conserve* helicity.**

(4) ***Only $\vec{A}$ couples to spin $\vec{\sigma}$.*** $\xrightarrow{\text{One Can Show}}$ Magnetic field couples to spin: $\vec{\sigma}\cdot\left(\nabla\times\vec{A}\right) = \vec{\sigma}\cdot\vec{B}$.

$\Longrightarrow$ Can change spin but not helicity!

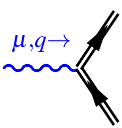
(g) e^- on Proton: Spin-$\frac{1}{2}$ Target With Structure [HM 8.2]

Electromagnetic current of point particle: $J^\mu_{s,s'} = -\mathrm{i}\, Ze\, \bar{u}_{s'}(k')\, \gamma^\mu\, u_s(k)$ from $\mathcal{L}_{\text{int}} = -A_\mu J^\mu$ (I.7.4C)

Proton has structure $\Longrightarrow$ What is most general form for J^μ_{proton}?

Must-Have: Lorentz-invariant, current conserved $q_\mu J^\mu_{\text{proton}} \overset{!}{=} 0$, parity invariant (elmag. interaction!)

Build it from combinations of $p^\mu, q^\mu, g^{\mu\nu}$, Dirac matrices γ^μ but not γ_5 (parity) [details [HM ex. 8.5/6.1]]



$$: \; J^\mu_{S,S'} = -\mathrm{i}\underbrace{e\, F_1(q^2)\, \bar{u}_{S'}(p')\, \gamma^\mu\, u_S(p)}_{\text{Dirac: modify point-form}} + \underbrace{\overbrace{\frac{e}{2M}}^{\text{Bohr magneton}} F_2(q^2)\, q_\nu\, \bar{u}_{S'}(p')\, \mathrm{i}\sigma^{\mu\nu}\, u_S(p)}_{\text{Pauli: anomalous mag. term}} \qquad \text{(I.7.5C)}$$

with $\sigma^{\mu\nu} := \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu]$ combination of Dirac matrices, current conserved since $q_\mu q_\nu \sigma^{\mu\nu} = 0$

$F_{1,2}$ are dimension-less Form Factors (FFs): functions of q^2 to be determined by experiment/theory.

Dirac FF $F_1(q^2)$: modifies ordinary "point-structure" of spin-$\frac{1}{2}$ (i.e. mag. moment with $g = 2$)

Normalisation to charge: $F_1^{\mathrm{p}}(q^2 = 0) = Z^{\mathrm{p}} = 1$, $F_1^{\mathrm{n}}(q^2 = 0) = Z^{\mathrm{n}} = 0$

Pauli FF $F_2(q^2)$: extended structure produces *anomalous* magnetic moment κ

Normalisation $F_2^{\mathrm{p}}(q^2 = 0) = \kappa_{\mathrm{p}} = +1.79$, $F_2^{\mathrm{n}}(q^2 = 0) = \kappa_{\mathrm{n}} = -1.91$ (in Bohr magnetons)

Common theory trick: parametrisation of ignorance by most general structure...

Helicity Conservation and Magnetic Effects

$$\overline{|\mathcal{M}|^2}\Big|_{\text{lab}} = \frac{e^4}{q^4} L_{\mu\nu} W^{\mu\nu} = 16M^2 EE' \Bigg[\underbrace{\left(F_1^2 - \frac{q^2}{4M^2}F_2^2\right)\cos^2\frac{\theta}{2}}_{\text{charge \& spin}} - \underbrace{\frac{q^2}{2M^2}(F_1+F_2)^2 \sin^2\frac{\theta}{2}}_{\text{spin flip (helicity): magnetic!}} \Bigg]$$

$$\Longrightarrow \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big|_{\text{lab}} = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big|_{\text{Mott}} \times \left[\left(F_1^2 + \tau F_2^2\right) + \underbrace{2\tau\,(F_1+F_2)^2\,\tan^2\frac{\theta}{2}}_{\text{spin flip: magnetic!}} \right] \text{ with } \tau := -\frac{q^2}{4M^2} > 0 \text{ L-inv.}$$

