PHYS 6610: Graduate Nuclear and Particle Physics I

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Spring 2023

III. Descriptions

4. Pions and Nucleons: Chiral Effective Field Theory

Or: What I Do for a Living

References: [(Goldstone: CL 5; Ryd 8.1-3); Scherer/Schindler: Primer χEFT;
CL 5; Ryd 8.1-2; Ber 2, 3; Ericson/Weise: Pions and Nuclei Chap. 9;
lectures 1-6 of Fleming's EFT online course at MIT; and much more – see me!]

(a) Matching Expectations

What Holds the Nucleus Together?

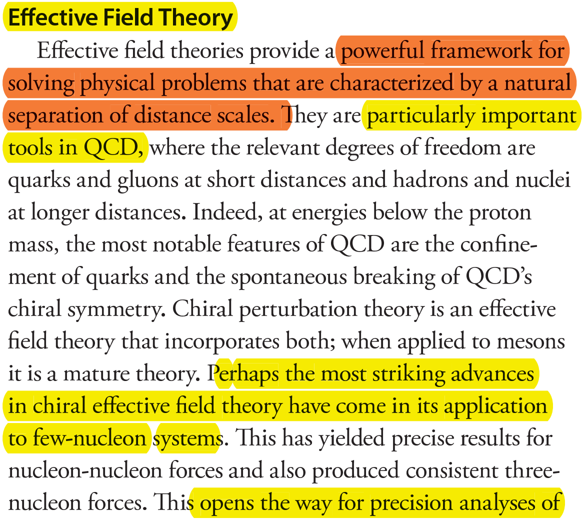
1953

In the past quarter century physicists have devoted a huge amount of experimentation and mental labor to this problem – probably more man-hours than have been given to any other scientific question in the history of mankind. [. . .]

The glue that holds the nucleus together must be a kind of force utterly different from any we yet know.

[Hans A. Bethe: “What holds the nucleus together?”, *Scientific American* **189** (**1953**), no. 2, p. 58]

2007



Effective Field Theory

Effective field theories provide a powerful framework for solving physical problems that are characterized by a natural separation of distance scales. They are particularly important tools in QCD, where the relevant degrees of freedom are quarks and gluons at short distances and hadrons and nuclei at longer distances. Indeed, at energies below the proton mass, the most notable features of QCD are the confinement of quarks and the spontaneous breaking of QCD’s chiral symmetry. Chiral perturbation theory is an effective field theory that incorporates both; when applied to mesons it is a mature theory. Perhaps the most striking advances in chiral effective field theory have come in its application to few-nucleon systems. This has yielded precise results for nucleon-nucleon forces and also produced consistent three-nucleon forces. This opens the way for precision analyses of

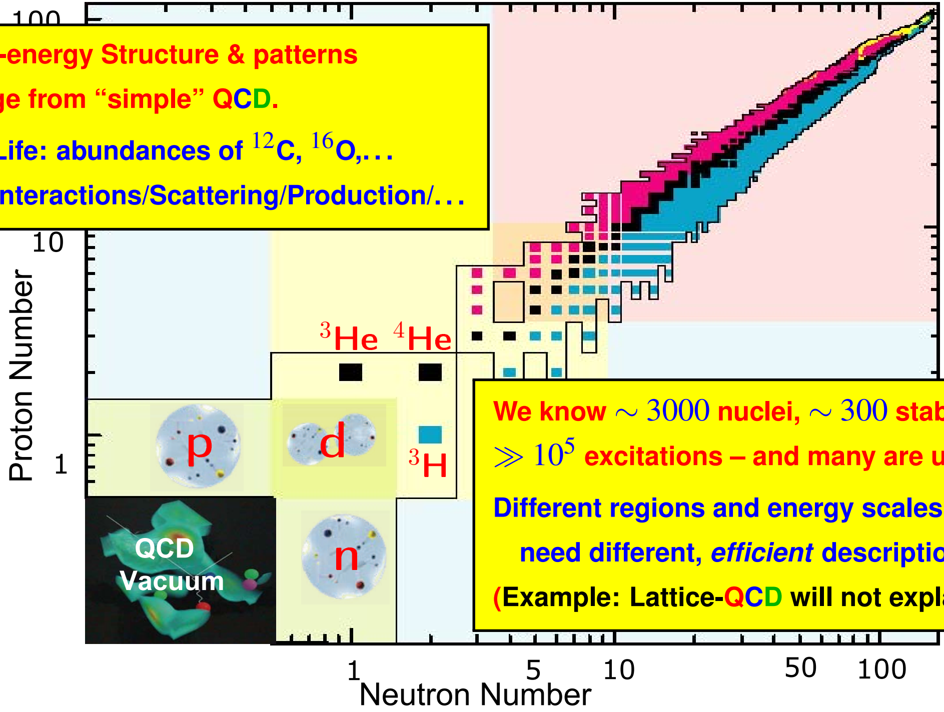
[US Nuclear Science Advisory Committee Long Range Plan **2007**, “QCD & Structure of Hadrons” p. 18/19]

We look for an approach which • connects to QCD; • is efficient;
• provides falsifiable predictions with reliable theoretical uncertainties.

The Bridge From QCD To Nuclear Physics

$\mathcal{L}_{QCD} = \sum_q \bar{\Psi}_q [i\not{\partial} + g\not{A} - m_q]\Psi_q - \frac{1}{2}\mathrm{tr}[F^{\mu\nu}F_{\mu\nu}]$ **with *few* parameters: $\alpha_s(Q_0^2) + 6$ masses.**

Nucleon & Few-N System: gateway to *quantitative* understanding of nuclear structure from QCD.



Rich low-energy Structure & patterns

all emerge from “simple” QCD.

Explain Life: abundances of ^{12}C, ^{16}O,. . .

Explain Interactions/Scattering/Production/. . .

We know ~ 3000 nuclei, ~ 300 stable,

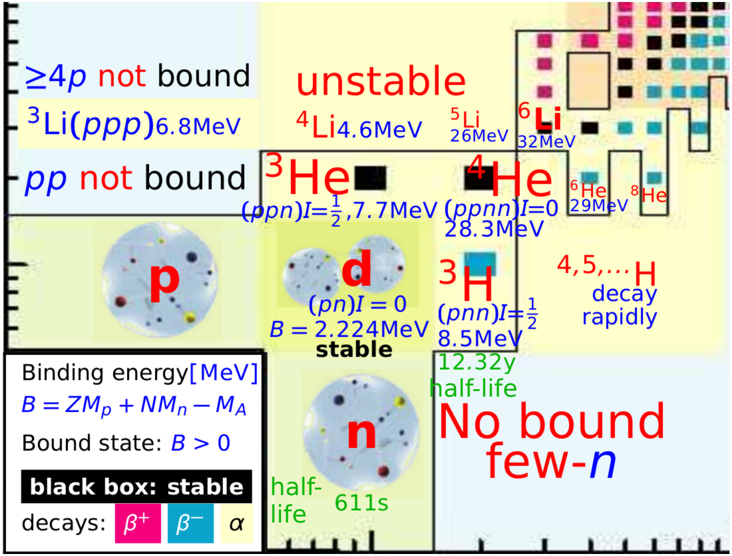
$\gg 10^5$ excitations – and many are unknown.

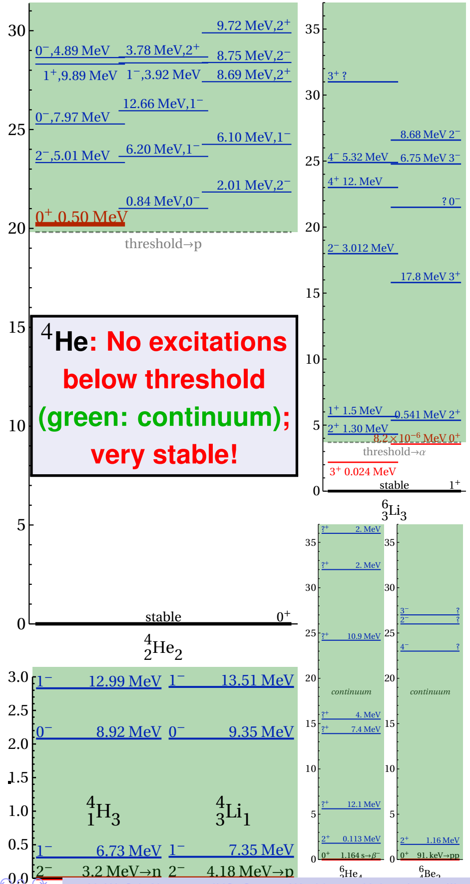
Different regions and energy scales

need different, *efficient* descriptions.

(Example: Lattice-QCD will not explain ^{235}U.)

(b) Few-Nucleon Systems: Complexity, Patterns, Bridge





^{4}He: No excitations below threshold (green: continuum); very stable!

Tally of States: 44 isospins allowed for $A \leq 6$

4(5) **stable**: d, ^{3}H, ^{3,4}He, ^{6}Li

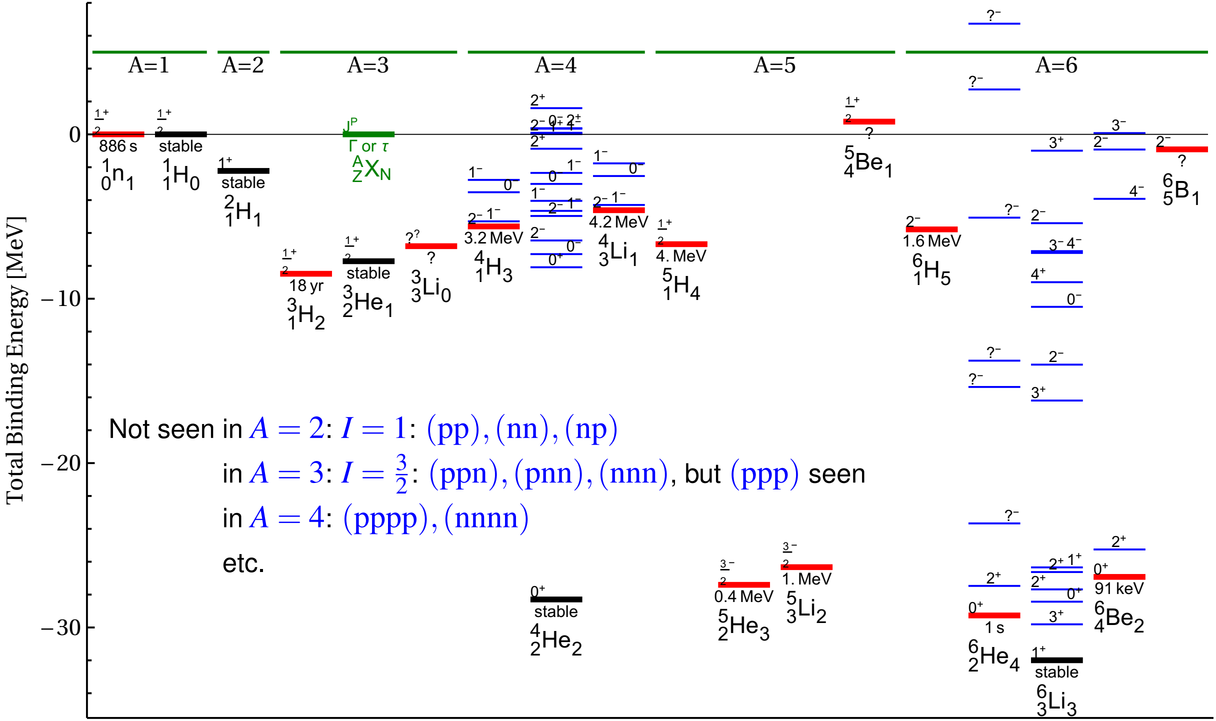
11 **unstable**: ^{3}Li & ^{5}(H,He,Li,Be) & ^{6}(H,B): 1 level

^{4}(H,Li), ^{6}(He,Be): ≥ 4 levels; 50 **excitations, only** 2 **bound**

Notable No-Shows: few-n, $pn(I=1)$, pp, $4p$, $A=5$ unstable,. . .

Whence the Patterns?

Few-Nucleon Spectra “Should” follow from QCD



Not seen in $A=2$: $I=1$: $(pp), (nn), (np)$

in $A=3$: $I=\frac{3}{2}$: $(ppn), (pnn), (nnn)$, but (ppp) seen

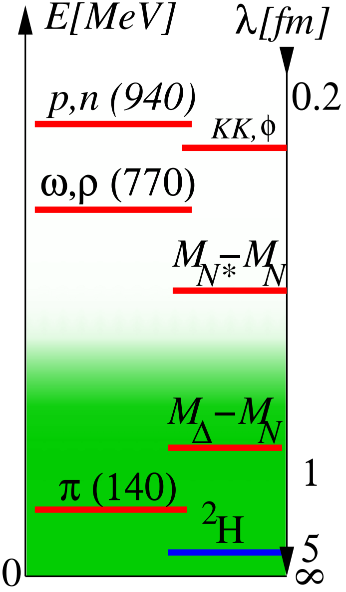
in $A=4$: $(pppp), (nnnn)$

etc.

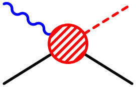
Whence the Patterns? How much is special to QCD?

(c) A Question of Resolution: Effective Field Theories

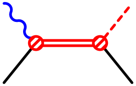
What Is “Low Energy”?: QCD Spectrum Above Nucleon Has Gap



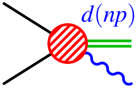
- **Low-energy excitations** at scales $p_{\text{typ}} \lesssim 300\text{MeV}$:



lightest mesons produced: pion $m_\pi \approx 140\text{MeV}$



lowest resonance: $\Delta(1232)$, $M_\Delta - M_N \approx 300\text{MeV}$



bound states of nucleons, e.g. $E_B[d(np)] \approx 2.2\text{MeV}$:

$$\frac{1}{\text{size}} = p_{\text{bind}} \approx \sqrt{ME_B} \approx 45\text{MeV}$$

- **High-energy excitations** at scales $p_{\text{typ}} \gtrsim 1000\text{MeV}$:

next-lightest meson $m_{\rho,\omega} \approx 770\text{MeV}$;

next-lowest excitation $M_{N^*} - M_N \approx 600\text{MeV}$;

strange excitations only as s-quark pair: $2M_K \approx 1000\text{MeV}$

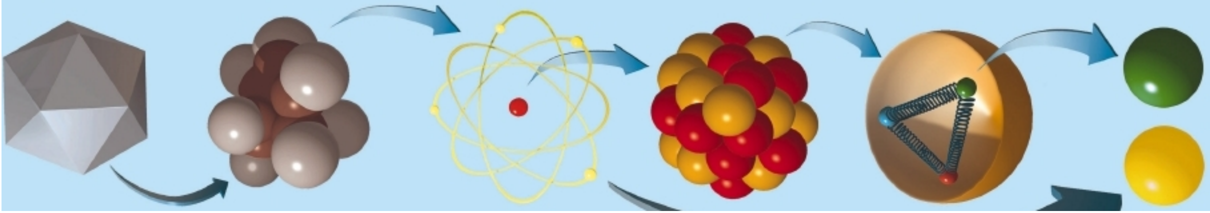
string constant $\sigma \approx 1\text{GeV}/\text{fm}$, $\alpha_s(1\text{GeV}^2) \to 1, \ldots$

QCD at low resolution/energy: Confinement/infrared slavery of quarks & gluons.

