PHYS 6610: Graduate Nuclear and Particle Physics I

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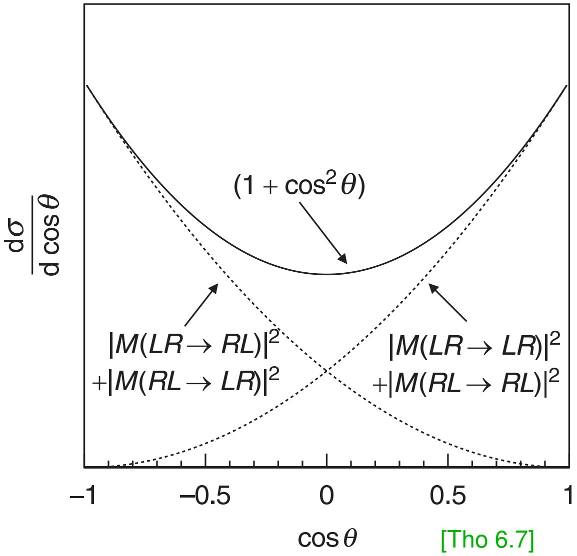
II. Phenomena

5. $e^+e^- \to$ Leptons and Quarks

Or: Why We Believe in Things We Don't See

References: [PRSZR 9.1/3; PRSZR 15/16 (cursorily); HG 10.9, 15.1-7; HM 11.1-3; Tho 9.6]

(a) Recap $e^+e^- \to \mu^+\mu^-$: Massless Point-Fermions

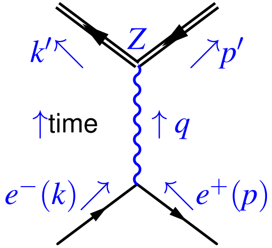


[Tho 6.7]

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{cm}} = \frac{(Z\alpha)^2}{4s}\,(1+\cos^2\theta)$$

Ang. distrib. characteristic of spin-$\frac{1}{2}$.

Crossing symmetry to $e\mu \to e\mu$.



$s = q^2 > 0$: timelike γ

Final state *not* electrons, has quantum numbers of virtual photon: $I(J^{PC}) = 0$ or $1(1^{--})$

⊕ $\frac{1}{137}$, simple to interpret, e^+e^- collider cheap

⊖ Directly probes only charges, not strong int.

$$\sigma_{\text{cm}} = \frac{4\pi(Z\alpha)^2}{3s} = Z^2\,\frac{21.7\text{nb}}{(E^e_{\text{cm}}[\text{GeV}])^2}$$

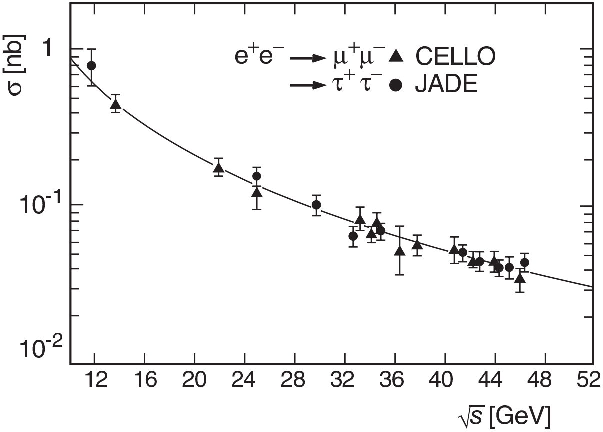
When producing massless point-fermions X of charge Z_X: $R := \dfrac{\sigma(e^+e^- \to X\bar{X})}{4\pi\alpha^2/(3s)} \to \sum\limits_{\substack{\text{finals } X \text{ with}\\ \sqrt{s}\geq 2M_X}} Z_X^2$

(b) Leptoproduction and Lepton Universality

Threshold $\sqrt{s_{\text{min}}} = 2M_l$: muon $M_\mu = 0.106$ GeV (1936), tau lepton $M_\tau = 1.777$ GeV (1975)

Lifetime $\tau(\tau^- \to e^- \bar{\nu}_e \nu_\tau \text{ or } \mu^- \bar{\nu}_\mu \nu_\tau) = 3 \times 10^{-13}\text{s} \gg \tau_{\text{elmag,strong}}$ $\Longrightarrow$ weak decay

Even at $\gamma = \dfrac{E \approx 100\text{GeV}}{M_\tau} \approx 50$, τ lepton travels $c\gamma\tau_\tau = 10^{-2}\text{m}$ before decay $\Longrightarrow$ not in detector



Experiments at $E \gg M_\tau, M_\mu$:

$R(\mu^+\mu^- \text{ or } \tau^+\tau^-) = 1$

$\Longrightarrow |Z_\mu| = |Z_\tau| = 1 = |Z_e|$

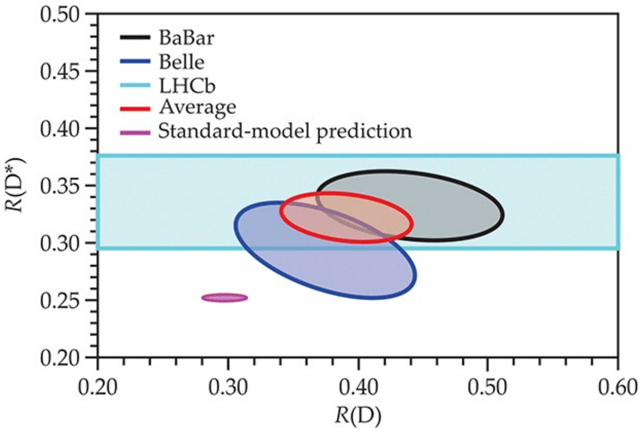
Fig. 9.3. Cross-sections of the reactions $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ as functions of the centre of mass energy $\sqrt{s}$ (from [Ba85] and [Be87]). The solid line shows the cross-section (9.6) predicted by quantum electrodynamics. [PRSZR]

$\Longrightarrow$ ***Lepton Universality Hypothesis:*** **Leptons couple with same form & strengths, and differ only by mass & charge (thresholds etc. different, but *not* by the fundamental couplings).**

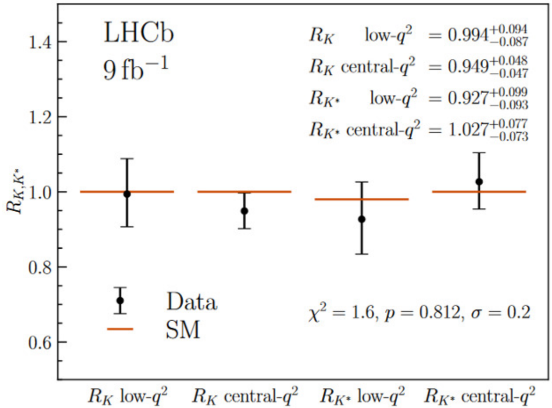
Is Lepton Universality Broken? Update 2022.