Sachs FFs also often used: $G_E(q^2) := F_1 + \frac{q^2}{4M^2}F_2$, $G_M(q^2) := F_1 + F_2$

$G_E(q^2 = 0) = 1$, $G_M(q^2 = 0) = Z + \kappa \equiv \mu$ magnetic moment of target

$$\Longrightarrow \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big|_{\text{lab}} = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big|_{\text{Mott}} \times \left[\underbrace{\frac{G_E^2 + \tau\,G_M^2}{1+\tau}}_{\text{el. \& mag.: no spin flip}} + \underbrace{2\tau\,G_M^2\,\tan^2\frac{\theta}{2}}_{\text{mag.: spin flip}} \right]$$

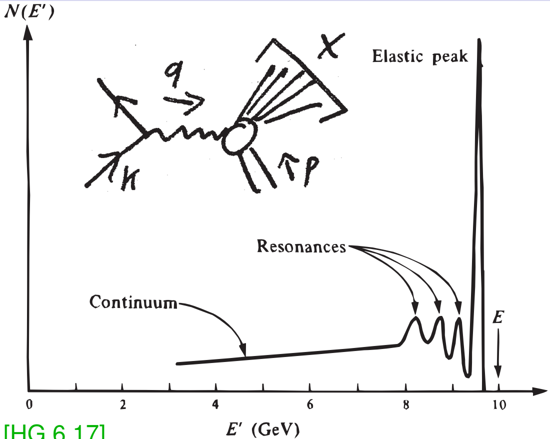
Experimentalists like Sachs FFs: separate by $\tan^2\frac{\theta}{2}$ (practical).

Can be interpreted as Fourier trafos of charge/current distribution (in Breit frame).

Theorists like Dirac/Pauli FFs: modification of point-like structure (concept).

We will explore them more next week...

(h) Preview of Inclusive Inelastic Scattering $\mathrm{p}(\mathrm{e},\mathrm{e}')X$



[HG 6.17]

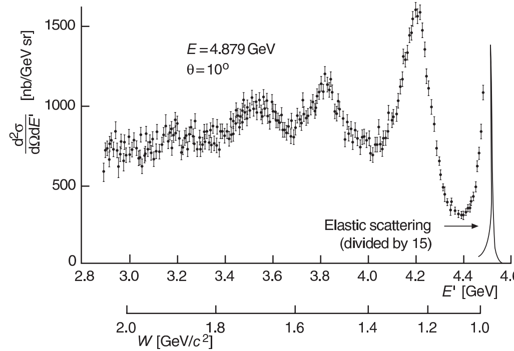


Fig. 7.1. Spectrum of scattered electrons from electron-proton scattering at an electron energy of $E = 4.9\,\mathrm{GeV}$ and a scattering angle of $\theta = 10^\circ$ (from [Ba68]).

[PRSZR 7.1]

scattered electron loses energy to target excitation/breakup

$\Longrightarrow$ **In lab frame:** $\dfrac{E'}{E} < \dfrac{1}{1+\frac{E}{M}(1-\cos\theta_{\mathrm{lab}})}$

$\Longrightarrow$ New independent kinematic variable E'_{lab} or **Invariant mass-squared of all fragments:**

$$W^2 := p'^2 = (p+q)^2 = M^2 + 2p\cdot q + q^2 = M^2 + 2p\cdot q(1 - \underbrace{\frac{-q^2}{2p\cdot q}}_{\text{Bjorken}-x}) \geq M^2$$

$\Longrightarrow x \in [0;1]$, elastic: $x = 1$

Preview of Inelastic Scattering: Most General Structure

Common theory trick: parametrisation of ignorance by most general structure...

Most general structure for hadron tensor $W^{\mu\nu}$: $W^{\mu\nu} = W^{\nu\mu}$ symmetric (since $L_{\mu\nu}$ is)

Must-Have: Lorentz-invariant, current conserved $q_\mu W^{\mu\nu} \stackrel{!}{=} 0$, parity invariant (elmag. interaction!)