$\Longrightarrow$ **Rearrange into “seen”/*effective low-energy degrees of freedom:*** $N = \binom{p}{n}$, $\Delta(1232)$, π^a.

$\mathcal{L}[N,\Delta,\pi^a]$: **any interaction imaginable.** $\Longrightarrow$ **Small, dimension-less expansion parameter?!?**

What You See Is What You Get: $\Delta x\,\Delta p \gtrsim \hbar$ Taken Seriously



To probes with wavelength λ, object of size R appears	point-like for $\lambda \gg R$,	blurry for $\lambda \gtrsim R$,	composed for $\lambda \lesssim R$.

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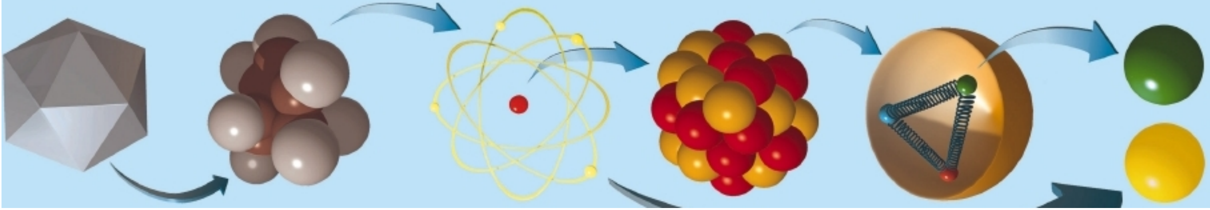

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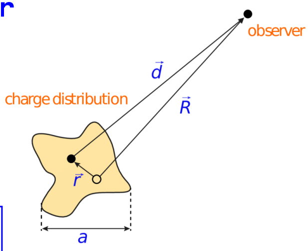
- **Example Electric Multipole Expansion of Localised Charge Distribution**

$$\Phi(\vec{R}) = \frac{1}{4\pi} \int \mathrm{d}^3 r' \, \frac{\rho(\vec{r}')}{|\vec{R} - \vec{r}'|}$$ usually impossible to do

$\Longrightarrow$ **Separation of Scales:** expand in **small dimension-less parameter**:

$$Q = \frac{a}{R} \ll 1 \implies \Phi(\vec{R}) = \frac{1}{4\pi} \left[\frac{Ze}{R} + \frac{\vec{d} \cdot \vec{\mathrm{e}}_R}{R^2} + \frac{Q_{ij} \mathrm{e}_R^i \mathrm{e}_R^j}{R^3} + \mathcal{O}\left(\frac{a^3 \text{or??}}{R^4} \right) \right]$$



[S. Babilon, ResearchGate]

Efficient: Save time and effort – **makes many calculations even just doable!**

Low-Energy Coefficients (LECs): charge Ze, dipole mom. $\vec{d} = \# \times Ze\, a \sim Ze\, a$, $Q_{ij} \sim Ze\, a^2, \dots$

Simple parameters resolve increasingly more detail of complicated short-distance Physics, calculate/fix by data – estimate from system size, charge, dimensions: 2^l multipole $Q_l \sim Zea^l$.

Error Estimate: Next term with **Naturalness Assumption**. **Breakdown Scale:** $Q \to 1$, i.e. $R \approx a$.

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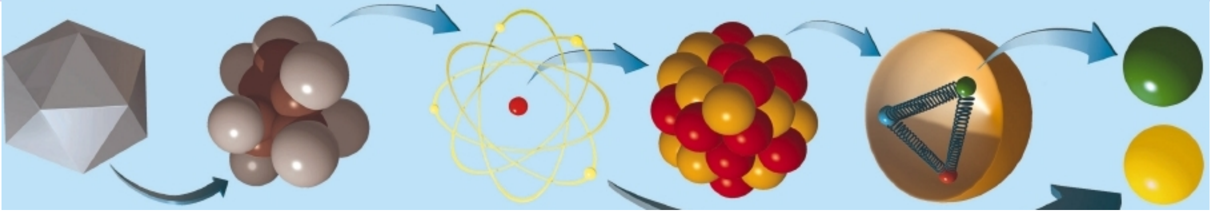

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EFT Tenet: **Short-distance physics does not have to be right for a good calculation, because a low-energy process cannot probe details of the high-energy structure.** **[e.g. Weinberg 1979]**

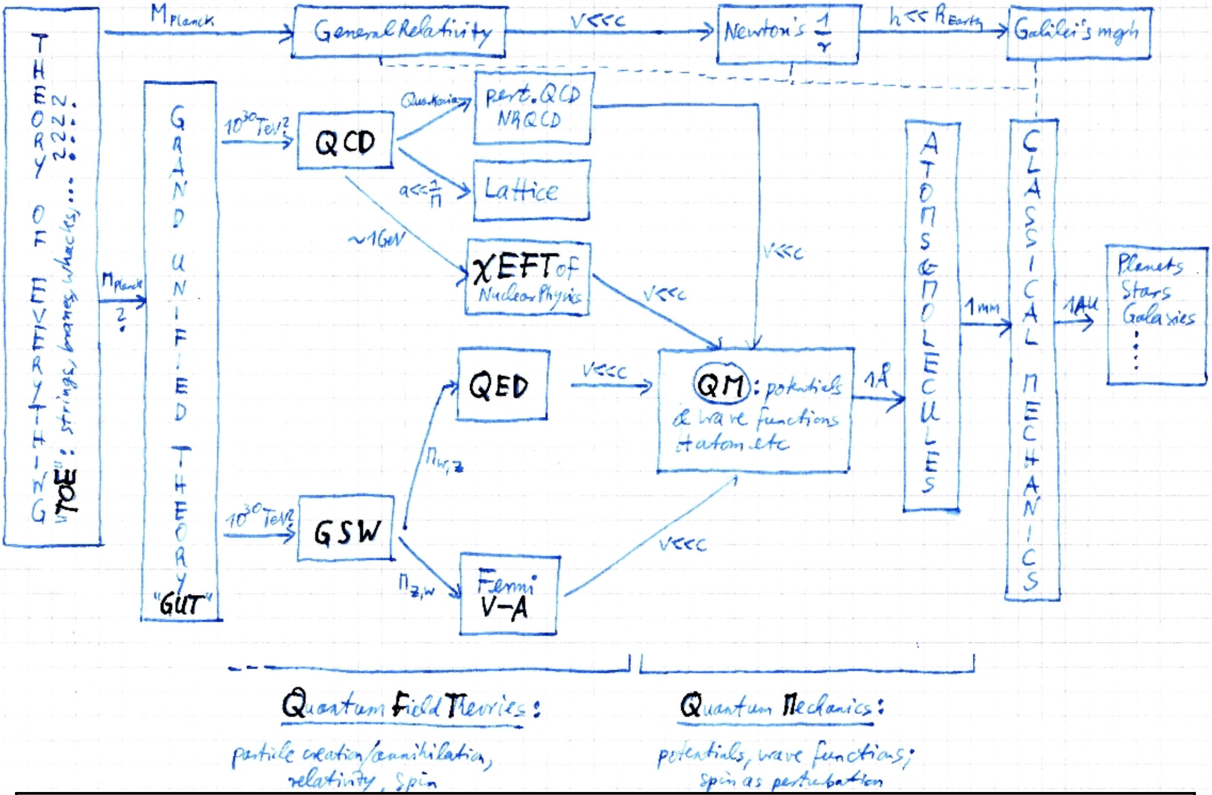
$\Longrightarrow$ ***Effective Field Theories***

Identify those degrees of freedom and symmetries which are *appropriate* to resolve the *relevant* Physics at the scale of interest.

Systematic approximation of real world

with **estimate of theoretical uncertainties.**

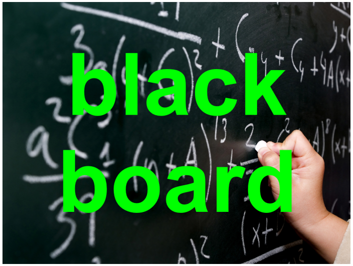
The Onion We Call Nature: The World Is Effective



All Physics Theories applicable only in a *limited energy range* (except in-effective TOE. . .).

And Another Example: Why The Sky Is Blue

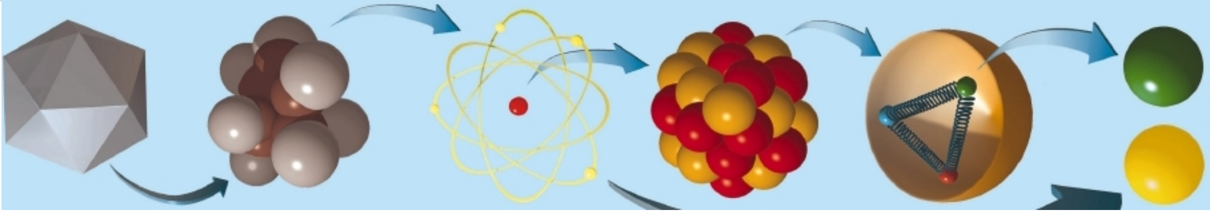
John William Strutt,
3rd Baron of Rayleigh 1871/99



What you Don’t See Can’t Hurt You

"QUARKS. NEUTRINOS. MESONS. ALL THOSE DAMN PARTICLES YOU CAN'T SEE. THAT'S WHAT DROVE ME TO DRINK. BUT NOW I CAN SEE THEM!"

What you Don’t See Can’t Hurt You



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An Effective Field Theory Cookbook

Wilson, Weinberg 1967, 1979;
Georgi, Manohar, … 1982-

Ingredients:

Separation of scales by breakdown-scale $\bar{\Lambda}_{\text{EFT}}$:

high momenta $q_{\text{high}} \gtrsim \bar{\Lambda}_{\text{EFT}} \longrightarrow$ simplify complicated/unknown UV into **Low-Energy Coefficients (LECs)**: contact interactions.

low momenta $q_{\text{low}} \ll \bar{\Lambda}_{\text{EFT}}$

Effective (i.e. relevant) degrees of freedom: ***What's Seen at That Scale*** $\longrightarrow$ correct IR-Physics

Symmetries at low scales constrain interactions. Lorentz, gauge, isospin, parity,…

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Recipe:

Write down **most general Lagrangean (set of interactions)** permitted by particles and symmetries.

Infinitely many terms $\Longrightarrow$ **Order in small, dimension-less expansion parameter**

$$\boxed{Q = \frac{\text{typ. low momenta } q_{\text{low}}}{\text{breakdown scale } \bar{\Lambda}_{\text{EFT}}} = \frac{1/(\text{resolution } \lambda)}{1/(\text{target size } R)} \ll 1:}$$

Power-counting for quantum loops & LECs (loops: usually simple; LECs: some fun!)

$\Longrightarrow$ Expand observable, **truncate** at desired predicted accuracy: $\mathcal{O} = c_0 Q^0 + c_1 Q^1 + c_2 Q^2 + \dots$

Estimate importance of LECs & terms before calculation by

Naïve Dimensional Analysis & Naturalness Assumption.

Determine LECs at desired accuracy from underlying theory or (simple) low-mom. observables.

Calculate Observable and learn, or extract unknown from data, or check QCD predictions, or. . .

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Result:

Model-independent, universal, systematic, unique: Predictions with estimate of uncertainties.

Truncation Error of Q-series: from short-distance details not captured. "Space for Improvement"
Finite accuracy with minimal number of parameters at each order. "Compress Unknown Information"

Weinberg's "Folk Theorem"

Original [Physica **96A** (1979) 327] – here 1997 version

Also Called "Swiss Basic Law"/"Totalitarian Principle"

often attributed to Gell-Mann

[. . .], **you're not really making any assumption that could be wrong, unless of course Lorentz invariance or quantum mechanics or cluster decomposition is wrong,** [. . .]

[. . .] **As long as you let it be the most general possible Lagrangian consistent with the symmetries of the theory, you're simply writing down the most general theory you could possibly write down.**

[Steven Weinberg: *What is quantum field theory, and what did we think it is?* [hep-th/9702027]]

"I know of no proof, but I am sure it's true. That's why it's called a »folk theorem«." [Weinberg, Chiral Dynamics 2009 (Bern)]

"EFT = Symmetries + Parameterisation of Ignorance"???

WHAT CAN POSSIBLY GO WRONG???

Know Your Limitations: Garbage-In, Garbage-Out!

Serve With Caution:

Check assumptions:

– $p_{\text{typ.}} \nearrow \bar{\Lambda}_{\text{EFT}} \Longrightarrow Q \not\ll 1$?

> **“EFTs carry seed of their own destruction.”**
> [D. R. Phillips]

– **No scale separation?** e.g. N^* jungle at $2\ \text{GeV}$

– **Wrong constituents/degrees of freedom?**

new d.o.f. e.g. QED at $100\ \text{GeV}$ without W, Z

change of d.o.f. over phase transition

e.g. $N, \pi \longrightarrow$ quarks, gluons

– **Wrong Symmetry Assumed?** Nature refuses it?

e.g. impose Parity in weak interactions

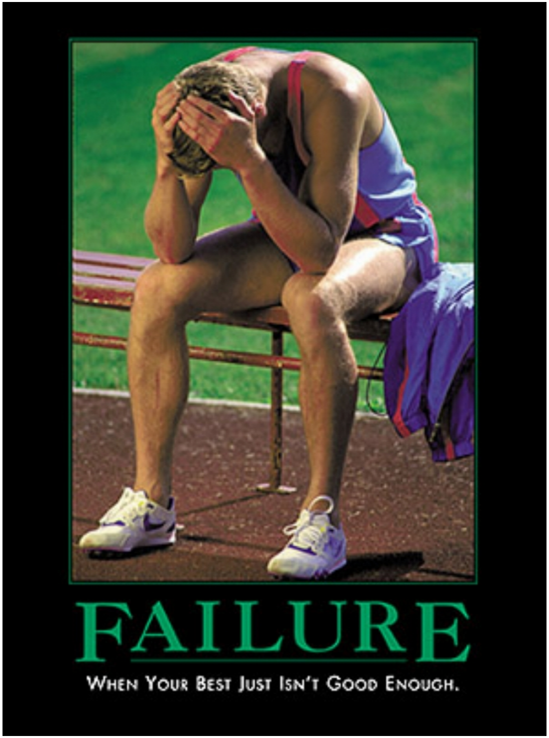
Check *Quantitatively Predicted* Convergence Pattern:

– Convergence? ***Coefficients of Natural Size?***

$\Longrightarrow$ Bayesian Statistics predicts “error-bars”. → later

– Order by order smaller corrections.