BaBar at SLAC 2012: branching ratio $B \to \mu$ or τ [Phys. Rev. Lett. 109 (2012) 101802]

LHCb at CERN 2015: branching ratio $D, D^* \to \mu$ or τ [Phys. Rev. Lett. 115 (2015) 111803]



[Heavy Flavour Averaging Group 2018]



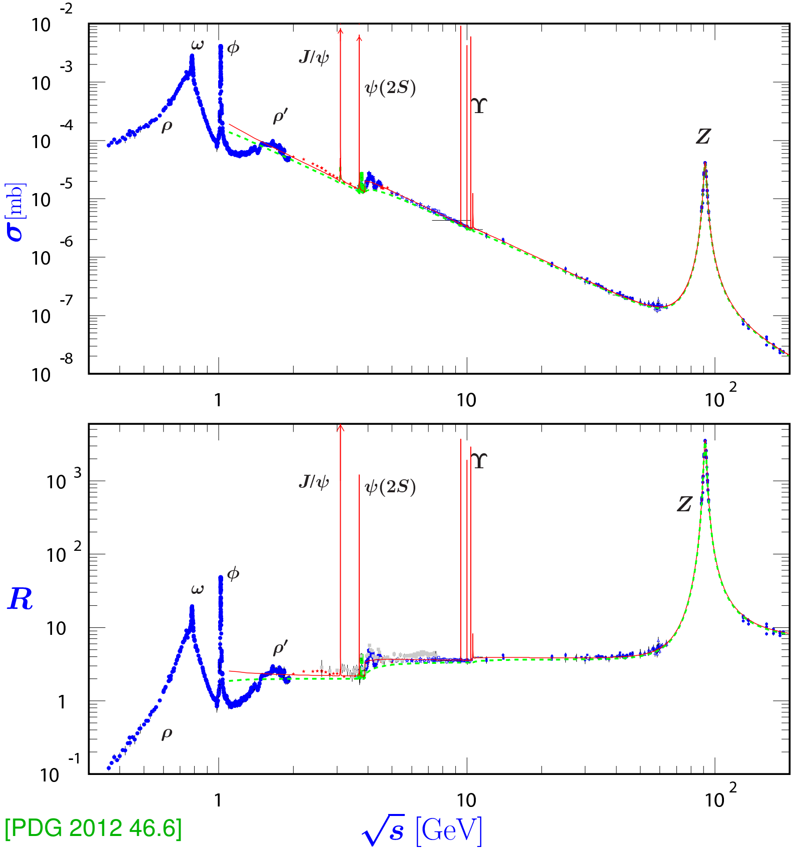
[LHCb seminar & CERN press release Dec 2022]

Lepton Universality may be broken – or Not: 3.9σ**?** 0.5σ**?**

If so, it is small (10^{-2}) **and definitely important: needed for Baryogenesis.**

$\Longrightarrow$ **Beyond-Standard-Model Physics?**

(c) $e^+e^- \to \text{Hadrons}$: Overview



[PDG 2012 46.6]

non-resonant:

well-reproduced by $\sigma \propto \frac{1}{s}$

resonances:

have quantum numbers of γ^*:

$I = 0$ or 1, $J^{PC} = 1^{--}$

wide at low s, *narrow* at high s

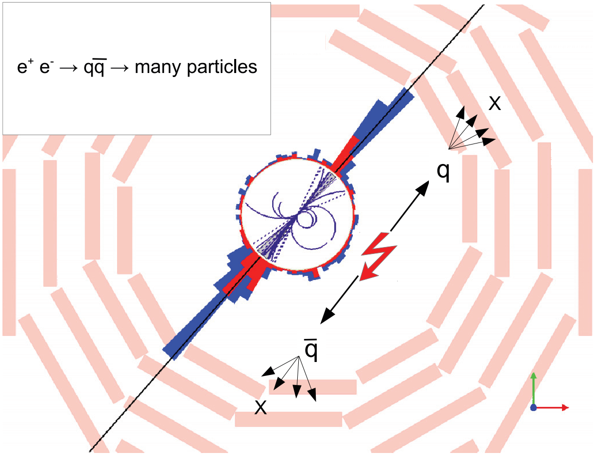
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

increases after each resonance region

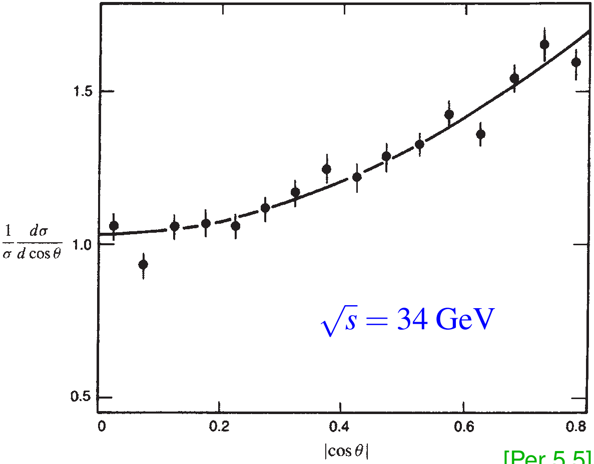
$\Longrightarrow$ **Hadron contains point-like, charged fermions with different masses.**