Build it from combinations of $p^\mu, q^\mu, g^{\mu\nu}$ – but *not* γ^μ's since we do (here) not prepare/detect spin.

$$W^{\mu\nu} = \frac{F_1(q^2,x)}{M}\left[\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right] + \frac{F_2(q^2,x)}{p\cdot q\, M}\left[p^\mu - \frac{p\cdot q}{q^2}q^\mu\right]\left[p^\nu - \frac{p\cdot q}{q^2}q^\nu\right]$$

Structure functions $F_{1,2}(Q^2,x)$ have nothing to do with Pauli/Dirac FFs

New variables: momentum-transfer $Q^2 := -q^2 > 0$, **Bjorken-x** $:= -\dfrac{q^2}{2p\cdot q}$

$\Longrightarrow$ inelastic cross section with independent variable E':

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\text{lab}} = \underbrace{\left(\frac{2\alpha}{q^2}\right)^2 \cos^2\frac{\theta}{2}\,E'^2}_{\text{like Mott, but no recoil}}\left[\frac{M\,F_2(Q^2,x)}{p\cdot q} + \underbrace{\frac{2\,F_1(Q^2,x)}{M}\tan^2\frac{\theta}{2}}_{\text{spin-flip: magnetic}}\right]$$

Recover elastic result for $x := \dfrac{-q^2}{2p\cdot q} \to 1 \Longrightarrow$ set $\delta(\dfrac{p\cdot q}{M} + \dfrac{q^2}{2M})$ and $\int \mathrm{d}E'$ of cross section.

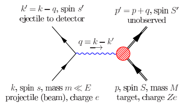
(i) Summary: Electron Scattering Cross Section

Link to handout on website here (p. 3 of "Essentials").

Summary Electron Scattering Cross Sections

cf. [HM 8]

Lowest-order Feynman graph



Mandelstam: $s = (k+p)^2$; $t = (k'-k)^2$; $u = (p'-k)^2$

$k = (E, \vec{k})$, $k' = (E', \vec{k}')$, $q^2 = -Q^2 < 0$

$M\nu = p \cdot q = M(E' - E)_{\text{lab}}$

scatt. angle θ: $\vec{k} \cdot \vec{k}' = |\vec{k}| \cdot |\vec{k}'| \cos\theta$

for elastic in lab frame $\quad \dfrac{E'}{E} = \dfrac{1}{1 + \frac{E}{M}(1 - \cos\theta)}$

$\left.\dfrac{d\sigma}{d\Omega}\right|_{\text{cm}} = \dfrac{1}{64\pi^2 s} |\overline{\mathcal{M}}|^2$ **elastic, unpolarised, cm-frame**

$\left.\dfrac{d\sigma}{d\Omega}\right|_{\text{lab}} = \dfrac{1}{64\pi^2 M^2} \left(\dfrac{E'}{E}\right)^2 |\overline{\mathcal{M}}|^2$ **elastic, unpolarised, lab-frame**

$|\overline{\mathcal{M}}|^2 = (e^2)^2 L_{\mu\nu} \dfrac{1}{q^4} W^{\mu\nu}$ **matrix element**: avg. initial, sum final spins

$e^2 L^{\mu\nu} = \dfrac{1}{2s+1} \sum\limits_{s,s'} j^\mu_{s,s'}(k,k') j^{\nu\dagger}_{s,s'}(k,k')$ **lepton tensor**: scatter off virtual γ

$j^\mu_{s,s'}(k,k') = \langle k', s' | j^\mu | k, s\rangle$ **lepton current** (electromagnetic)

$j^\mu_{s,s'} = -\mathrm{i}e\, \bar{l}_{s'}(k')\, \gamma^\mu\, l_s(k)$ for electron

$L^{\mu\nu} = 2[k^\mu k'^\nu + k'^\mu k^\nu - g^{\mu\nu}\, k \cdot k']$ for electron ($m = 0, s = \frac{1}{2}$); cf. (I.7.4W)

$e^2 W^{\mu\nu} = \dfrac{1}{2S+1} \sum\limits_{S,S'} J^\mu_{S,S'}(p,p') J^{\nu\dagger}_{S,S'}(p,p')$ **hadron tensor**: scatter off virt. γ