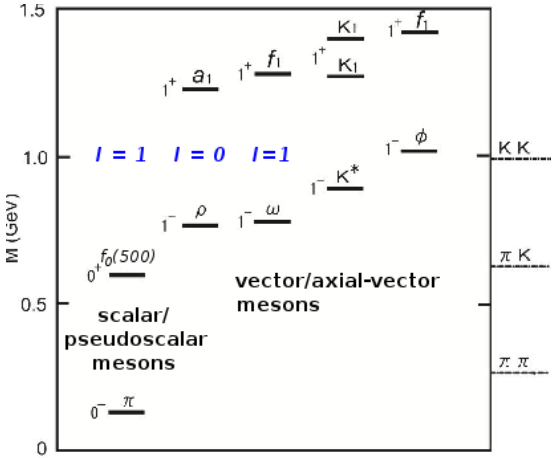
– Order by order less cut-off/RScheme dependence.



Falsifiability: Convergence to Nature tests assumptions. – *After* theory uncertainties determined.

(d) Chiral Symmetry: Nambu-Goldstone Theorem

The Pion Is Special



typical hadronic/ QCD scale $\sim 1\text{GeV} \gg$ $m_\pi^\pm = 139.57\text{MeV}$

$m_\pi^0 = 134.97\text{MeV}$

pseudo-scalar iso-triplet

$I(J^P) = 1(0^-)$

Next non-strange hadron mass $m[f_0(500)] \approx 3.5 m_\pi$ with $I(J^P) = 1(0^+)$: opposite parity.

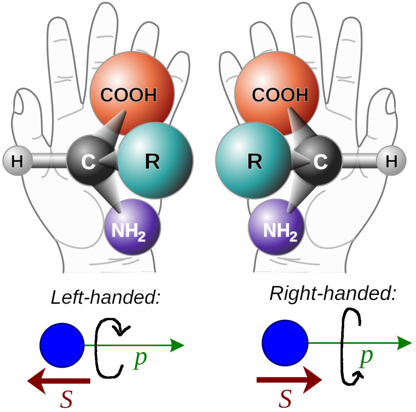
$\Longrightarrow$ ***Pion by far lightest hadron*** $\Longrightarrow$ **mediates interaction with longest-range** $R \sim \frac{1}{m_\pi} \sim 1.4\text{fm}$**;**

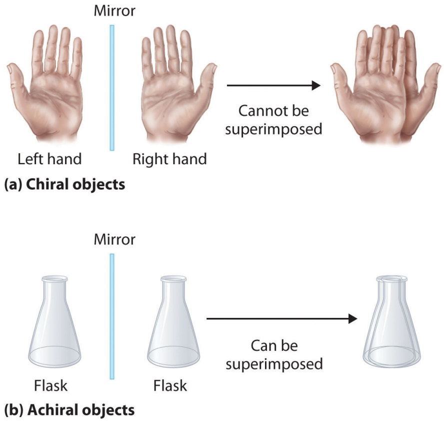
decays only weakly $\pi^\pm \to \mu^\pm \nu_\mu$**.** $\to$ HW later

Why is the pion mass so much smaller than any typical QCD scale $\sim 1\text{GeV}$?

No “King’s Way” to Chiral Symmetry in QCD

Chirality: Mirror image and object differ





(a) Chiral objects

(b) Achiral objects

Helicity: spin projection $\frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|} = \pm 1$ not conserved for $m > 0$: overtaking with $v < c$ reverses $\vec{p}$.

for massless particles, helicity $=$ chirality

Fact Of Nature: None of the forces we know mixes chiralities/helicities – only the mass does!

QCD in chiral basis for γ^μ with $q_{R/L} = \begin{pmatrix} u_{R/L} \\ d_{R/L} \end{pmatrix}$ eigenstates to $\gamma_5\, q_{R/L} = \pm q_{R/L}$

$$\bar{q}[\mathrm{i}\partial\!\!\!/ + g\, A\!\!\!/ - m_q]q = \left(q_R^\dagger, q_L^\dagger\right) \begin{pmatrix} E - gA_0 + \vec{\sigma}\cdot(\vec{p} + g\vec{A}) & m_q \\ m_q & E + gA_0 - \vec{\sigma}\cdot(\vec{p} - g\vec{A}) \end{pmatrix} \begin{pmatrix} q_R \\ q_L \end{pmatrix}$$

Review: Chiral Symmetry in QCD

QCD in chiral γ^μ basis: $\begin{pmatrix} q_R^\dagger, q_L^\dagger \end{pmatrix} \begin{pmatrix} E - gA_0 + \vec{\sigma}\cdot(\vec{p}+g\vec{A}) & m_q \to 0 \\ m_q \to 0 & E + gA_0 - \vec{\sigma}\cdot(\vec{p}-g\vec{A}) \end{pmatrix} \begin{pmatrix} q_R \\ q_L \end{pmatrix}$, $q_{R/L} = \begin{pmatrix} u_{R/L} \\ d_{R/L} \end{pmatrix}$

$\Longrightarrow$ For $m=0$ invariant under **separate transformations** $R \in SU_R(2)$, $L \in SU_L(2)$ in flavour space:

$$q_R \to Rq_R = \mathrm{e}^{-\mathrm{i}\theta_R^a \frac{\tau^a}{2}} q_R \quad q_L \to Lq_L = \mathrm{e}^{-\mathrm{i}\theta_L^a \frac{\tau^a}{2}} q_L$$

$\Longrightarrow SU_R(2) \times SU_L(2)$ **chiral symmetry**

$\Longrightarrow$ **Noether Theorem:** 2×3 conserved currents & charges:

$$j_R^{\mu a} = \bar{q}_R \gamma^\mu \frac{\tau^a}{2} q_R \;:\; \partial_\mu j_R^{\mu a} = 0 \qquad \text{and} \qquad j_L^{\mu a} = \bar{q}_L \gamma^\mu \frac{\tau^a}{2} q_L \;:\; \partial_\mu j_L^{\mu a} = 0$$

More convenient are these linear combinations:

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More convenient are these linear combinations:

Vector Current: $V_\mu^a := j_R^{\mu a} + j_L^{\mu a} = \bar{q}\gamma^\mu \frac{\tau^a}{2} q$ $\Longrightarrow$ **Vector Charges:** $Q^a = \int \mathrm{d}^3r\, q^\dagger \frac{\tau^a}{2} q$

Symmetry in Nature? **YES:** $SU_V(2)$ **isospin**, e.g. $m_{\pi^+} = m_{\pi^-} \approx m_{\pi^0} \Longrightarrow I, I_3 = Q^3$ label states ✓

Review: Chiral Symmetry in QCD

QCD in chiral γ^μ basis: $$(q_R^\dagger, q_L^\dagger)\begin{pmatrix} E-gA_0+\vec{\sigma}\cdot(\vec{p}+g\vec{A}) & m_q\to 0 \\ m_q\to 0 & E+gA_0-\vec{\sigma}\cdot(\vec{p}-g\vec{A}) \end{pmatrix}\begin{pmatrix} q_R \\ q_L \end{pmatrix},\quad q_{R/L}=\begin{pmatrix} u_{R/L} \\ d_{R/L} \end{pmatrix}$$

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$\Longrightarrow SU_R(2)\times SU_L(2)$ **chiral symmetry**

$\Longrightarrow$ **Noether Theorem:** 2×3 conserved currents & charges:

$$j_R^{\mu a}=\bar{q}_R\gamma^\mu\frac{\tau^a}{2}q_R\ :\ \partial_\mu j_R^{\mu a}=0 \qquad \text{and} \qquad j_L^{\mu a}=\bar{q}_L\gamma^\mu\frac{\tau^a}{2}q_L\ :\ \partial_\mu j_L^{\mu a}=0$$

More convenient are these linear combinations:

Vector Current: $V_\mu^a:=j_R^{\mu a}+j_L^{\mu a}=\bar{q}\gamma^\mu\frac{\tau^a}{2}q$ $\Longrightarrow$ **Vector Charges:** $Q^a=\int \mathrm{d}^3r\, q^\dagger\frac{\tau^a}{2}q$

Symmetry in Nature? **YES:** $SU_V(2)$ **isospin**, e.g. $m_{\pi^+}=m_{\pi^-}\approx m_{\pi^0}$ $\Longrightarrow I, I_3=Q^3$ label states ✓

Axial Current: $A_\mu^a:=j_R^{\mu a}-j_L^{\mu a}=\bar{q}\gamma^\mu\ \gamma_5\ \frac{\tau^a}{2}q$ $\Longrightarrow$ **Axial Charges:** $Q^a=\int \mathrm{d}^3r\, q^\dagger\ \gamma_5\ \frac{\tau^a}{2}q$

Symmetry realised in Nature?

$$H\left(Q_5^a|\text{state}\rangle\right)=Q_5^a\left(H|\text{state}\rangle\right)=E\left(Q_5^a|\text{state}\rangle\right)$$

$$P\left(Q_5^a|\text{state}\rangle\right)=-Q_5^a\left(P|\text{state}\rangle\right)$$

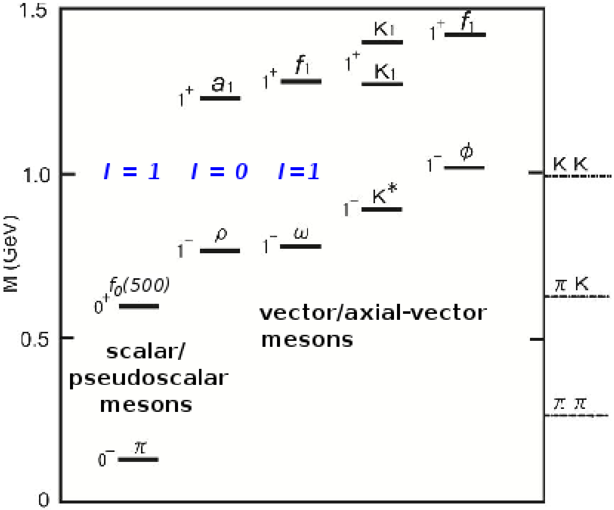
$\Longrightarrow Q_5^a|$**state**$\rangle$ **should have same mass, opposite parity.**

Chiral Symmetry Is Broken!

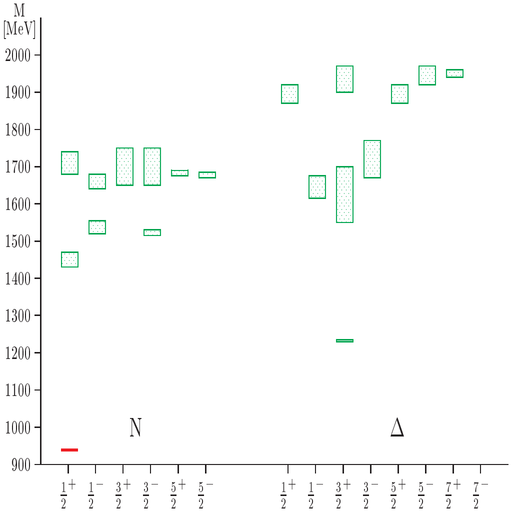
$\Longrightarrow Q_5^a|\textbf{state}\rangle$ **should have same mass, opposite parity.**

but we see no low-lying parity doublets in Nature: $m[I(J^+)] \neq m[I(J^-)]$

Meson Mass Spectrum



Baryon Mass Spectrum



[Cohen/Glozman IJMPA**17** (2002) 1327]

$\Longrightarrow$ ***Axial*** $SU_A(2)$ ***symmetry*** **generated by** Q_5^a **is broken although** $\mathcal{L}$ **is invariant: Noether???**

Spontaneous Symmetry Breaking: Strange But Not Uncommon

Sometimes, ground state / vacuum of a physical system does not share symmetries of $\mathcal{L}$.

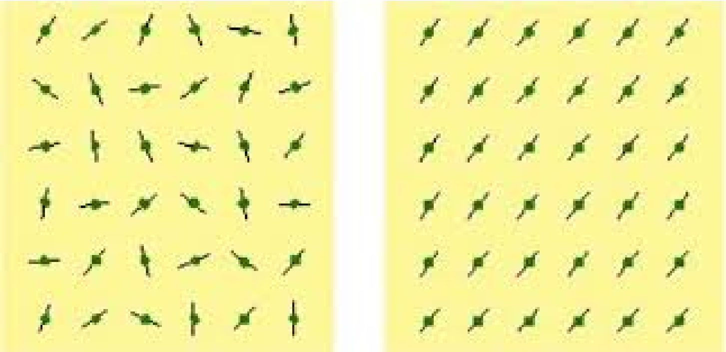
Classical example: pencil ⊥ table is rotationally symmetric, but bends when force applied:

tiny asymmetry in force ⇒ huge breaking of rotational invariance

Example Ferromagnet:

Spin-spin interactions $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ rotationally symmetric (isotropy), but ground state wants all spins aligned: point in same direction.

$\Longrightarrow$ Preferred orientation.



Nambu-Goldstone Symmetry Breaking [Ryder 8.1-2, EW App. 14]

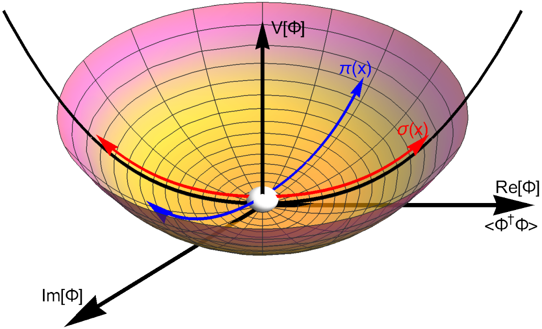
Example: Global $U(1)$ Symmetry Breaking in the Nonlinear σ Model

Landau-Ginzburg model of complex scalar Φ: $\mathcal{L}_{\text{LG}} = (\partial_\mu\Phi)^\dagger(\partial^\mu\Phi) - \lambda \left[\Phi^\dagger\Phi - a^2\right]^2$

depends on magnitude $|\Phi|$, not on phase: continuous global symmetry $\mathcal{L}_{\text{LG}}(\mathrm{e}^{\mathrm{i}\alpha}\Phi) = \mathcal{L}_{\text{LG}}(\Phi)$ ($\alpha\in\mathbb{R}$).

Field Theory: Potential at every point x, particles are excitations moving between points $x_1 \to x_2$.

Noether?? Semi-classical: quantum fluctuations around classical ground state/state of least action.