(d) Nonresonant Hadron Production (High Energies)

Produce point spin-$\frac{1}{2}$ particle: $\left.\frac{d\sigma}{d\Omega}\right|_{\text{cm}} = \frac{(Z\alpha)^2}{4s}\,(1+\cos^2\theta)$



$e^+ e^- \to q\bar{q} \to$ many particles



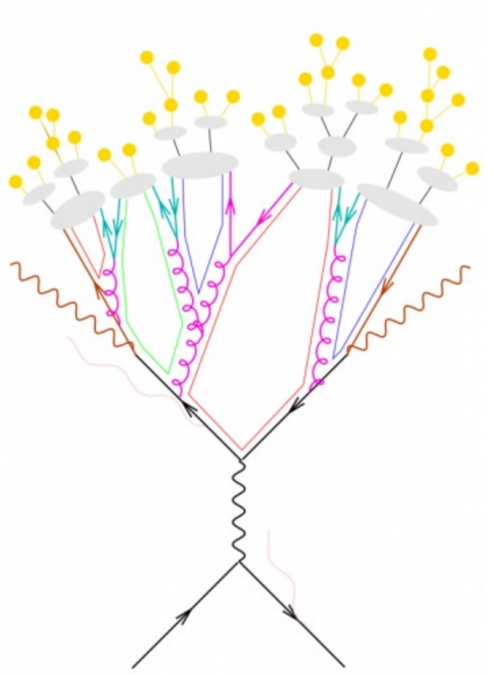
[Per 5.5]

Angular distribution of 2-jet event consistent with $1+\cos^2\theta$ $\Longrightarrow$ Evidence for point-fermions.

Hadronisation from Vacuum: Avoiding Free Quarks & Gluons

No free quarks seen. Each quark of initial $q\bar{q}$ pair carries energy $E \gg m_q$; fly in *opposite directions*.

$\Longrightarrow$ Generate light $q\bar{q}$ pairs out of vacuum.



Rearrange to **dress** bare quarks into **baryons & mesons**:

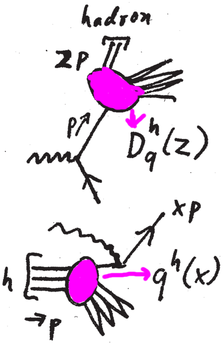
hadronisation timescale $\gg$ pair-production timescale

$\Longrightarrow$ Production & hadronisation: 2-step process:

incoherent sum of $q\bar{q}$-pair productions like in DIS

time $\longrightarrow$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(ee \to hX) = \sum_q \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(ee \to q\bar{q})\,[D_q^h(z) + D_{\bar{q}}^h(z)]$$



which are weighted by

Quark-fragmentation functions

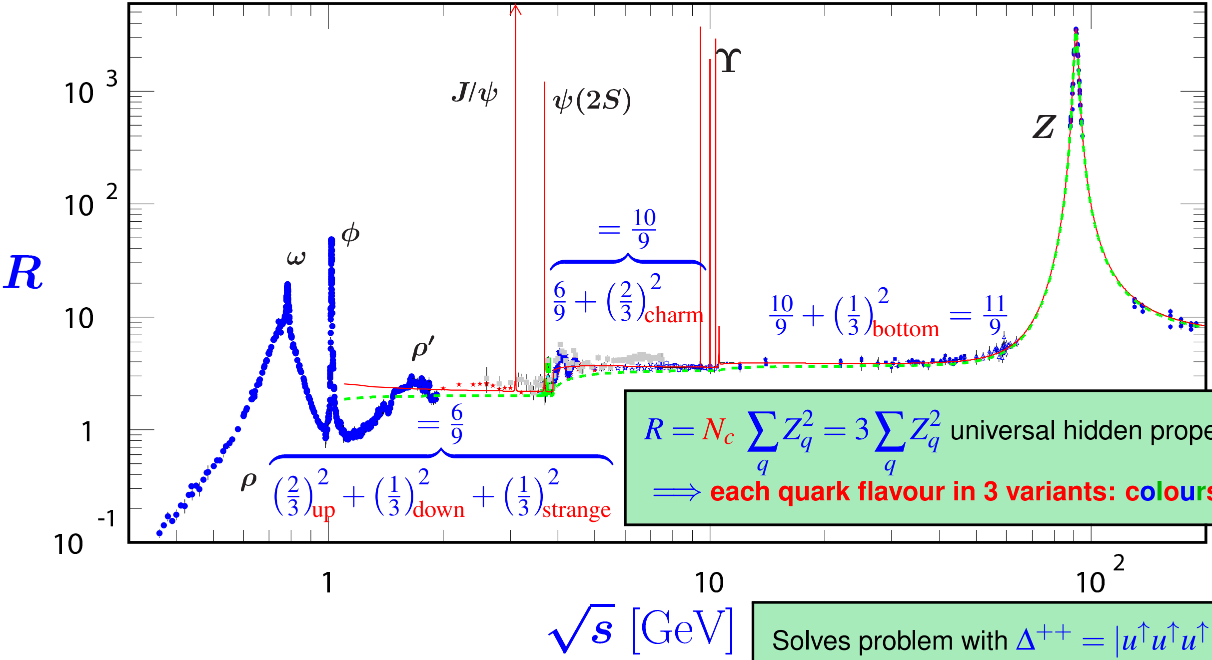
$D_q^h, D_{\bar{q}}^h(z = \frac{E_h}{E_q})$ related to PDFs $q(x)$

by crossing & time-reversal symmetries;

$z = \frac{E_h}{E_q}$: fraction of energy of produced quark which is carried by hadron; cf. Bjørken-x.