$J^\mu_{S,S'}(p,p') = \langle p', S' | J^\mu | p, S\rangle$ **hadron current** (electromagnetic)

$q_\mu j^\mu = 0 = q_\mu J^\mu \implies q_\mu L^{\mu\nu} = 0 = q_\mu W^{\mu\nu}$ **elmag. current conservation**

Relativistic Rutherford: Coulomb of massless, spin-0 on infinitely heavy, spin-0 point-target ($s = 0; M = \infty, S = 0$)

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \qquad J^\mu = -\mathrm{i}Ze\delta^{\mu 0}\text{, i.e. point charge at rest} \qquad \text{(I.7.1) (I.7.1C)}$$

$e^\pm$ **Coulomb scattering on infinitely heavy, composite spin-0 target:** ($s = \frac{1}{2}; M = \infty, S = 0$)

helicity forbids back-scattering; isotropic charge density $\rho(r)$

$$\left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2} \times |F(\vec{q}^2)|^2 \qquad \overset{\text{Fourier}}{\Longrightarrow} \text{ form factor } F(\vec{q}^2) := \frac{4\pi}{Ze}\int_0^\infty dr\, \frac{r}{q}\, \sin(qr)\, \rho(r) \qquad \text{(I.7.2)}$$

normalisation: $F(0) = 1$; charge radius: $\langle r^2\rangle = -6 \left.\dfrac{dF(\vec{q}^2)}{d\vec{q}^2}\right|_{\vec{q}^2=0}$

$e^\pm$ **full electromagnetic scattering on massive composite spin-0 target:** ($s = \frac{1}{2}; M$ finite, $S = 0$)

$$\underbrace{\left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}\, \frac{E'}{E}}_{\textbf{Mott: }\text{no structure}} \times |F(q^2)|^2$$

adds target recoil; $\vec{q}^2 \to q^2$: 4-momentum transfer (I.7.3)

$J^\mu = -\mathrm{i}Ze\, F(q^2)\, (p^\mu + p'^\mu)$ (most general for $S = 0$) (I.7.3C)

$\implies W^{\mu\nu} = Z^2 (p+p')^\mu\, (p+p')^\nu\, |F(q^2)|^2$ (I.7.3W)

$e^\pm\mu^\pm \to e^\pm\mu^\pm$ **scattering on massive spin-$\frac{1}{2}$ target without structure:** ($s = \frac{1}{2}; M, S = \frac{1}{2}$)

$$\left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}\, \frac{E'}{E} \times \left[1 - \frac{q^2}{2M^2}\tan^2\frac{\theta}{2}\right] \qquad \text{(I.7.4)}$$

back-scattering by helicity transfer (spin-spin/mag. moment interaction)

Massive $S = \frac{1}{2}$ hadr. tensor, no structure: $W^{\mu\nu} = 2Z^2 \left[p^\mu p'^\nu + p'^\mu p^\nu - g^{\mu\nu}(p \cdot p' - M^2)\right]$ (I.7.4W)

$e^\pm$ **on composite, massive spin-$\frac{1}{2}$ target:** form factors $F_1(q^2)$: Dirac; $F_2(q^2)$: Pauli ($s = \frac{1}{2}; M, S = \frac{1}{2}$)

$$\left(\frac{\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}\, \frac{E'}{E} \times \left[\left[F_1^2(q^2) + \tau F_2^2(q^2)\right] + 2\tau\, \left[F_1(q^2) + F_2(q^2)\right]^2 \tan^2\frac{\theta}{2}\right] \qquad \tau := -\frac{q^2}{4M^2} \qquad \text{(I.7.5)}$$