For $\lambda < 0$, ground state has same symmetry:

$\langle\Phi\rangle = 0$: Zero Vacuum Expectation Value (VEV)

Re-parametrise oscillations around ground state:

$$\Phi(x) = \frac{\sigma(x) + \mathrm{i}\pi(x)}{\sqrt{2}}$$ with real fields σ, π

Nambu-Goldstone Symmetry Breaking

[Ryder 8.1-2, EW App. 14]

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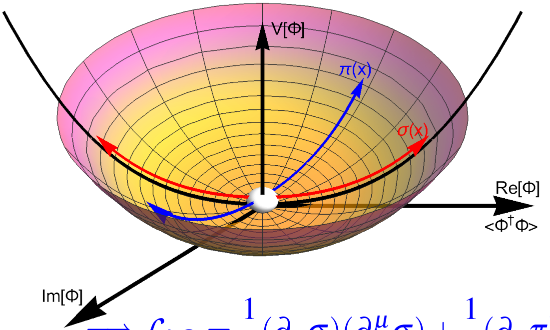

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Re-parametrise oscillations around ground state:

$$\Phi(x) = \frac{\sigma(x) + \mathrm{i}\pi(x)}{\sqrt{2}}$$ with real fields σ, π

$$\Longrightarrow \mathcal{L}_{\text{LG}} = \frac{1}{2}(\partial_\mu \sigma)(\partial^\mu \sigma) + \frac{1}{2}(\partial_\mu \pi)(\partial^\mu \pi) - \underbrace{a^2|\lambda|}_{= m^2/2} (\sigma^2 + \pi^2) - \underbrace{\frac{\lambda}{4}(\sigma^2 + \pi^2)^2}_{\sigma\sigma,\ \pi\sigma,\ \pi\pi \text{ forces symmetric}}$$

σ, π see same curvature $\Longrightarrow m_\sigma^2 = m_\pi^2 = 2a^2|\lambda|$

Wigner-Weyl (Symmetric) mode: ground and particle states share symmetry of Lagrangean.

Nambu-Goldstone Symmetry Breaking

[Ryder 8.1-2, EW App. 14]

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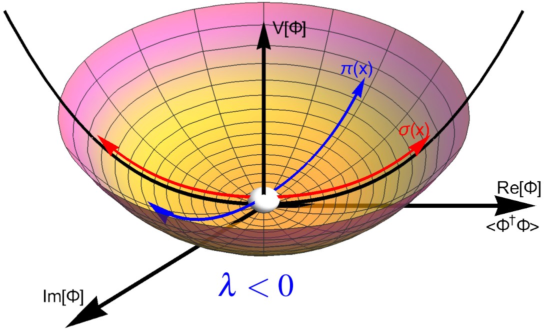
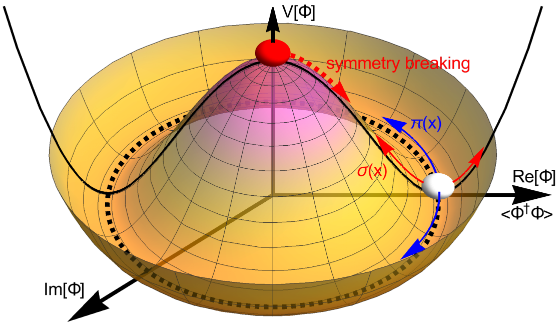
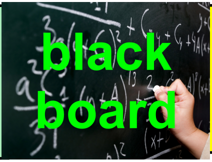


$\lambda < 0$



$\lambda > 0, a \neq 0$

Wigner-Weyl (Symmetric) Mode: ground and particle states share symmetry of Lagrangean.



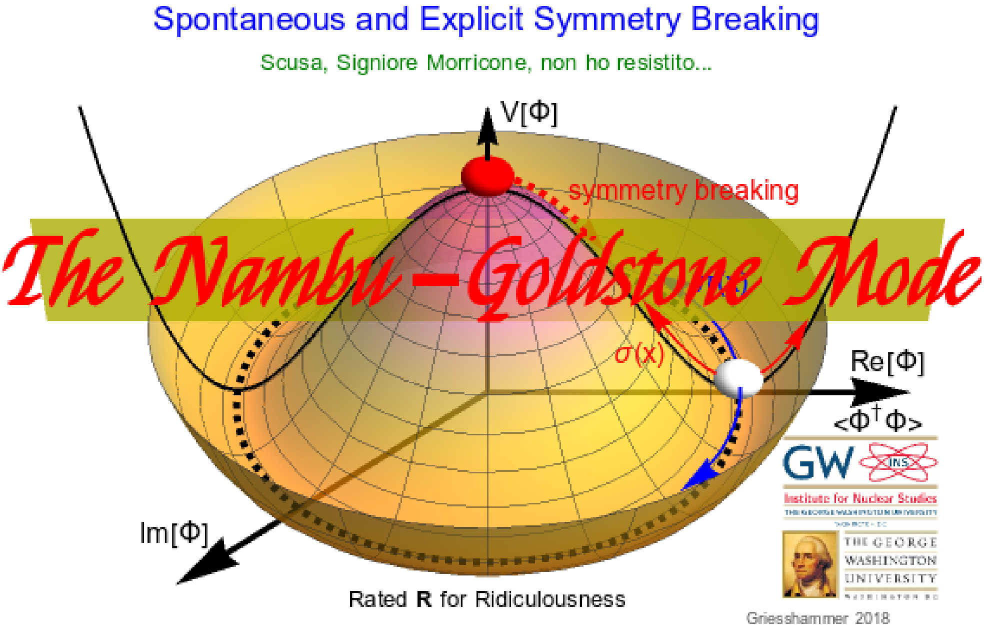
Nambu-Goldstone (Symmetry-Broken) Mode: states do *not* share symmetry of Lagrangean. Pion becomes massless: *Goldstone boson*.

Nambu-Goldstone Symmetry Breaking

[Ryder 8.1-2, EW App. 14]

Example: Global $U(1)$ Symmetry Breaking in the Nonlinear σ Model

And here as a movie with sound (click picture):



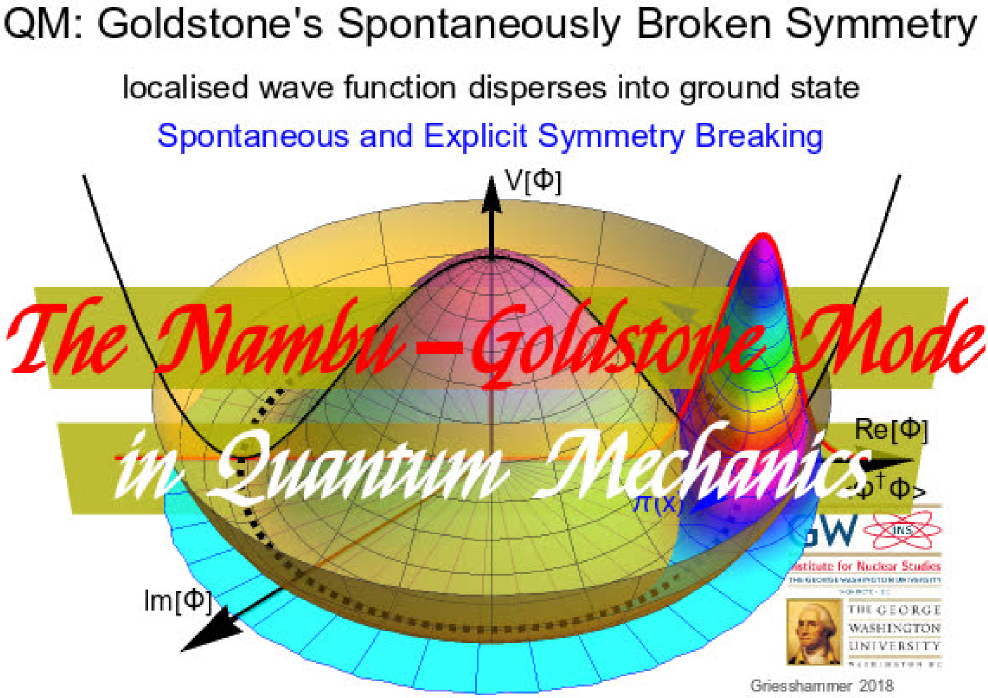
Nambu-Goldstone Symmetry Breaking

[Ryder 8.1-2, EW App. 14]

Example: Global $U(1)$ Symmetry Breaking in the Nonlinear σ Model

Quantum Mechanics and QFT: (Sloppy) Semiclassical Reasoning

Movie (click picture): collapse and dissipation of QM wave function.



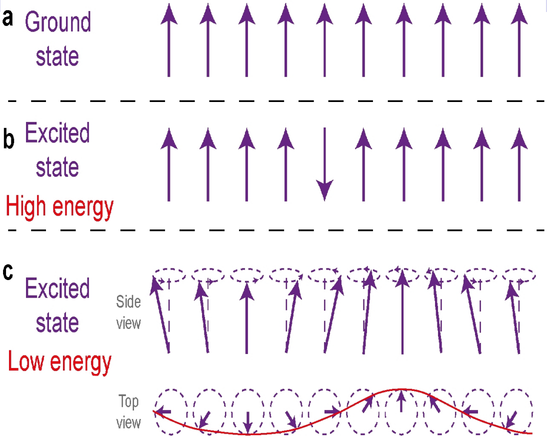
QM: Goldstone's Spontaneously Broken Symmetry

localised wave function disperses into ground state

Spontaneous and Explicit Symmetry Breaking

The Nambu – Goldstone Mode in Quantum Mechanics

From QM To QFT: Circumventing Noether



Intuitively: Excitation at $E \to 0 \implies$ wave length $\lambda \to \infty$:

$\implies \pi(x) \to$ constant: $\mathrm{e}^{\mathrm{i}\pi(x)/(\sqrt{2}a)} \to \mathrm{e}^{\mathrm{i}\alpha}$ just symmetry.

Rotate the whole ferromagnet, not just one individual spin:
Need not overcome interaction energy between neighbours.

For $\lambda \to \infty \mathrel{\widehat{=}} p \to 0$, neighbours nearly aligned

$\implies$ "friction" only at surface of spin wave

$\implies$ energy difference between excited and ground state:

$$\Delta E_{01}(\lambda \to \infty) \propto \frac{\text{surface}}{\text{volume}\,\mathcal{V}} \xrightarrow{\mathcal{V}\to\infty} 0$$

For $p \to 0$: $E^2(p) - p^2 = 0$, Goldstone Boson is "massless excitation".

Localised spin wave is superposition of eigenstates. $\implies$ Dissipates with time into symmetric form.

$$\textbf{Transition amplitude} \propto \sum_{\text{states } k} a_k(t)\,\mathrm{e}^{-\mathrm{i}E^{(k)}t} \text{ with } \textbf{dissipation timescale } \frac{1}{\Delta E_{01}} \overset{\lambda\to\infty}{\xrightarrow{\mathcal{V}\to\infty}} \infty.$$

$\implies$ Zero transition for momentum $p \propto \dfrac{1}{\lambda} \to 0$.

Goldstone Bosons Decouple At Zero Momentum.

Strictly speaking, there is no Spontaneous Continuous Symmetry Breaking for finite number of degrees of freedom (QM, finite volume, lattice). In practise, metastable when dissipation timescale $\gg$ lifetime of universe.

QFT: Nambu-Goldstone Theorem (Sloppy Version)

Nambu 1960 (Nobel 2008),
Jeffrey Goldstone 1961
cond-mat: Anderson/Bogoliubov 1958

Quantum **Mechanics: Noether's Theorem**: Symmetries of $\mathcal{L}$ must be symmetries of the ground state.

Quantum **Field Theory**: ∞ many degrees of freedom $\Longrightarrow$ not necessarily true (loophole in Noether).

Take a **global, continuous symmetry of $\mathcal{L}$** which is generated by N conserved charges.

Here: $SU_R(2) \times SU_L(2)$: $N = 6$ charges: 3 Q^a_R & 3 Q^a_L

Let the vacuum state show only $n < N$ symmetries (i.e. VEV has n of these symmetries).

Here: isospin $SU_V(2)$: $n = 3$ charges $Q^a = Q^a_R + Q^a_L$ $\Longrightarrow$ flavour symmetry

Then vacuum only annihilated by n Noether charges — Here: $Q^a|\text{vac}\rangle = 0$ with $a = 1,2,3$

and one finds

- n **massive fields**, $m_\sigma \propto$VEV — Here: $f_0(500)^{\pm,0}(I[J^P]=1[0^+])$: $m_f \gg m_\pi$
- $Q^a_A|\text{vac}\rangle \neq 0$ *not symmetries of states* — Here: $Q^a_A|\text{vac}\rangle$ creates a pion.
- and $(N-n)= 3$ **massless fields** — Here: $\pi^{\pm,0}(I[J^P] = 1[0^-])$ pseudoscalar

carry the **quantum numbers of the $(N-n)$ broken symmetries**

and **do not interact for momenta $p \to 0$**.

Symmetry Scenarios of Vacuum in QFT with Noether Charges Q^a_N

Either WIGNER-WEYL MODE: unbroken/ invariant: $e^{iQ^a_N\theta_a}|\text{vac}\rangle = |\text{vac}\rangle \Leftrightarrow Q^a_N|\text{vac}\rangle = 0$, symmetry in spectrum

Or NAMBU-GOLDSTONE MODE: **spontaneously broken**: $e^{iQ^a_N\theta_a}|\text{vac}\rangle \neq |\text{vac}\rangle \Leftrightarrow Q^a_N|\text{vac}\rangle = |\text{Goldstones}\rangle \neq 0$

generates massless fields $\pi^a(x)$, symmetry not seen in spectrum.

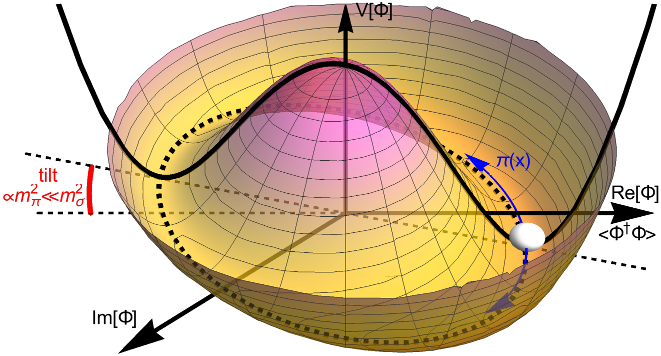
Explicit Chiral Symmetry Breaking: Tilting the Hat

$m_\pi = 140\,\text{MeV} \ll 1\,\text{GeV}$ typical QCD scale, *but not zero.*

$\Longrightarrow$ Need additional small explicit breaking of chiral symmetry.

Analogy in Landau-Ginzburg: small tilt ϵ leaves σ alone, only affects π, keeps minimum at a:

$$V_{\text{LG}} \to V_{\text{LG}} \underbrace{-\epsilon a^4\left[\mathrm{e}^{\mathrm{i}\pi/(\sqrt{2}a)} + \mathrm{e}^{-\mathrm{i}\pi/(\sqrt{2}a)}\right]}_{= -2\epsilon a^4 \cos\frac{\pi(x)}{\sqrt{2}a}} \to V_{\text{LG}} - 2\epsilon a^4 + \frac{1}{2}\underbrace{\epsilon a^2}_{= m_\pi^2} \pi^2 - \underbrace{\frac{\epsilon}{48}}_{= m_\pi^2/a^2} \pi^4 + \mathcal{O}(\pi^6)$$



$$m_\pi = \sqrt{\epsilon a^2} \approx 140\,\text{MeV}(\neq 0) \ll 1\,\text{GeV} \approx m_\sigma = \sqrt{4\lambda a^2}$$ for tilt $\epsilon \ll \lambda$

Also predicts/fixes more interactions to/from m_π.