R Counts Quark Charges AND Colours

$$R(s) := \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_{\text{quarks } i \text{ with } 2M_i \leq \sqrt{s}} Z_i^2$$

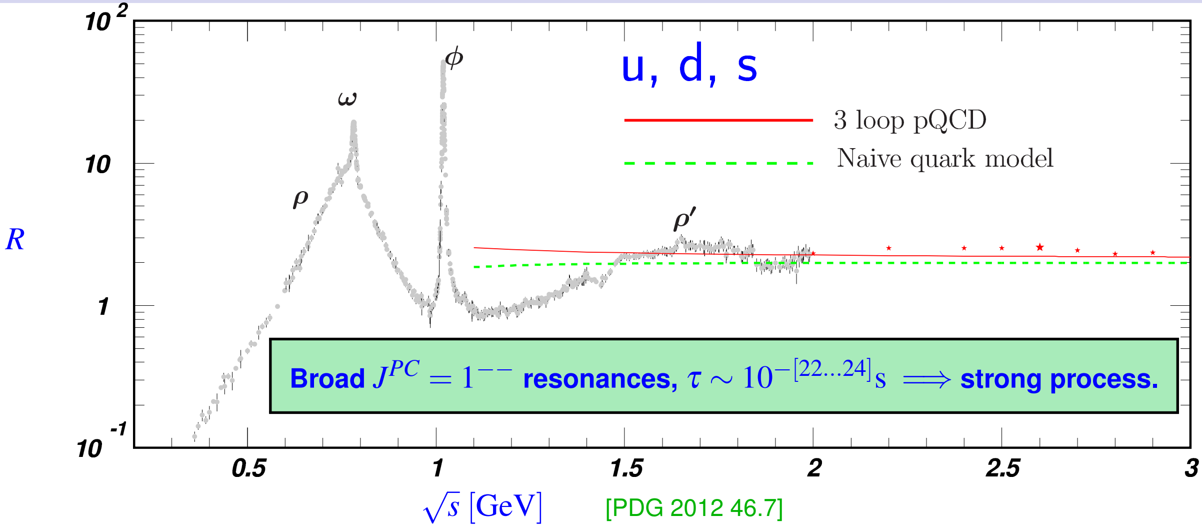


$R = N_c \sum_q Z_q^2 = 3 \sum_q Z_q^2$ universal hidden property

$\Longrightarrow$ **each quark flavour in 3 variants: colours**

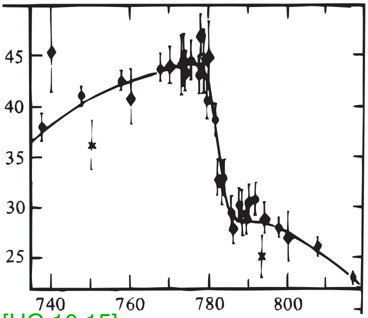
Solves problem with $\Delta^{++} = |u^\uparrow u^\uparrow u^\uparrow\rangle$: antisymmetric if colour wf is. ✓

(e) Resonant Hadron Production at Low Energies



Broad $J^{PC} = 1^{--}$ resonances, $\tau \sim 10^{-[22\ldots24]}\text{s} \implies$ strong process.

[PDG 2012 46.7]



[HG 10.15]

$\omega^0(782\text{ MeV})$: decay $\to \pi^+\pi^0\pi^-$; no isospin partners $\implies I = 0$

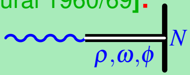
$\rho^0(770\text{ MeV})$: decay $\to \pi^+\pi^-$; isospin partners $\rho^{\pm,0}$ $\implies I = 1$

spin-isospin-constituent-quark content e.g. $|\rho^+\rangle = -|u^\uparrow\bar{d}^\uparrow\rangle$

ω, ρ resonances in close proximity $\implies$ substantial interference!

$\implies$ **Vector Meson Dominance (VMD) Model** [Sakurai 1960/69]:

Elmag. dominated by these mesons, e.g. in γN

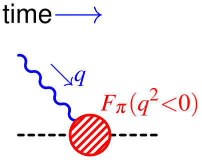


Post-Dicting Vector Mesons (VMD Endorsement)

Something interesting should indeed happen around 800MeV:

Lowest hadron production threshold: $\sqrt{s} = 2m_\pi$ from $e^+e^- \to \pi\pi$.

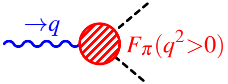
$\pi\gamma$ coupling from pion form factor for space-like $q^2 < 0$ (see II.2.g):



$$J_\pi^\mu = -\mathrm{i}e\,(p'^\mu - k'^\mu)\;\overbrace{\frac{a^2}{a^2-q^2}}^{F_\pi(q^2):\ \text{pion FF}} \quad \text{with } a^2 = \frac{6}{\langle r_\pi^2\rangle} \approx (740\text{MeV})^2 \text{ (exp)}$$

Apply **crossing symmetry/analytic continuation** into **time-like region** $q^2 = s > 0$:

$\Longrightarrow$ Expect pole/very large amplitude/resonance in $J^{PC} = 1^{--}$ processes around



$$q^2 = s = a^2 \approx (740\text{MeV})^2$$

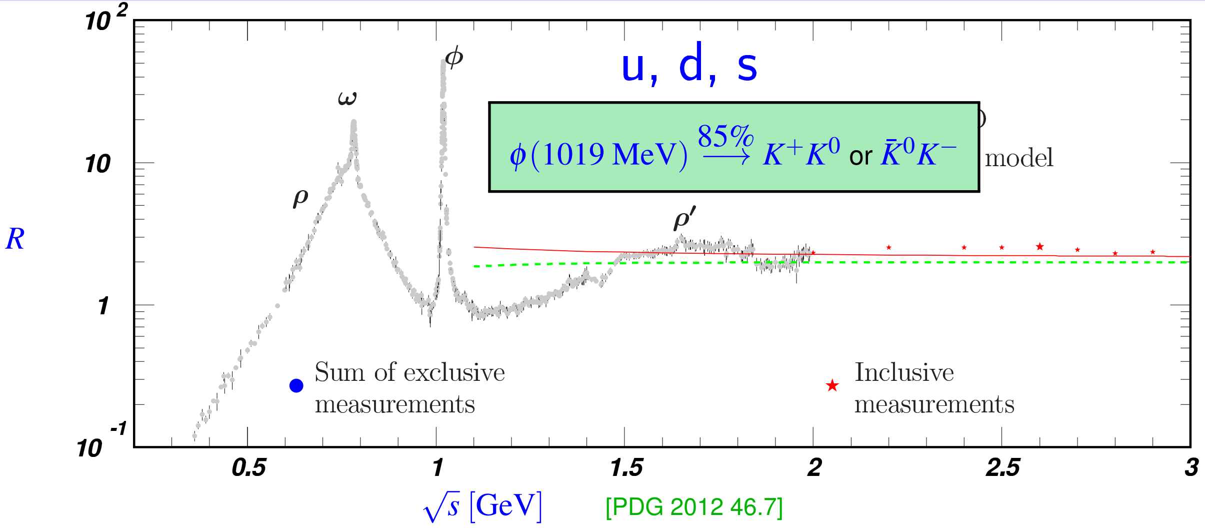
Agrees with ω/ρ-meson quantum numbers $J^{PC} = 1^{--}$ (inherited from photon)