Variant: **Rosenbluth/Sachs formula** uses **Sachs form factors** $G_E = F_1 - \tau F_2$, $G_M = F_1 + F_2$

$$= \qquad [\dots] \qquad \times \left[\frac{G_E^2(q^2) + \tau\, G_M^2(q^2)}{1+\tau} + 2\tau\, G_M^2(q^2)\, \tan^2\frac{\theta}{2}\right] \qquad \text{(I.7.5)}$$

$$J^\mu = -\mathrm{i}\underbrace{e\, F_1(q^2)\, \bar{u}(p')\gamma^\mu u(p)}_{\substack{\text{modify point form}\\ F_1(0) = Z = G_E(0)}} + \frac{e}{2M} \underbrace{F_2(q^2)\, q_\nu\, \bar{u}(p')\mathrm{i}\sigma^{\mu\nu} u(p)}_{\substack{\text{anomalous mag. term, } F_2(0) = \kappa = G_M(0) - Z;\\ \kappa \text{ anomalous mag. moment, } \mu = Z + \kappa \text{ mag. moment}}} \quad \text{(most general for } S = \tfrac{1}{2}) \qquad \text{(I.7.5C)}$$

$e^\pm$ ***inelastic*, inclusive scattering:** E' independent variable, p' not detected ($s = \frac{1}{2}; M, S =$ any [sic!])

$$\left.\frac{d\sigma}{d\Omega\, dE'}\right|_{\text{lab}} = \left(\frac{2\alpha}{q^2}\right)^2 E'^2 \cos^2\frac{\theta}{2} \times \left[\frac{F_2(q^2,x)}{\nu} + \frac{2\, F_1(q^2,x)}{M}\tan^2\frac{\theta}{2}\right] \qquad \text{(I.7.6)}$$

inelasticity measure: **Bjorken-x** $:= -\dfrac{q^2}{2p \cdot q} = \dfrac{Q^2}{2M\nu} \in [0;1]$; elastic: $x = 1$, i.e. $F_{1,2}(q^2,x) \propto \delta(\nu + \dfrac{q^2}{2M})$

Most general elmag. hadronic ME (symmetric $\mu \leftrightarrow \nu$, charge conservation; any spin [sic!]):

$$W^{\mu\nu} = \frac{F_1(q^2,x)}{M}\left[\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right] + \frac{F_2(q^2,x)}{M^2\nu}\left[p^\mu - \frac{p \cdot q}{q^2}\, q^\mu\right]\left[p^\nu - \frac{p \cdot q}{q^2}\, q^\nu\right] \qquad \text{(I.7.6W)}$$

Structure functions F_1, F_2 *are not the Dirac, Pauli FFs of Eq.* (I.7.5)!

Summary: Electron Scattering Cross Section $\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}}$ [HM 8.4]

rel. Rutherford: Coulomb $(s=0; M=\infty, S=0)$ $\left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2$ (I.8.1)

$e^\pm$ **Coulomb on composite** $(s=\frac{1}{2}; M=\infty, S=0)$ $\left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2} \quad \left|F(\vec{q}^2)\right|^2$ (I.8.2)

$e^\pm$ **full elmag on composite** $(s=\frac{1}{2}; M \text{ finite}, S=0)$
called **Mott** for $F(q^2)=1$

$$\underbrace{\left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}\,\frac{E'}{E}} \left|F(\vec{q}^2)\right|^2 \Longrightarrow \text{recoil} \quad \text{(I.8.3)}$$

$e\mu \to e\mu$**: no structure** $(s=\frac{1}{2}; M, S=\frac{1}{2})$ $[\ldots] \times \left[1-\frac{q^2}{2M^2}\tan^2\frac{\theta}{2}\right]$ (I.8.4)

$e^\pm$ **on composite spin-**$\frac{1}{2}$ $(s=\frac{1}{2}; M, S=\frac{1}{2})$ $[\ldots] \times \left[\left(F_1^2+\tau F_2^2\right) + 2\tau\,(F_1+F_2)^2\tan^2\frac{\theta}{2}\right]$ (I.8.5)

F_1: Dirac FF; F_2: Pauli FF

(moved Z from prefactor to $F_1(0)=Z=G_E(0)$, $F_2(0)=\kappa=G_M(0)-Z$ anom. mag. mom.)