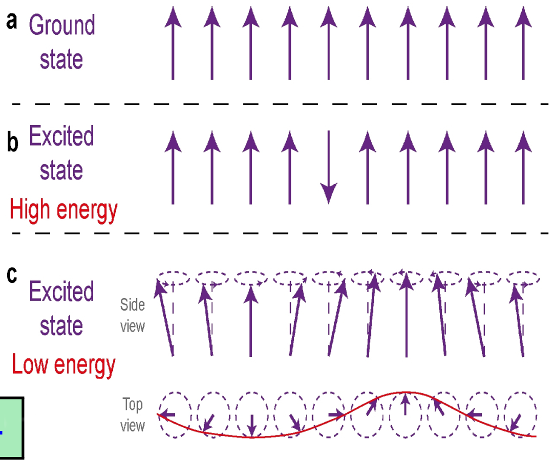
3 Important Take-Home Messages: Nambu-Goldstone (χ)SSB

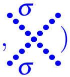
Goldstone Bosons are “massless” excitations of the vacuum/ground state: $E(p \to 0) \to 0$!

Goldstone Bosons Decouple At Zero Momentum: “No Interactions as $p \to 0$”.

Feynman rules: interactions depend on $p^2, p^4 \dots$

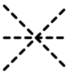
(Broken) Symmetries Relate Some (Not All) Interactions.



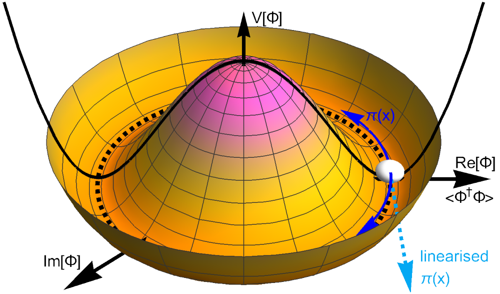
$\mathcal{L}_{\text{LG}}$: all interactions determined by $(\lambda, a) \leftrightarrow (m_\sigma,$  $)$

Interactions “bend” Goldstone Boson in angular direction.

In a moment: *self-interactions* “bend” pion to stay on **Chiral Circle**.

 $\sim \frac{(\partial_\mu \pi)^2 \pi^2}{f_\pi^2}$,  $\sim \frac{(\partial_\mu \pi)^2 \pi^4, (\partial_\mu \pi)^4 \pi^2}{f_\pi^4}$ etc.

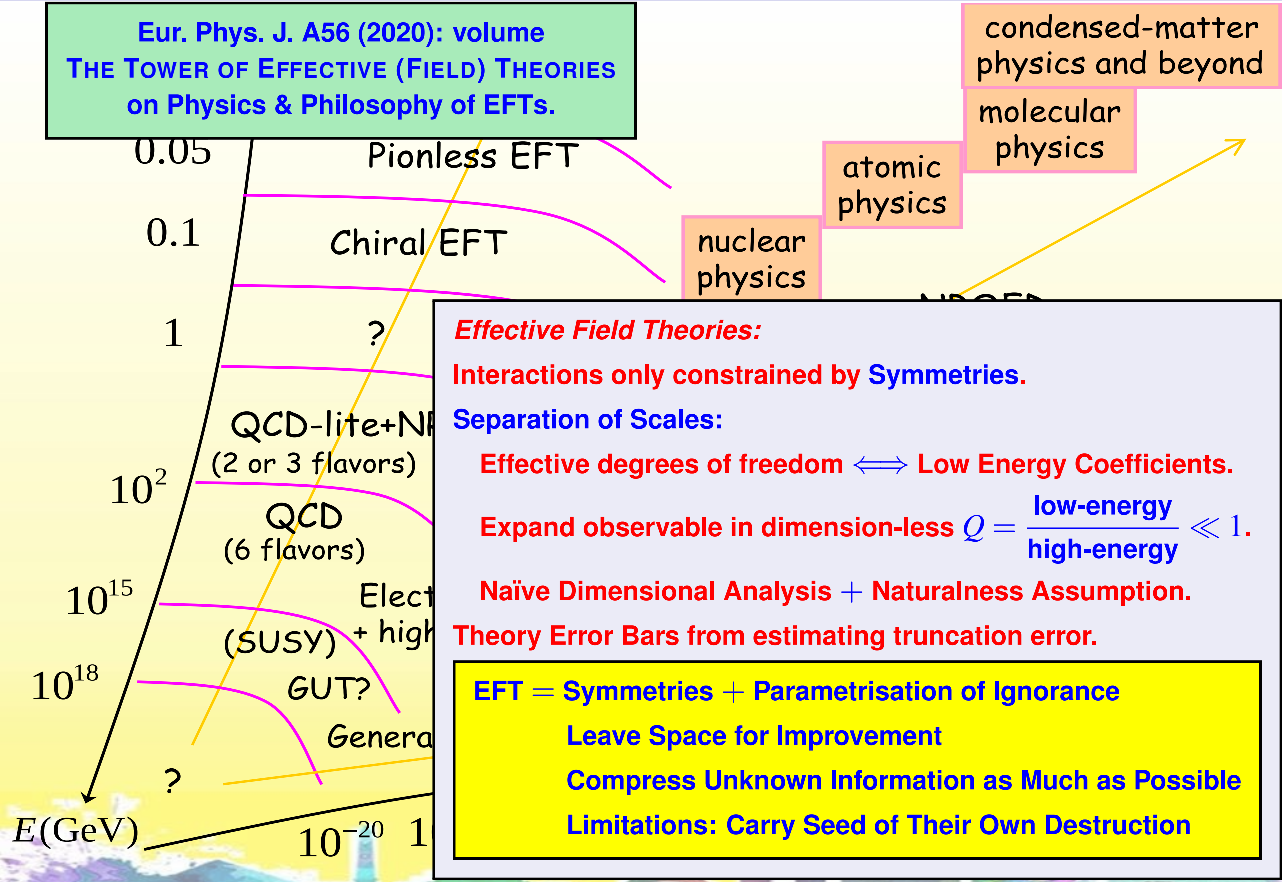
$$e^{i\frac{\pi^a(x)\tau^a}{f_\pi}} = 1 + \frac{i\pi^a(x)\tau^a}{f_\pi} - \frac{\pi^a(x)\pi^b(x)\overbrace{\tau^a\tau^b}^{= \delta^{ab} + i\underbrace{\epsilon^{abc}\tau_c}_{SU(2)\text{ curvature}}}}{2f_\pi^2} + \dots$$



EFT Take-Home Message: “Bounded In A Nutshell”

Hamlet, II.ii
background: U. van Kolck

Eur. Phys. J. A56 (2020): volume
THE TOWER OF EFFECTIVE (FIELD) THEORIES
on Physics & Philosophy of EFTs.



Effective Field Theories:

Interactions only constrained by Symmetries.

Separation of Scales:

- **Effective degrees of freedom $\Longleftrightarrow$ Low Energy Coefficients.**
- **Expand observable in dimension-less $Q = \dfrac{\text{low-energy}}{\text{high-energy}} \ll 1$.**
- **Naïve Dimensional Analysis $+$ Naturalness Assumption.**

Theory Error Bars from estimating truncation error.

EFT $=$ Symmetries $+$ Parametrisation of Ignorance

Leave Space for Improvement

Compress Unknown Information as Much as Possible

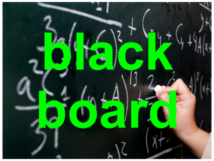
Limitations: Carry Seed of Their Own Destruction

(e) Chiral Perturbation Theory: Mesons in χEFT

Weinberg 1979,
Gasser/Leutwyler,
...

The Most Important QCD Symmetry: Chiral Symmetry

[PDG 2014]



χ**EFT relates quark parameters ($m_q, \ldots$) and pion/low-energy parameters ($m_\pi, f_\pi, \ldots$).**

Prior: "Current Algebra": hard, unknown corrections. **[Gell-Mann,... 1964-]**

Now: *Chiral Low Energy Theorems*: simpler, systematic corrections.

[Weinberg, Gasser/Leutwyler/... 1979-]

Pion Decay: Mass, Fixing Parameter $f_\pi = [92.21 \pm 0.15]\mathrm{MeV}$ [PDG 2014] more: script

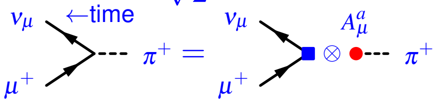
LO χEFT: $\mathcal{L}_\pi = \frac{f_\pi^2}{4}\mathrm{tr}[(\partial_\mu U^\dagger)(\partial^\mu U)]$ separately invariant under $U \to LUR$, $L, R \in SU(2)$.

$\Longrightarrow$ Noether: axial current conserved (indeed axial: $P\pi^a = -\pi^a$)

$$A_\mu^a(x) = \dots = -\mathrm{i}\frac{f_\pi^2}{2}\mathrm{tr}[(\partial_\mu U^\dagger)\{\frac{\tau^a}{2}, U\}] = \dots = -f_\pi(\partial_\mu \pi^a) + \dots \to \mathrm{i}f_\pi\, q_\mu \pi^a(q) \text{ odd in } \pi^a$$

Spontaneous Symmetry Breaking: Currents/operators still conserved – vacuum/states break symmetry!

$\pi^+ \to \mu^+ \nu_\mu$ in $\tau = 26.033\mathrm{ns}$: Let $\pi^+ = \frac{\pi^1 - \mathrm{i}\pi^2}{\sqrt{2}}$ decay by coupling pion current A_μ^a to leptons:



$$\langle \mu^+ \nu_\mu | \text{create lepton} \otimes \text{annihilate pion} | \pi^+ \rangle = \text{lepton} \otimes \langle \text{no hadron} | \mathrm{i}f_\pi q_\mu \pi^a | \pi^+(q) \rangle = \mathrm{i}f_\pi \sqrt{2} q_\mu$$

$\Longrightarrow \frac{1}{\tau} \propto |\mathcal{M}(\pi^+ \to \mu^+ \nu_\mu)|^2 \propto f_\pi^2\, q^2 = f_\pi^2 m_\pi^2$ (pion on-shell).

$\Longrightarrow$ **Pion Decay Constant** f_π parametrises pion decay! (Duh!)

No Decay for $m_\pi = 0$: Goldstone boson decouples✓; $\partial^\mu A_\mu^a \to q_\mu f_\pi q^\mu \pi^a = q^2 f_\pi \pi^a = 0$ conserved✓.

$\Longrightarrow$ **Real World:** Measure pion decay to find $f_\pi = \begin{cases} [92.21 \pm 0.15]\mathrm{MeV} & \text{[PDG 2014]} \\ [92.32 \pm 0.30]\mathrm{MeV} & \text{[PDG 2022, eq. (72.23)]} \end{cases}$

Some, including PDG, use *same symbol* but mean $\sqrt{2} f_\pi \approx 130\mathrm{MeV}$.

Explicit Symmetry Breaking, Chiral Condensate as Order Parameter

QCD: Quark-mass term breaks axial symmetry: $\mathcal{L}_{m\text{QCD}} = -m_q \bar{q} q = -\left(q_R^\dagger, q_L^\dagger\right) \begin{pmatrix} 0 & m_q \\ m_q & 0 \end{pmatrix} \begin{pmatrix} q_R \\ q_L \end{pmatrix}$

measures degree of chirality mixture; symmetric under T, C, P, isospin $SU_V(2)$ – *not* under $SU_A(2)$.

$\Longrightarrow$ **Order parameter of χSSB** is the **condensate of $\bar{q}q$ pairs in vacuum**: vacuum is not empty!

$$\langle \bar{q} q \rangle := \langle \text{vac} | [\bar{q}_L q_R + \bar{q}_R q_L] | \text{vac} \rangle = \langle [\bar{u} u + \bar{d} d] \rangle = 2 \, \langle \bar{u} u \rangle \neq 0$$

Landau-Ginzburg order parameter: VEV $\langle \text{vac} | \Phi^\dagger \Phi | \text{vac} \rangle = a^2 \neq 0$ parametrises degree of SSB

$\mathcal{L}_{\text{exLG}} = -\epsilon a^4 [\mathrm{e}^{\mathrm{i}\pi/(\sqrt{2}a)} + \mathrm{e}^{-\mathrm{i}\pi/(\sqrt{2}a)}]$ breaks symmetry $\Phi \to \mathrm{e}^{\mathrm{i}\alpha} \, \Phi$ explicitly by tilt

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χEFT: $\mathcal{L}_{\text{exSB}} = B\,\dfrac{f_\pi^2 m_q}{2}\,\text{tr}[U(x) + U^\dagger(x)]$

not invariant under $SU_R(2) \times SU_L(2)$: $U \to RUL^\dagger$

but under $SU_V(2)$ ($V = L = R$): $U \to VUV^\dagger$

$$= 2\,B\,f_\pi^2\,m_q - \underbrace{B\,m_q}_{\text{pion mass: } = m_\pi^2/2 \neq 0}\,\pi^a(x)\pi_a(x) + (\pi^4\dots)$$

sets again some higher-order interactions

even # of pions: parity-even ✓

$\text{tr}[U(x) - U^\dagger(x)]$ would be parity-odd ×

Equate VEVs: $\langle\mathcal{L}_{\text{exSB}}\rangle = \langle 2\,B\,f_\pi^2\,m_q + \mathcal{O}(Q^2)\rangle \overset{!}{=} -m_q\langle\bar{q}q\rangle = \langle\mathcal{L}_{m\text{QCD}}\rangle \Longrightarrow B = -\dfrac{\langle\bar{q}q\rangle}{2f_\pi^2} > 0$

$$\Longrightarrow m_\pi^2 = -\frac{m_q}{f_\pi^2}\,\langle\bar{q}q\rangle + \dots$$ **Gell-Mann–Oakes–Renner relation**

$\Longrightarrow m_\pi \neq 0$ by small quark mass $+$ vacuum condensate of $\bar{q}q$ pairs (cf. superconductor: Cooper pairs)

Explains "quadratic mass formula" for light meson octet. →HW

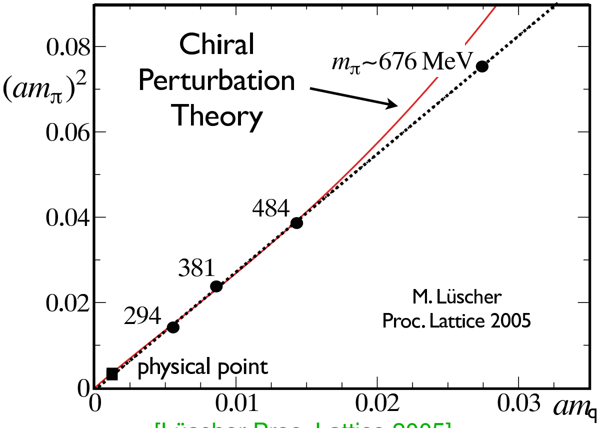
Chiral Condensate and Gell-Mann–Oakes–Renner in Lattice QCD

m_π **from quark condensate (χSB order param.):** $m_\pi^2 = -\frac{m_q}{f_\pi^2}\,\langle\bar{q}q\rangle\; + \;\mathcal{O}(m_q^2 \to \frac{m_\pi^2}{\sim 1\mathrm{GeV}^2} \sim 3\%)$.

isospin symmetry of $q = \binom{u}{d}$: $\langle\bar{q}q\rangle = 2\langle\bar{u}u\rangle = 2\langle\bar{d}d\rangle$

phenomenology: $\langle\bar{u}u\rangle \approx -(250\ \mathrm{MeV})^3 \approx -2\mathrm{fm}^{-3} \gg \rho_{\text{nucl. matter}} = 0.17\ \mathrm{fm}^{-3} = (100\ \mathrm{MeV})^3$

lattice QCD: $\langle\bar{u}u\rangle = -([251 \pm 7 \pm 11]\ \mathrm{MeV})^3$ [JLQCD: Phys. Rev. Lett. **98** (2007) 172001]



[Lüscher Proc. Lattice 2005]

Predicted m_π-dependence consistent with lattice QCD. $\Longrightarrow$ ***Confirms χEFT, chiral symmetry.***

χEFT ok up to $m_\pi \lesssim 600\ \mathrm{MeV}$, in line with breakdown scale $\Lambda_{\chi\text{EFT}} \sim m_\rho \sim 1\ \mathrm{GeV}$.