and with $m_\omega \approx m_\rho \approx 775\text{MeV}$.

But be careful: Exp. only gives rough form factor with uncertainties.

$\Longrightarrow$ Analytic continuation needs "reasonable" assumptions.

(f) ϕ Resonance: The Strange Quark



$\phi(1019\,\text{MeV}) \xrightarrow{85\%} K^+K^0$ or $\bar{K}^0K^-$

[PDG 2012 46.7]

Narrow resonance: $\Gamma_\phi = 4.4\,\text{MeV}$ since $2m_K = 990\,\text{MeV} \Longrightarrow$ only $30\,\text{MeV}$ of phase space!

$\phi \to \pi\pi\pi$ decay very small, although $m_\phi - 3m_\pi = 600\,\text{MeV}$ much bigger $\Longrightarrow$ weak int.?!

$\Longrightarrow$ Attribute to new quark flavour: **Strange Quark**; strangeness S conserved in strong int.

K^+K^0, $K^-\bar{K}^0$ isospin doublets $\Longrightarrow$

New charge formula: $Q = \dfrac{\text{Baryon}}{2} + I_3 + \dfrac{S}{2}$

(original) **Gell-Mann–Nishijima relation**

$Q_s = -\dfrac{1}{3}$, $B_s = \dfrac{1}{3}$ But strangeness of strange is $S(s) = -1$: That's strange! (but a definition. . .)

(g) Extending Isospin: The Eightfold Way

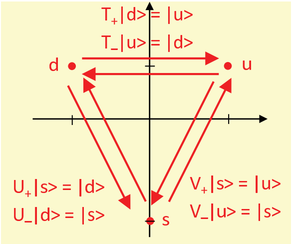
Gell-Mann 1962
good explanation:
[Tho 9.6]

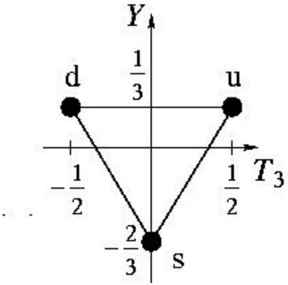
"Tamed" the Particle Zoo in the 1960's.

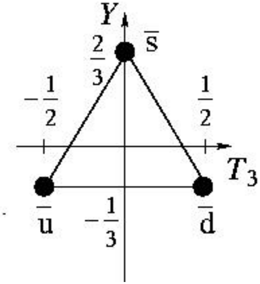
Multiplet classification scheme still used for nomenclature.

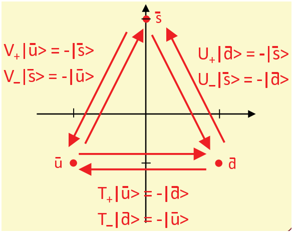
Interpret Strangeness (or **Strong Hypercharge** $Y = S + B$) as quantum number, orthogonal to Isospin.

One Can Show: symmetry group in Nature extends from $SU_I(2) \to SU_{\text{flavour}}(3)$ acting on $\begin{pmatrix} u \\ d \\ s \end{pmatrix}$:



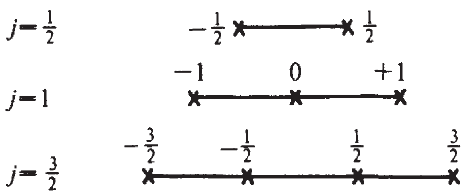


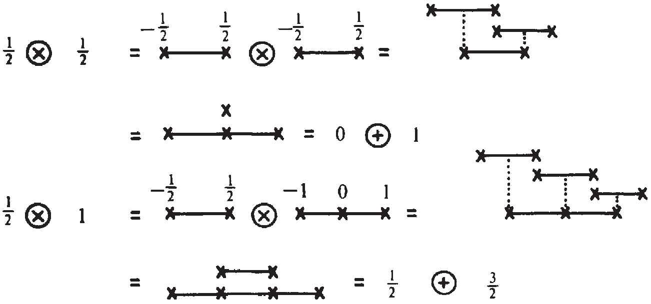




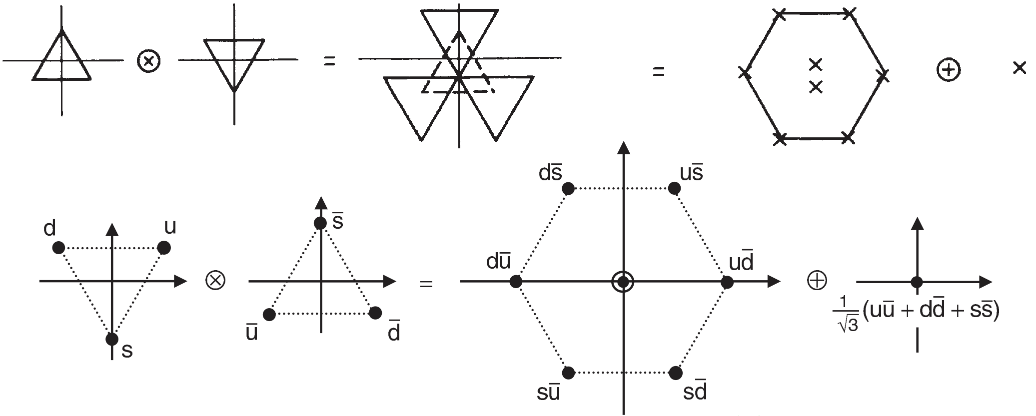
Constructing Multiplets from The Fundamental Representations