$\tau=-\frac{q^2}{4M^2}$; $G_{E,M}$: Sachs FFs $[\ldots] \times \left[\frac{G_E^2+\tau\,G_M^2}{1+\tau} + 2\tau\,G_M^2\tan^2\frac{\theta}{2}\right]$

$e^\pm$ **inelastic inclusive** $x\in[0;1[$

$$\left.\frac{d\sigma}{d\Omega\,dE'}\right|_{\text{lab}} = \left(\frac{2\alpha}{q^2}\right)^2 \cos^2\frac{\theta}{2}\,E'^2\left[\frac{F_2(Q^2,x)}{\nu} + \frac{2\,F_1(Q^2,x)}{M}\tan^2\frac{\theta}{2}\right] \quad \text{(I.8.6)}$$

Structure functions $F_{1,2}(Q^2,x)$ have nothing to do with Pauli/Dirac FFs

Summary: Electron Scattering, Hadron Currents & Tensors [HM 8.4]

hadr. tensor, current
scatter off virtual γ

$$e^2 W^{\mu\nu} = \frac{1}{2S+1} \sum_{S,S'} J^{\mu}_{S,S'}(p,p') J^{\nu\dagger}_{S,S'}(p,p') \text{ with } J^{\mu}_{S,S'}(p,p') = \langle p', S' | J^{\mu} | p, S \rangle$$

electromagnetic current conservation $q_\mu J^\mu = 0 \implies q_\mu W^{\mu\nu} = 0 = q_\nu W^{\mu\nu}$

rel. Rutherford: Coulomb
$(s=0; M=\infty, S=0)$

$$J^\mu = -\mathrm{i} Z e \delta^{\mu 0} \quad \text{i.e. point charge at rest} \qquad \text{(I.7.1C)}$$

$e^\pm$ **Coulomb on composite**
$(s=\frac{1}{2}; M=\infty, S=0)$

$$J^\mu = -\mathrm{i} Z e \delta^{\mu 0} F(|\vec{q}|) \text{ with charge form factor: Fourier of } \rho(\vec{r})$$

$$F(|\vec{q}|) := \frac{4\pi}{Ze} \int_0^\infty \mathrm{d}r \, \frac{r}{q} \, \sin(qr) \, \rho(r)$$

$e^\pm$ **full elmag on composite**
$(s=\frac{1}{2}; M \text{ finite}, S=0)$
(most general for $S=0$)

$$J^\mu = -\mathrm{i} Z e \, F(q^2) \, (p^\mu + p'^\mu) \qquad \text{(I.7.3C)}$$

$$\implies W^{\mu\nu} = Z^2 \, (p+p')^\mu \, (p+p')^\nu \, \left| F(q^2) \right|^2 \qquad \text{(I.7.3W)}$$

$e\mu \to e\mu$**: no structure**
$(s=\frac{1}{2}; M, S=\frac{1}{2})$

$$J^\mu_{S,S'} = -\mathrm{i} Z e \, \bar{u}_{S'}(p') \, \gamma^\mu \, u_S(p) \qquad \text{(I.7.4C)}$$

$$\implies W^{\mu\nu} = 2Z^2 \left[p^\mu p'^\nu + p'^\mu p^\nu - g^{\mu\nu}(p \cdot p' - M^2) \right] \quad \text{(I.7.4W)}$$

$e^\pm$ **on composite spin-$\frac{1}{2}$**
(most general elast. $S=\frac{1}{2}$)
$F_1(0) = Z$, $F_2(0) = \kappa$

$$J^\mu_{S,S'} = -\mathrm{i} \underbrace{e \, F_1(q^2) \, \bar{u}_{S'}(p') \gamma^\mu u_S(p)}_{\text{Dirac: modify point-form}} + \underbrace{\frac{e F_2(q^2)}{2M} \, q_\nu \, \bar{u}_{S'}(p') \mathrm{i} \sigma^{\mu\nu} u_S(p)}_{\text{Pauli: anomalous mag. term}} \qquad \text{(I.7.5C)}$$