LO Lagrangean and $\pi\pi$ S-Wave Scattering Lengths

$$\mathcal{L}^{\text{LO}}_{\chi\text{EFT}} = \frac{f_\pi^2}{4}\text{tr}[(\partial_\mu U)^\dagger(\partial^\mu U)] + \frac{m_\pi^2 f_\pi^2}{4}\text{tr}[U+U^\dagger]$$

$$\xrightarrow{U=e^{\frac{i\pi^a\tau_a}{f_\pi}}} \text{free} + \left[\frac{(\partial\pi)^2\pi^2}{f_\pi^2}, \frac{m_\pi^2\pi^4}{f_\pi^2}\right]\text{-terms}$$

Parameter-free prediction for LO $\pi\pi$ scattering lengths $[m_\pi^{-1}]$:

$$a_{I=0} = \frac{7\pi}{2}\left(\frac{m_\pi}{4\pi f_\pi}\right)^2 = +0.16$$

$$a_{I=2} = -\pi\left(\frac{m_\pi}{4\pi f_\pi}\right)^2 = -0.05$$

unnaturally *small* because

vanish in chiral limit $m_\pi \to 0$:

Goldstone bosons decouple. ✓

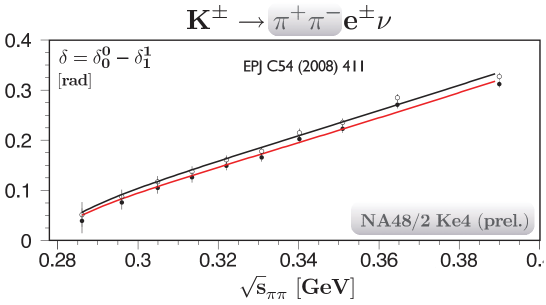
Now calculated to 2-loop order,

i.e. $\mathcal{O}\left(Q = \frac{m_\pi}{4\pi f_\pi}\right)^6$ [Bijnens].

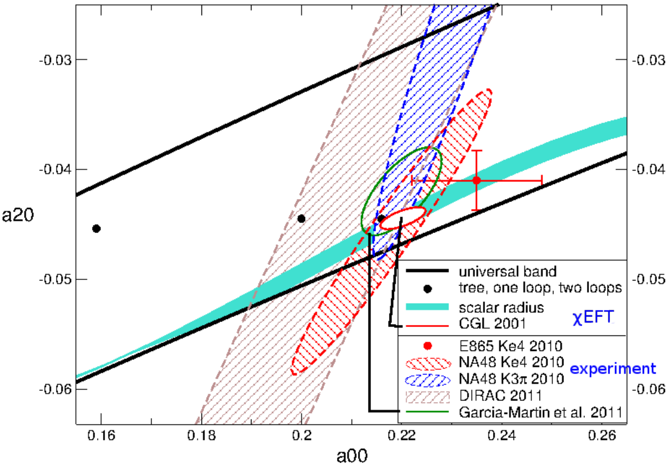
Confirms chiral symmetry.

Theory: Chiral Symmetry + Roy Equations

G. Colangelo et al. Nucl. Phys. B 603 (2001) 125



	Theory (ChPT)	Exp (NA48/2)
a_0	0.220 ± 0.005	0.218 ± 0.013
a_2	-0.044 ± 0.001	-0.0457 ± 0.0084



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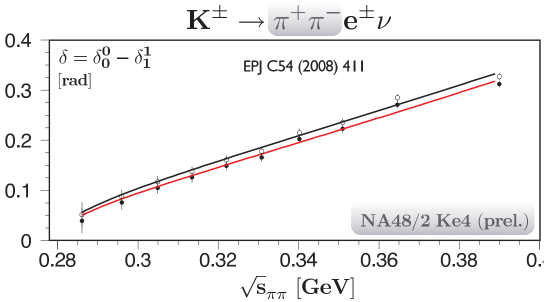

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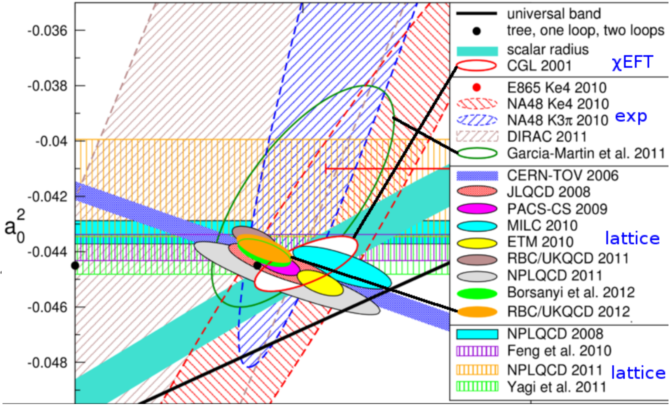


(f) $1N$-χEFT: (Heavy) Baryon χPT

Gasser/Sainio 1988,
Jenkins/Manohar 1994,
Bernard/Kaiser/Meißner 1995,. . .

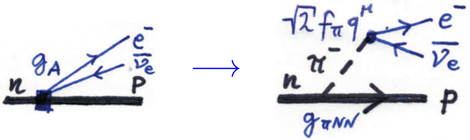
The Goldberger-Treiman Relation

[details in notes]

Decay $n \to p\, e^- \, \overline{\nu}_e$ dominated by axial contribution $g_A\, \bar{u}_p\, \gamma^\mu \gamma_5\, u_n$ with $I[J^P] = 1[0^-]$ as pion.

Idea: microscopically saturated

by decay of virtual pion in cloud.



Other particles: separation of scales $\Longrightarrow$ not resolved: suppressed in Q and/or absorbed in LECs.

Rigorous calculation in LO χEFT relates πN coupling $g_{\pi NN} = g_A \dfrac{M_N}{f_\pi} + \text{corrections.}$

Data:

$g_{\pi NN} = 13.21^{+0.11}_{-0.05}$ pion-photoproduction $\gamma N \to \pi N$ [SS 4.3]

$g_A = 1.2695(29)$ axial coupling in neutron decay $n \to p e^- \bar{\nu}_e$ [SS 4.3]

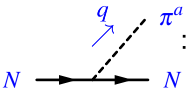
$f_\pi = 92.21(15)$ MeV pion decay $\pi^+ \to \mu^- \nu_\mu$ [PDG 2014]

$\Longrightarrow$ **Goldberger-Treiman Discrepancy $\Delta_{GT} = 1 - \dfrac{g_A\, M_N}{g_{\pi NN}\, f_\pi} = [2.15^{+0.89}_{-0.51}]\%$ indeed tiny.**

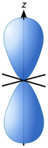
Another consequence of chiral symmetry and its breaking!

Chirally Symmetric Interactions in the πN System

$\Longrightarrow$ **Chiral symmetry derives prefactor of isospin-symmetric πN interaction stated in [ResReg]:**



$: -\frac{g_A}{2f_\pi}\not{q}\gamma_5\,\tau^a \;\to\; -\frac{g_A}{2f_\pi}\,\vec{\sigma}\cdot\vec{q}\tau^a$ for particle in Dirac basis of γ^μ.



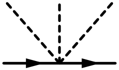
Needed in HW!

$\xrightarrow{q^\mu\to 0} 0$: decouples by χSSB ✓; P-wave interaction

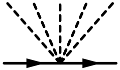
χ**symmetry**: $U = \mathrm{e}^{\frac{\mathrm{i}\pi^a\tau_a}{f_\pi}} = 1 + \frac{\mathrm{i}\pi^a(x)\tau^a}{f_\pi} - \frac{\pi^a(x)\pi^b(x)\overbrace{\tau^a\tau^b}^{=\delta^{ab}+\mathrm{i}\epsilon^{abc}\tau_c}}{2f_\pi^2} + \dots$

$\Longrightarrow$ $SU_A(2)$ **curvature ϵ^{abc} of SSB potential** prescribes more interactions with odd # of π^a, cf. .

qualitatively:



$\sim \frac{g_A}{2f_\pi}\not{q}_{123}\epsilon^{abc}\,\gamma_5$



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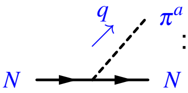

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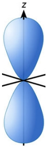

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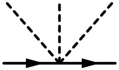
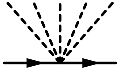
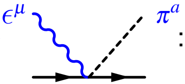
qualitatively:  $\sim \frac{g_A}{2f_\pi}\not{q}_{123}\epsilon^{abc}\,\gamma_5$  $\sim \frac{g_A}{2f_\pi}\not{q}_{12345}\epsilon^{abc\dots}\,\gamma_5$

Gauge $q_\mu \to q_\mu + eZ_\pi A_\mu$ $\Longrightarrow$ 

: $-\mathrm{i}\,Z_\pi e\,\frac{g_A}{2f_\pi}\,\underbrace{\epsilon^\mu\gamma_\mu\,\gamma_5}_{\mathrm{nonrel.}:\,\vec{\epsilon}\cdot\vec{\sigma}}$

“Kroll-Rudermann term”

$\Longrightarrow$ **χsym. quantitatively predicts charged-pion photoproduction.** Used to determine $g_{\pi NN}$.

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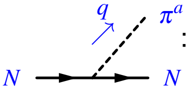

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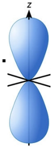

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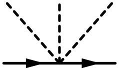

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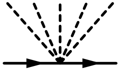

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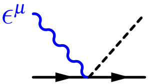

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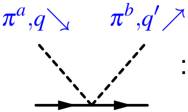
"Kroll-Rudermann term"

$\Longrightarrow$ **χsym. quantitatively predicts charged-pion photoproduction.** Used to determine $g_{\pi NN}$.

First $N\pi\pi$ interaction comes from **curvature of SSB potential**, cf. :



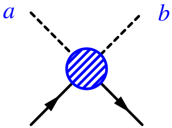
: $\frac{1}{4f_\pi^2}\left(\not{q}+\not{q}'\right)\epsilon^{abc}\,\tau_c$ charge-transfer, **no** g_A!

"Weinberg-Tomozawa term"

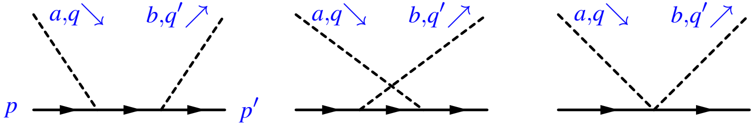
decouples by χSSB for $q\to 0$ ✓

Sketch of the HW Problem: πN Scattering Length

[more details in notes]
[SS 4.3]



LO χEFT amplitudes:



Isospinology in [ResReg: II.3.f]: $N\otimes\pi^a\ :\ \frac{1}{2}\otimes 1=\frac{1}{2}\oplus\frac{3}{2}\Longrightarrow 2$ amplitudes $\mathcal{M}_{2I+1=4}$, $\mathcal{M}_2$.

- Diagrams$=\mathrm{i}\times$amplitude $=\mathrm{i}T^{ab}=\mathrm{i}[\underbrace{\delta^{ab}\,T^+}_{\text{symmetric}}\underbrace{-\mathrm{i}\epsilon^{abc}\tau_c\,T^-}_{\text{antisymmetric}}]$.

- Most interested in prediction involving SSB curvature: **Weinberg-Tomozawa** $\frac{1}{4f_\pi^2}\,(\not q+\not q')\,\epsilon^{abc}\,\tau_c$.

$\Longrightarrow$ Consider **charge-transfer or charged pion scattering**; **track WT term** (the one without g_A!).