Combine **Weight Diagrams** like in $SU(2)$:



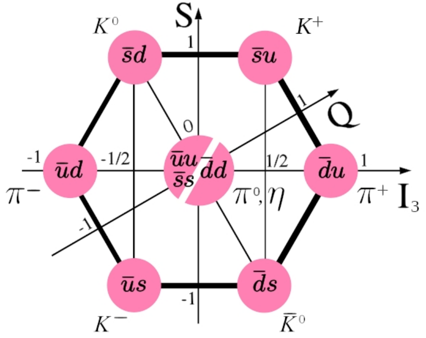


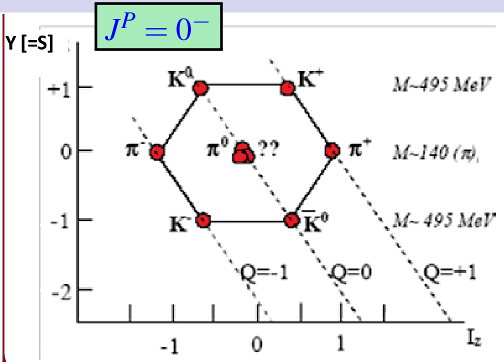
Example: $q\bar{q}$ combinations give Meson Octet & Singlet.



More?: Clebsch-Gordan analogues for $SU(3)$…

Lowest-Mass Meson Octets: Natural Isospin Doublets K^+K^0 & $K^-\bar{K}^0$





Constituent picture $\Longrightarrow$ **Gell-Mann–Okubo mass formula:**

$$m_{\text{meson}} = M_{\text{bind}}^{\text{meson}} + \sum_i m_{qi}?$$

$$m_s \approx 360\text{MeV}?$$

$SU_f(3)$**-Breaking in Ground-State Octet:** $\dfrac{\textbf{diff. } \pm 350\text{MeV}}{\textbf{avg. } 320\text{MeV}} \approx 1!$; **in Excited Octet:** $\dfrac{\pm 80\text{MeV}}{850\text{MeV}} \approx \dfrac{1}{10}$

Example Large Breaking in ϕ of Excited Octet:

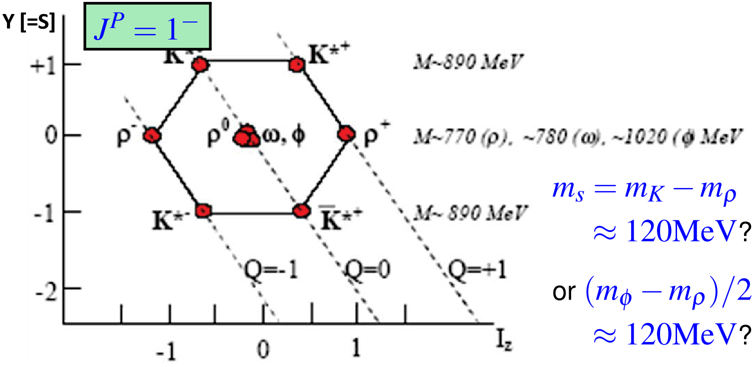
$$m_\phi - 3m_\pi \approx 600\text{MeV}$$
$$\gg m_\phi - 2m_K \approx 30\text{MeV}$$

but π decay tiny (weak $s \to d + \ldots$),

while decays to 85% into *Kaons*

$$\Longrightarrow |\phi\rangle \approx |s\bar{s}\rangle]$$

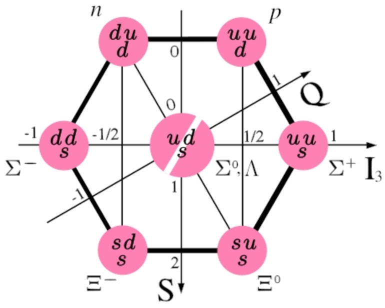
not $|\phi\rangle = \frac{1}{\sqrt{3}}[|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle]$

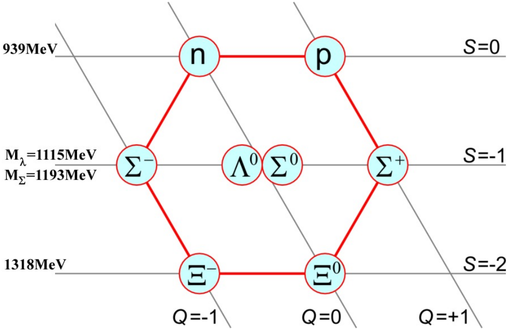


$$m_s = m_K - m_\rho \approx 120\text{MeV}?$$

or $(m_\phi - m_\rho)/2 \approx 120\text{MeV}?$

$\pi N/KN \to X$: Lowest-Mass Baryon Multiplets: Octet & Decouplet





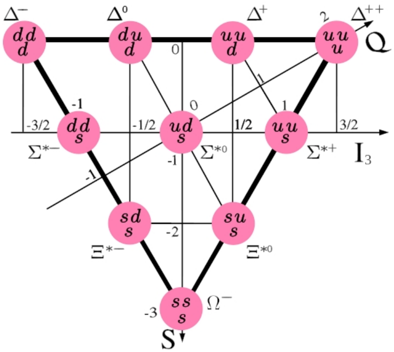
GMO: $M_{\text{baryon}} = M_{\text{bind}}^{\text{baryon}} + \sum_i m_{qi}$

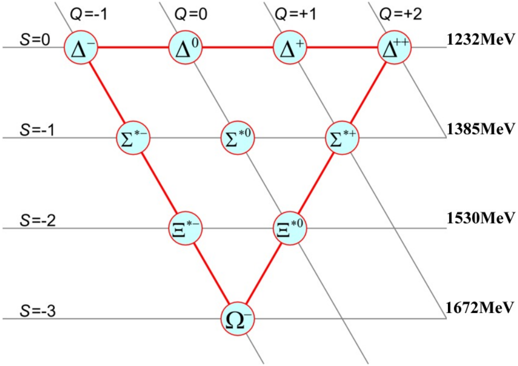
$\Downarrow$ add 1 strange

$\Longrightarrow$ lin. mass incr.