$e^\pm$ **inelastic inclusive**
(most general inel. hadronic)

$$W^{\mu\nu} = \frac{F_1(q^2,x)}{M} \left[\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right] + \frac{F_2(q^2,x)}{M^2 \nu} \left[p^\mu - \frac{p \cdot q}{q^2} q^\mu \right] \left[p^\nu - \frac{p \cdot q}{q^2} q^\nu \right] \qquad \text{(I.7.6W)}$$

Preview: Electron Scattering on Hadronic Systems

see II.1-3,5

[Martin fig. 2.4]

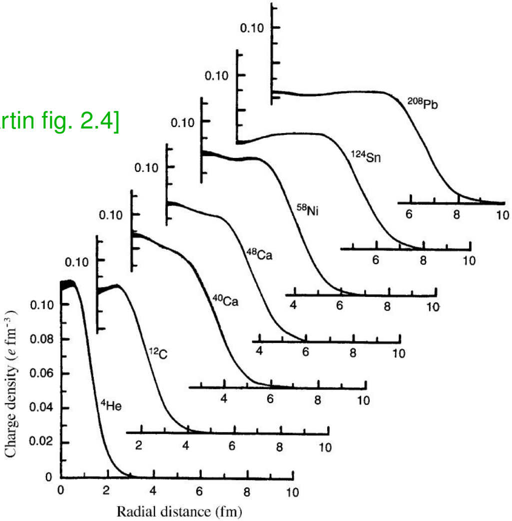
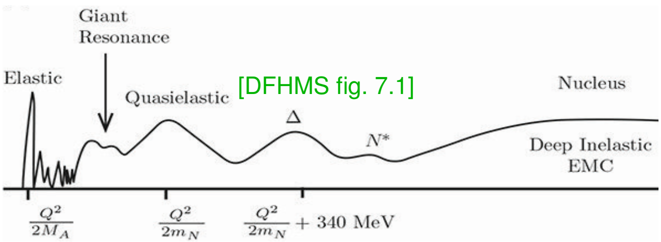


Figure 2.4 Radial charge distributions $\rho_{\rm ch}$ of various nuclei, in units of e fm^{-3}; the thickness of the curves near $r = 0$ is a measure of the uncertaintity in $\rho_{\rm ch}$ (adapted from Fr83)



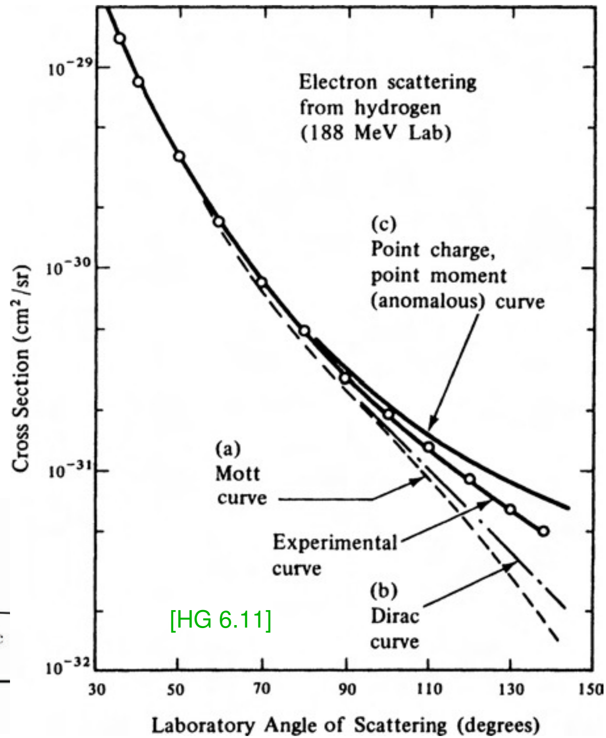
[DFHMS fig. 7.1]

nucleus levels excited

quasi-elast. on bound N in nucleus

inelast. on nucleon

on quarks



[HG 6.11]

Next: II. Phenomena

1. Shapes and Masses of Nuclei

Familiarise yourself with: [PRSZR 5.4, 2.3, 3.1/3; HG 6.3/4, (14.5), 16.1; cursorily PRSZR 18, 19]