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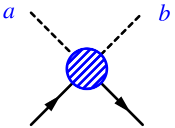

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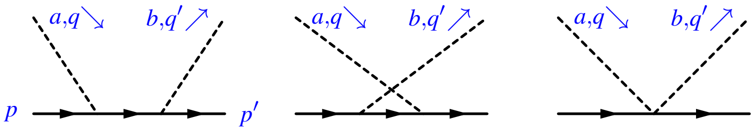

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- For cross sections, sum all 3 amplitudes, square total: QM interference matters! (not in HW)

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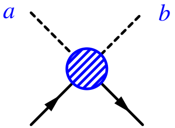

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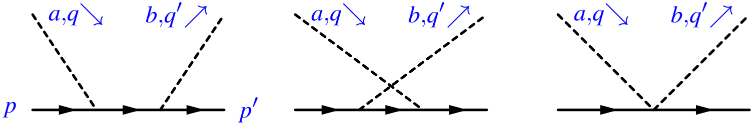

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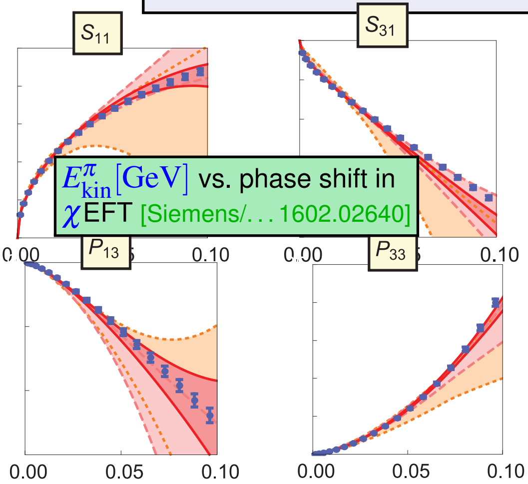
- Use crossing symmetry $q\longleftrightarrow -q'$ for "crossed" diagram: interaction sequence $a\to b$ vs. $b\to a$.
- For cross sections, sum all 3 amplitudes, square total: QM interference matters! (not in HW)
- ***Scattering length* is easier (HW)**: Zero-momentum scattering $q=q'=(m_\pi,\vec{0})\ ,\ p=p'=(M_N,\vec{0})$.
- Use norm $\bar{u}u=2M$ and without proof at $q=q'=(m_\pi,\vec{0})$: $\bar{u}\not{q}u=2M\,m_\pi$.
- Use without proof: scatt. lengths $a^\pm=\frac{1}{8\pi\sqrt{s}}\,T^\pm$ since $\sigma(\vec{q}=0)=4\pi\,a^2=\int\mathrm{d}\Omega\,\frac{|T|^2}{64\pi^2 s}$
- **Chiral limit, compare to experiment, error-assessment.**

πN Scattering: χEFT and Data

[SS 4.3]

	a^+ $[10^{-4}\mathrm{MeV}^{-1}]$	a^- $[10^{-4}\mathrm{MeV}^{-1}]$
χEFT **without WT** (i.e. not really “LO”)	-0.680	$+5.06$
LO χEFT (really, with WT) parameter-free	$-0.680 \pm \dots$	$+5.71 \pm \dots$
PWA-I [Koch 1986]	-0.7 ± 0.1	$+6.6 \pm 0.1$
PWA-II [Matsinos 1997]	$+0.20 \pm 0.12$	$+5.8 \pm 0.1$
pionic hydrogen [Schröder 2001]	-0.27 ± 0.36	$+6.59 \pm 0.30$
N^{2}LO χEFT (data extraction)[Siemens/...1602.02640]	$+0.2 \pm 0.1$	$+5.91 \pm 0.04$

χEFT with Weinberg-Tomozawa favoured. $\Longrightarrow$ Curvature of χSSB potential!

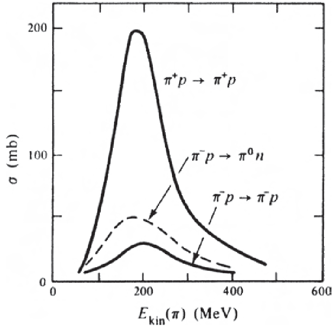


E_{kin}^{π} $[\mathrm{GeV}]$ vs. phase shift in χEFT [Siemens/...1602.02640]

Partial wave analysis tricky: $\Delta(1232)$!

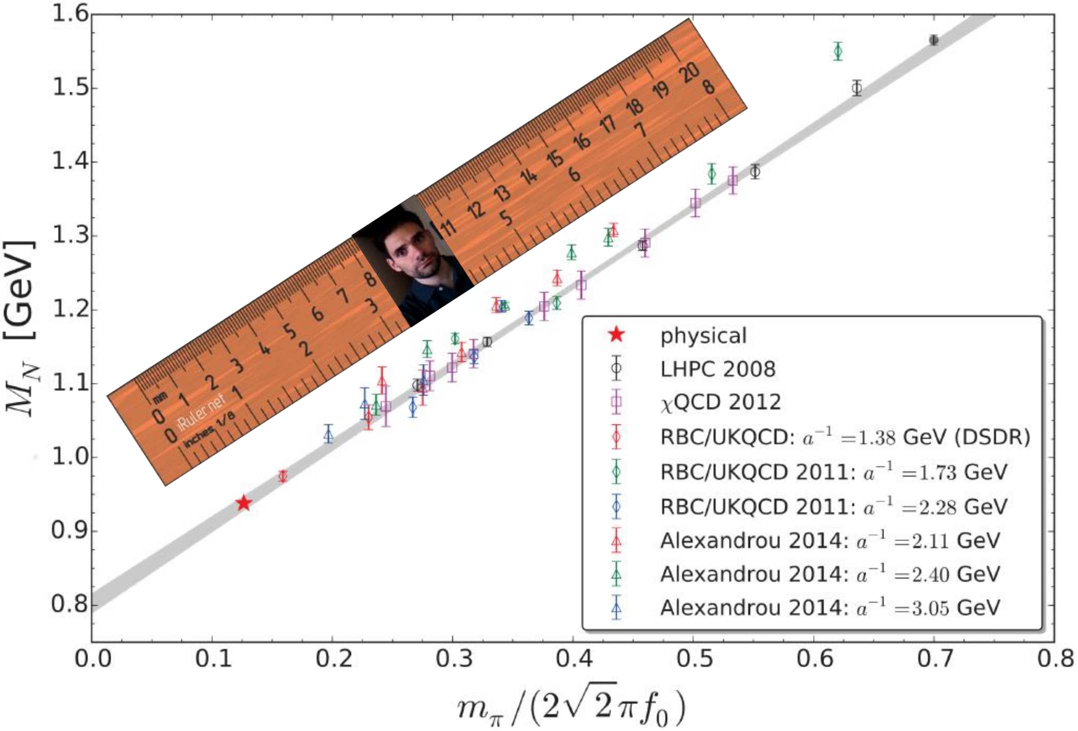
$\Longrightarrow$ Substantial uncertainties.

Still, higher-order χEFT agreement ok.



When χEFT Does Not Work: "Ruler Plots"

0810.0663; name: B. Tiburzi
populariser: A. Walker-Loud
this version after Bernard 1510.02180

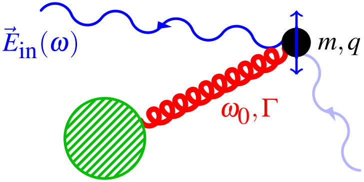


χEFT: $M_N(m_\pi)-M_N(m_\pi=0)\propto m_q\propto m_\pi^2$.

Lattice at $m_\pi\gtrsim 200\text{MeV}$: $M_N=800.0\text{MeV}+1.0m_\pi$! **WHY??**

Polarisabilities: Stiffness of Charged Constituents in El.-Mag. Fields

Example: induced electric dipole radiation off harmonically bound charge, damping Γ [Lorentz/Drude 1900/1905]



$$\vec{d}_{\text{ind}}(\omega) = \underbrace{\frac{q^2}{m}\,\frac{1}{\omega_0^2-\omega^2-\mathrm{i}\Gamma\omega}}_{=:\,4\pi\,\alpha_{E1}(\omega)} \vec{E}_{\text{in}}(\omega)$$

$=: 4\pi\,\alpha_{E1}(\omega)$ "displaced volume" $[10^{-4}\ \text{fm}^3]$

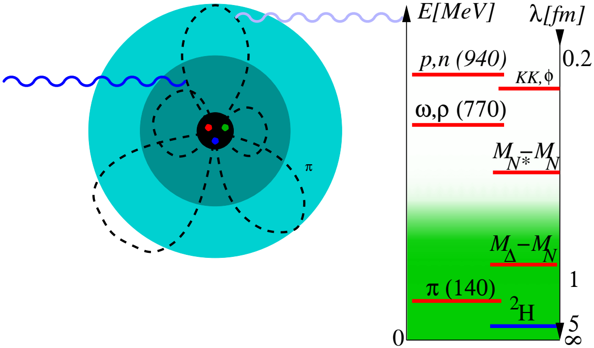
electric scalar dipole polarisability

$$\mathcal{L}_{\text{pol}} = 2\pi\left[\alpha_{E1}\,\vec{E}^2 + \beta_{M1}\,\vec{B}^2 + \dots\right]$$ **el./mag. scalar polarisabilities** (cf. Rayleigh scattering)

Dis-entangle ***interaction scales, symmetries & mechanisms*** with & among constituents.

$\Longrightarrow$ **Clean, perturbative probe of χiral symmetry of pion-cloud & its breaking.**

Fundamental hadron properties, like charge, mass, mag. moment, $\langle r_N^2\rangle\dots$ [PDG]



$\Delta(1232)$ is low-energy excitation.

$\Longrightarrow$ **Use numerical fact to expand in**

$$\delta = \frac{p_{\text{typ}}}{\Lambda_\chi} \to \frac{M_\Delta - M_N}{\Lambda_\chi \approx 1\,\text{GeV}} \approx \sqrt{\frac{m_\pi}{\Lambda_\chi}} \approx 0.4 \ll 1(?)$$

[Pascalutsa/Phillips 2002]

All 1N Contributions to N^{4}LO

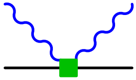
Bernard/Kaiser/Meißner 1992-4, Butler/Savage/Springer 1992-3, Hemmert/… 1998
McGovern 2001, hg/Hemmert/Hildebrandt/Pasquini 2003
McGovern/Phillips/hg 2013

Unified Amplitude: gauge & RG invariant set of all contributions which are

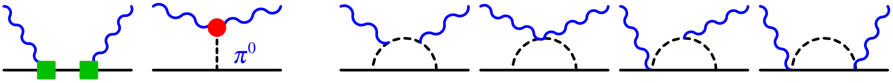
in low régime $\omega \lesssim m_\pi$ at least N^{4}LO $(e^2\delta^4)$: accuracy $\delta^5 \lesssim 2\%$;

or in high régime $\omega \sim M_\Delta - M_N$ at least NLO $(e^2\delta^0)$: accuracy $\delta^2 \lesssim 20\%$.

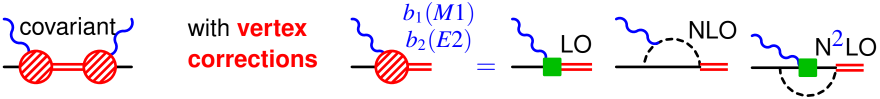
$\omega \lesssim m_\pi$ | $\sim M_\Delta - M_N \approx 300\text{MeV}$



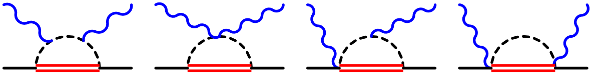
$e^2\delta^0$ ***LO*** | $e^2\delta^0$ ↘NLO



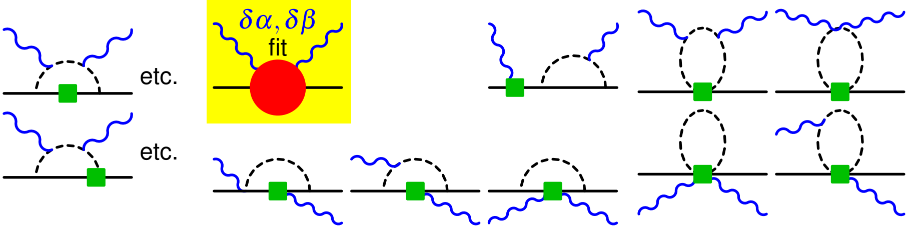
$e^2\delta^2$ N^{2}LO | $e^2\delta^1$ N^{2}LO



$e^2\delta^3$ N^{3}LO | $e^2\delta^{-1}$ ↗***LO***



$e^2\delta^3$ N^{3}LO | $e^2\delta^1$ N^{2}LO



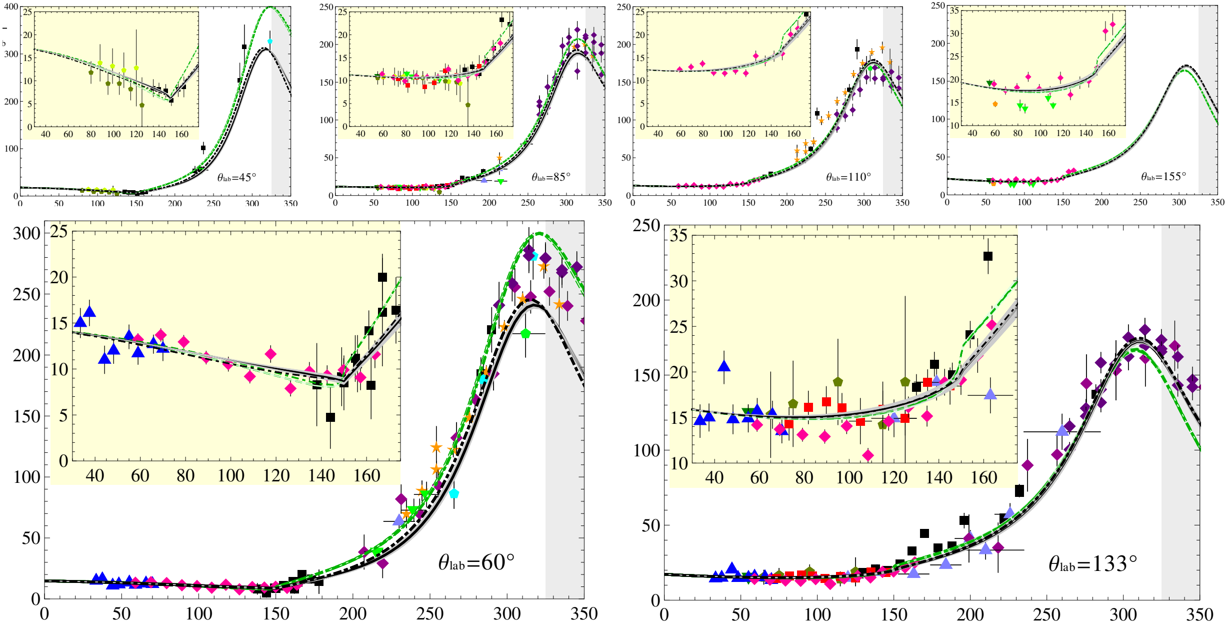
$e^2\delta^4$ N^{4}LO | $e^2\delta^2$ N^{3}LO

Unknowns: short-distance $\delta\alpha, \delta\beta \Longleftrightarrow$ Fit static α_{E1}, β_{M1} (offset). $\Longrightarrow$ *Predict* ω-dependence.