$m_s \approx 210\text{MeV}$?

$SU_f(3)$ **Breaking in Baryon Octet:** $\dfrac{\pm 210\text{MeV}}{1100\text{MeV}} \approx \dfrac{1}{5}$ **in Baryon Decouplet:** $\dfrac{\pm 140\text{MeV}}{1400\text{MeV}} \approx \dfrac{1}{10}$





$\Downarrow$ linear increase

$m_s \approx 140\text{MeV}$?

Gell-Mann 1962: predict quantum numbers & mass of Ω^-. Dedicated experiment found it.

What and How Good Is The Constituent Quark Model?

Assumptions: (cf. ”dressed” electron in solid)

– “Naked” QCD quarks dressed into constituent quarks inside hadrons.

– Constituent quarks determine bulk of quantum numbers;

– still point-like/”elementary”, but with anomalous magentic moments.

Add a spin-spin term $H_{\text{spin}} \propto \dfrac{\vec{\sigma}_{q1} \cdot \vec{\sigma}_{q2}}{m_{q1} m_{q2}}$ with “universal” prefactor

to match hadron masses & magnetic moments.

But conflicting answers:

– Different mesons & baryons give different constituent masses for same quark.

– Point-like but not fundamental: context-dependent masses, anom. mag. mom.

$\Longrightarrow$ constituent quarks $\neq$ QCD (current) quarks of DIS.

– Couplings > 1. $\Longrightarrow$ Perturbative treatment of non-perturbative interaction *inconsistent*!

– Confinement problem unsolved: If perturbative, then no confinement.

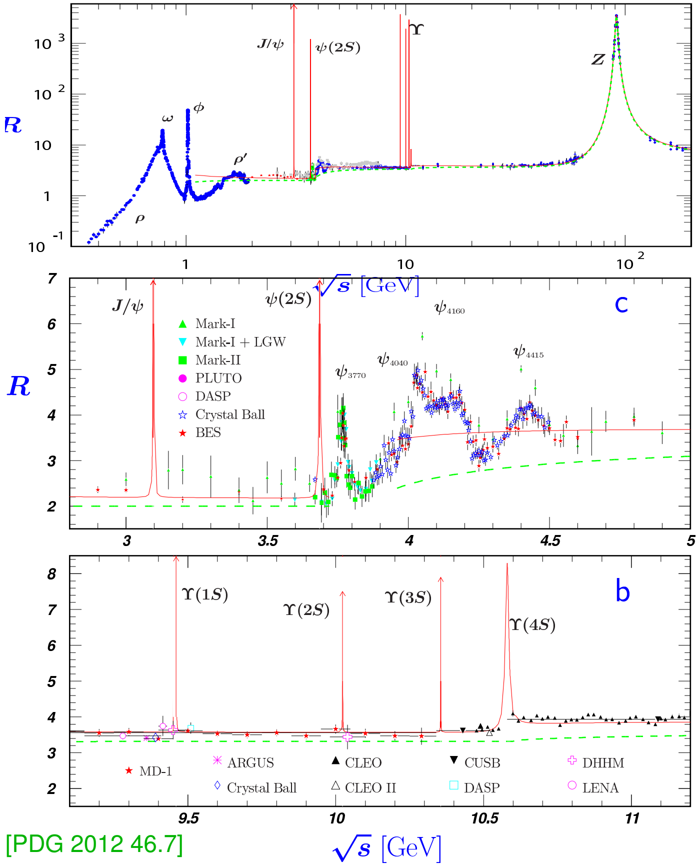
The Constituent Quark* Model *is* not *QCD– It is a QCD-inspired* MODEL*, at best!

Misconceptions lead to *self-inflicted puzzles/crises* (spin-puzzle, missing resonances, EMC effect,. . .).

Predictions for low-mass baryons & mesons misleading (e.g. inconsistent m_s).

Predictions for high-mass baryons & mesons adequate → quarkonia: QCD perturbative.

(h) High-E Resonant Hadron Production: Quarkonia



[PDG 2012 46.7]

Huge cross section, tiny width.

$J/\psi(3097\text{ MeV})$ (November Revolution 1974)

width $\Gamma = 0.093\text{ MeV}$

indicates **electromagnetic decay**.

$$\Delta R = \frac{4}{3} = N_c Z_c^2 \implies Z_c = \pm\frac{2}{3}$$

charm quark: $J/\psi = c\bar{c}$ charmonium

$\Upsilon(9470\text{ MeV})$ (1977)

width $\Gamma = 0.052\text{ MeV}$

$$\Delta R = \frac{1}{3} = N_c Z_b^2 \implies Z_b = \mp\frac{1}{3}$$

bottom quark: $\Upsilon = b\bar{b}$ bottomium

$$\implies Q = \frac{\text{Baryon}}{2} + I_3 + \frac{S+C+B+T}{2}$$

generalised Gell-Mann–Nishijima

"Toponium" (Fermilab 1995): resonance at

$\sqrt{s} = 2M_t \approx 340\text{GeV}$ in $p\bar{p}$ collisions.

Example of Excited States: $\psi \to \gamma X$ Photon Decay Spectrum

Very narrow states, decaying electromagnetically.

reconstruct

level structure

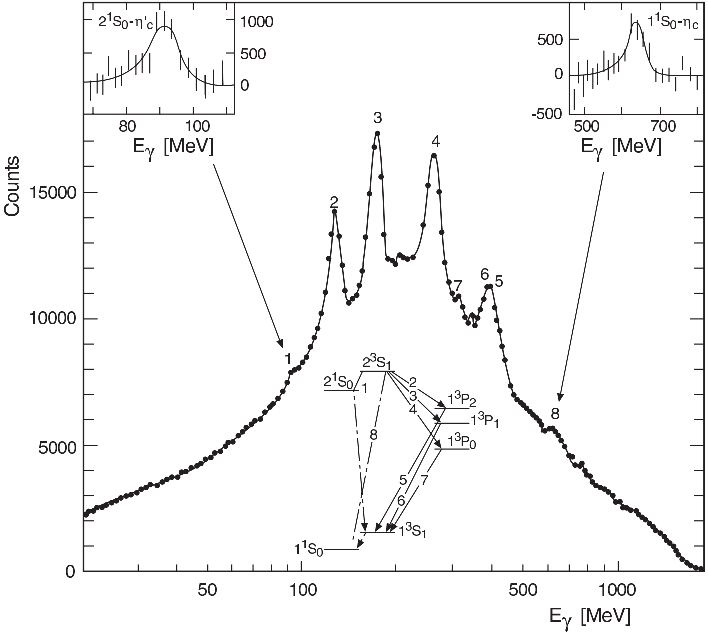


Fig. 13.5. The photon spectrum in the decay of $\psi\,(2^3S_1)$, as measured in a crystal ball, and a sketch of the so extracted charmonium energy levels. The strong peaks in the photon spectrum represent the so numbered transitions in the sketch. The continuous lines in the sketch represent parity changing electric dipole transitions and the dashed lines denote magnetic dipole transitions which do not change parity [Kö86].

[PRSZR]

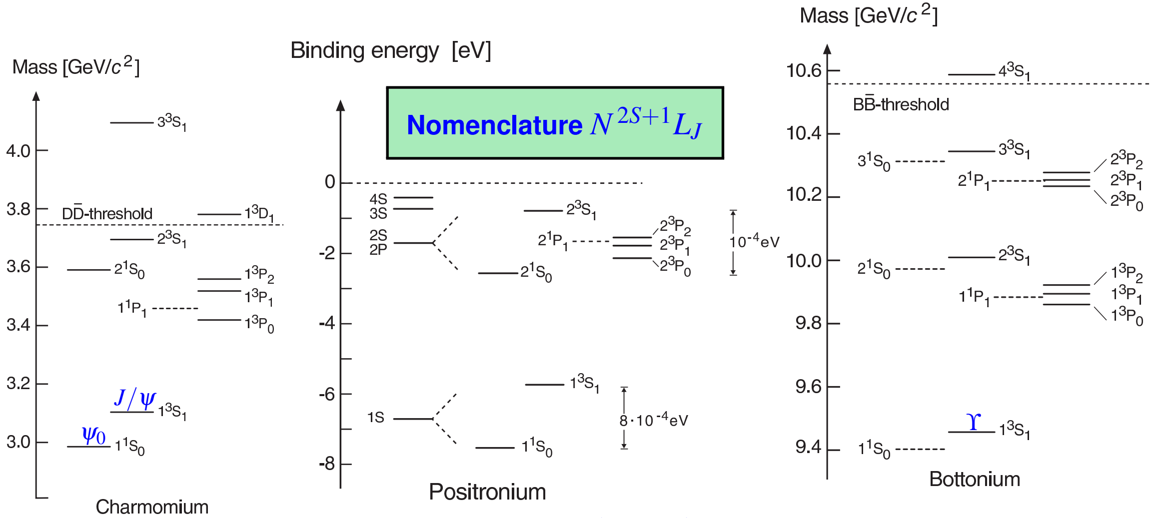
Comparing Quarkonium Spectra to Positronium in QED

J/ψ and Υ are $J^P = 1^-$ resonances. $\Longrightarrow$ $1^3\mathrm{S}_1$ excited states, *not* $J=0$ ground states $1^1\mathrm{S}_0$.

M(quarkonium)$<2M$(heavy-light meson) $\Longrightarrow$ strong decay forbidden $\Longrightarrow$ elmag., small rates

D mesons: $(\bar{c}u)$ etc B mesons: $(\bar{b}u)$ etc K mesons: $(\bar{s}u)$ etc.

Does not apply to ϕ: $2m_K < m_\phi \Longrightarrow$ strong decay but small phase space.



Nomenclature $N^{2S+1}L_J$

Spectra adjusted to same $1^1\mathrm{S}_0 - 2^1\mathrm{S}_0$ gap. [PRSZR]

$\Longrightarrow$ *Quarkonia* (heavy-heavy): Coulombic potential for low states – different for high states.

6. Summary: The Path to QCD

- Hadrons contain near-massless charged spin-$\frac{1}{2}$ point-particles. **partons ⟶ quarks**
- Parton masses do not set masses of light mesons, nucleons. **hadron masses from strong int.**
- 6 quark flavours: u, d, s, c, b, t – only charges $\pm\frac{2}{3}, \pm\frac{1}{3}$, $Q = \frac{\text{Baryon}}{2} + I_3 + \frac{S+C+B+T}{2}$.
- Approx. hadron mass multiplets: SU$_f(2)$ $\begin{pmatrix} u \\ d \end{pmatrix}$; less well for SU$_f(3)$ $\begin{pmatrix} u \\ d \\ s \end{pmatrix}$. **flavour symmetry generalises isospin**
- Quarks come in 3 colours ($\Delta^{++}(u^\uparrow u^\uparrow u^\uparrow)$, R in $e^+e^- \to$ hadr). **colour degree of freedom**
- Quarks only differ by mass & charge (and related effects). **flavour & colour universality**
- Neutral, strongly int. hadron constituents carry large fractions of its momentum & spin. **gluons**
- Strong int. QED-like & perturbative for $E, m_q \nearrow \infty$ (quarkonia, 3-jet event). **Asymptotic Freedom**

Identify gluons with colour carriers?

- No free quarks seen. **Quark Confinement Hypothesis** plausible, unproven
- No free gluons seen. **Gluon Confinement Hypothesis** plausible, unproven
- No states with net nonzero colour seen. **Colour Neutrality Hypothesis** plausible, unproven

Find a theory which explains all this, and Nuclear Physics – *quantitatively*!

Next: III. Descriptions

1. Non-Abelian Gauge Theories

Familiarise yourself with: [HM 14.1-4, 2.15; HG 12.3; CL 8.1]