Nucleon Polarisabilities from Consistent Database

McGovern/Phillips/hg 2013

database: +Feldman PPNP 2012

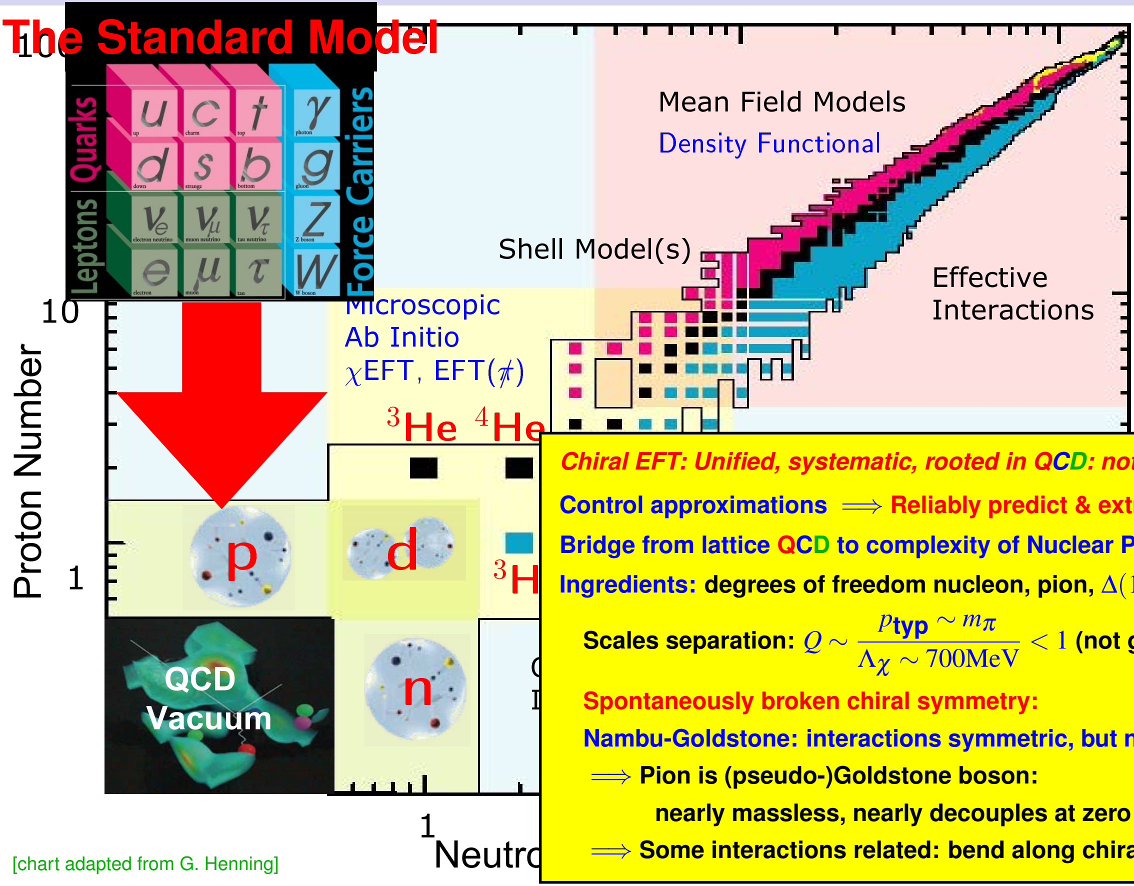


$\omega \not\ll m_\pi$: more than "static$+$slope"! $\Longrightarrow$ Understand *dynamics* to extrapolate from data to $\omega = 0$.

$\Delta(1232)$ clearly needed as effective degree of freedom: bump at $\omega \sim M_\Delta - M_{\rm N}$.

$\Longrightarrow$ **Compress rich dynamics into few numbers.**

(g) Error-Bars for Nuclear Physics!



[chart adapted from G. Henning]

Chiral EFT: Unified, systematic, rooted in QCD: not a model.

Control approximations $\Longrightarrow$ Reliably predict & extract.

Bridge from lattice QCD to complexity of Nuclear Physics.

Ingredients: degrees of freedom nucleon, pion, $\Delta(1232)$

Scales separation: $Q \sim \dfrac{p_{\text{typ}} \sim m_\pi}{\Lambda_\chi \sim 700\text{MeV}} < 1$ **(not great)**

Spontaneously broken chiral symmetry:

Nambu-Goldstone: interactions symmetric, but not states.

$\Longrightarrow$ **Pion is (pseudo-)Goldstone boson:**

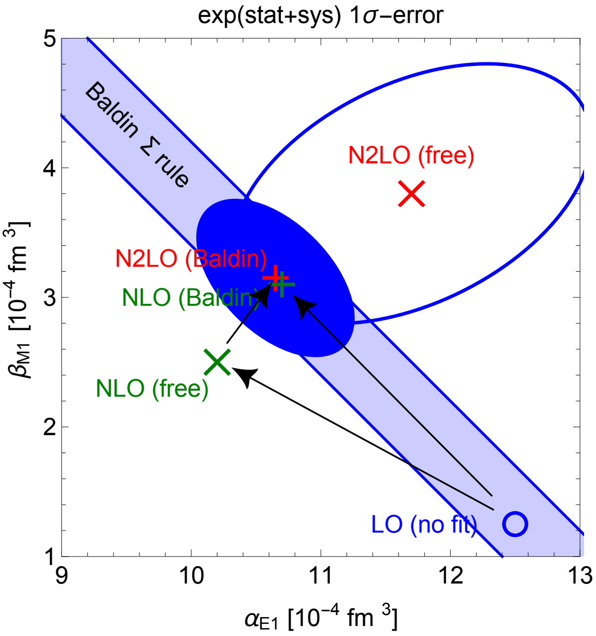
nearly massless, nearly decouples at zero energy.

$\Longrightarrow$ **Some interactions related: bend along chiral valley.**

Fit Discussion: Parameters and Uncertainties

McGovern/Phillips/hg 2013

Fit to LECs $\delta\alpha_{E1},\delta\beta_{M1} \widehat{=}$	$\alpha^p_{E1}(\omega=0)\ [10^{-4}\mathrm{fm}^3]$	$\beta^p_{M1}(\omega=0)\ [10^{-4}\mathrm{fm}^3]$	χ^2/d.o.f.
LO parameter-free [Bernard/Kaiser/Meißner 1992-4]	12.5	1.25	no fit
N^{2}LO Baldin constrained $\alpha^p_{E1}+\beta^p_{M1}=13.8\pm0.4$	$10.65\pm0.35_{\text{stat}}\pm0.2_{\Sigma}\pm0.3_{\text{theory}}$	$3.15\mp0.35_{\text{stat}}\pm0.2_{\Sigma}\mp0.3_{\text{theory}}$	$\frac{113.2}{135}$



Easy: Statistical (Experimental) Error

Traditional Tests of Fit Stability:

floating norms within exp. sys. errors;

vary dataset, parameter b_1, vertex dressing,. . .

Check consistency with Baldin Σ Rule

$$\alpha_{E1}+\beta_{M1}=\frac{1}{2\pi^2}\int\limits_{\nu_0}^{\infty}\mathrm{d}\nu\;\frac{\sigma(\gamma p\to X)}{\nu^2}$$

$$=13.8\pm0.4$$ [Olmos de Leon 2001]

Harder: χEFT Truncation Error

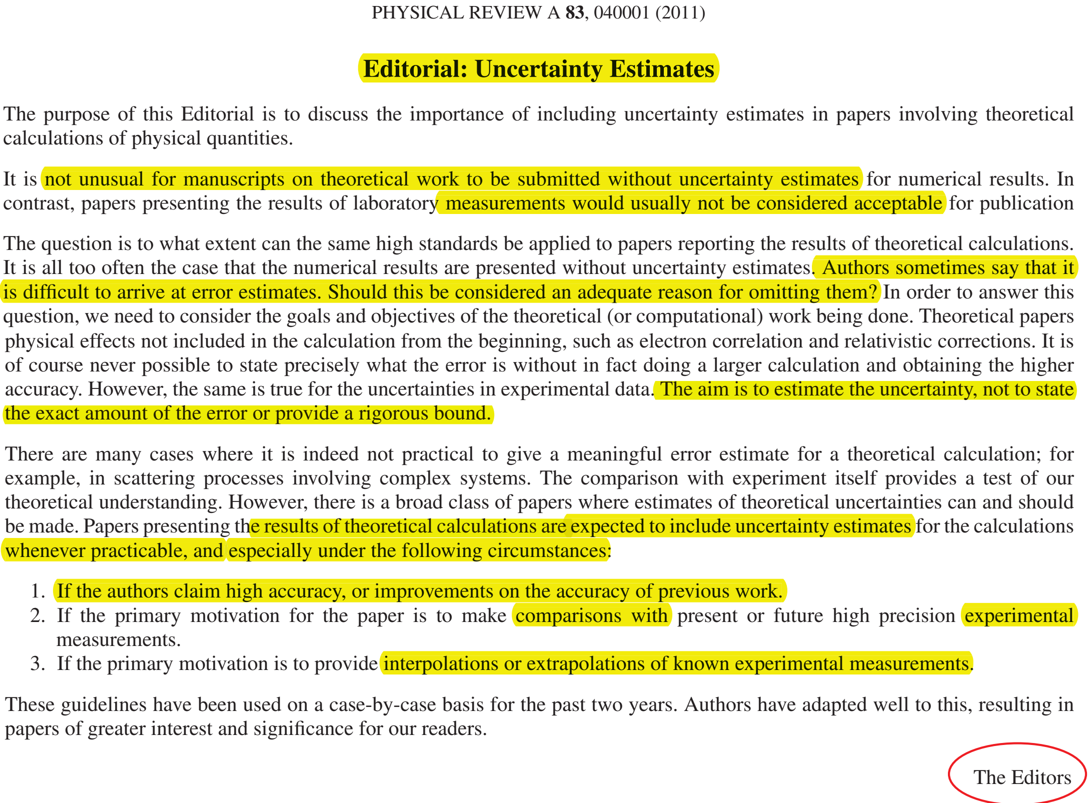
***A-priori* Assessment: $\delta^3\sim7\%$ of 10?/of 3?**

***A-posteriori* Assessment:**

Corrections smaller at higher orders,
but what does that mean?

(Dis)Agreement Significant Only When All Error Sources Explored

Editorial PRA 83
(2011) 040001



PHYSICAL REVIEW A **83**, 040001 (2011)

Editorial: Uncertainty Estimates

The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities.

It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication

The question is to what extent can the same high standards be applied to papers reporting the results of theoretical calculations. It is all too often the case that the numerical results are presented without uncertainty estimates. Authors sometimes say that it is difficult to arrive at error estimates. Should this be considered an adequate reason for omitting them? In order to answer this question, we need to consider the goals and objectives of the theoretical (or computational) work being done. Theoretical papers physical effects not included in the calculation from the beginning, such as electron correlation and relativistic corrections. It is of course never possible to state precisely what the error is without in fact doing a larger calculation and obtaining the higher accuracy. However, the same is true for the uncertainties in experimental data. The aim is to estimate the uncertainty, not to state the exact amount of the error or provide a rigorous bound.

There are many cases where it is indeed not practical to give a meaningful error estimate for a theoretical calculation; for example, in scattering processes involving complex systems. The comparison with experiment itself provides a test of our theoretical understanding. However, there is a broad class of papers where estimates of theoretical uncertainties can and should be made. Papers presenting the results of theoretical calculations are expected to include uncertainty estimates for the calculations whenever practicable, and especially under the following circumstances:

1. If the authors claim high accuracy, or improvements on the accuracy of previous work.
2. If the primary motivation for the paper is to make comparisons with present or future high precision experimental measurements.
3. If the primary motivation is to provide interpolations or extrapolations of known experimental measurements.

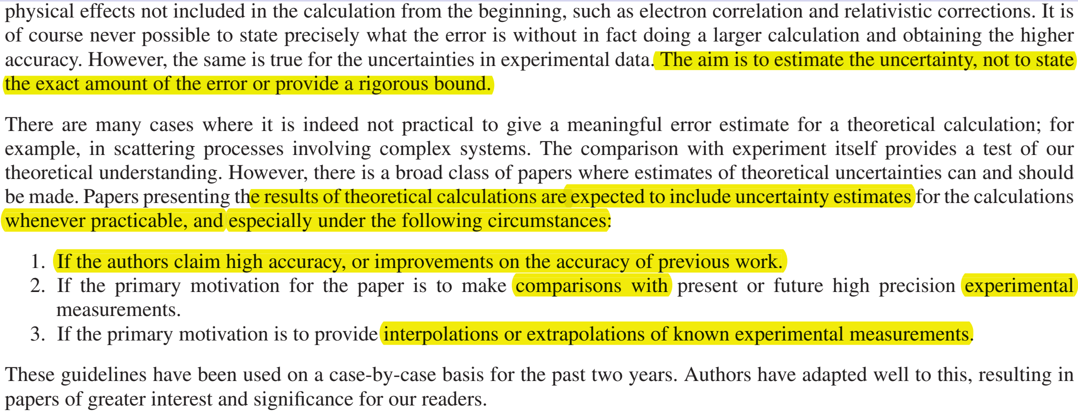
These guidelines have been used on a case-by-case basis for the past two years. Authors have adapted well to this, resulting in papers of greater interest and significance for our readers.

The Editors

Non-Theory Errors: Numerical $\Longrightarrow$ better computers. Statistical/parameter $\Longrightarrow$ better data.

(Dis)Agreement Significant Only When All Error Sources Explored

Editorial PRA 83 (2011) 040001



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These guidelines have been used on a case-by-case basis for the past two years. Authors have adapted well to this, resulting in papers of greater interest and significance for our readers.

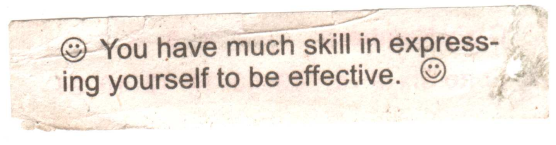
The Editors

$$\boxed{\alpha^{p}_{E1} = 10.65 \pm 0.35_{\text{stat}} \pm 0.2_{\Sigma} \pm 0.3_{\text{theory}}}$$

Non-Theory Errors: Numerical $\Longrightarrow$ better computers. Statistical/parameter $\Longrightarrow$ better data.

Theoretical uncertainty: Truncation of Physics

EFT claim: systematic in $Q = \dfrac{\textit{typ. low scale } p_{\textit{typ}}}{\textit{typ. high scale } \overline{\Lambda}_{\textit{EFT}}}$



☺ You have much skill in expressing yourself to be effective. ☺

Scientific Method: Quantitative results with corridor of theoretical uncertainties for *falsifiable predictions*.

Need procedure which is established, economical, reproducible: room to argue about "error on the error".

"Double-Blind" Theory Errors: Assess with pretense of no/very limited data.

What Does “Conservative” Uncertainty Mean?

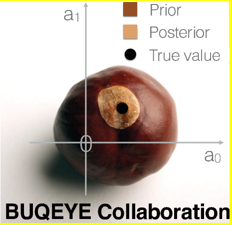
max-criterion: lore since ”time immemorial”
Bayes: e.g. Cacciari/Houdeau 1105.5152
BUQEYE 1506.01343+1511.03618
applied in hg/JMcG/DRP 1511.01952

χEFT $\alpha_{E1}^{(\mathrm{p})}-\beta_{M1}^{(\mathrm{p})}\,[10^{-4}\mathrm{fm}^3]: 7.5\pm ???_{\text{th}} = 11.2_{\text{LO}} - 3.6_{\text{NLO}} - 0.1_{\text{N}^2\text{LO}} \pm ???_{\text{th}}$

Observable as series: $\mathcal{O} = c_0 \quad + c_1\,\delta^1 \quad + c_2\,\delta^2 \quad +$ unknown $c_3\times\delta^3$

Assuming $\delta\simeq 0.4$: $11.2 \quad -9.1\,\delta^1 \quad -0.6\,\delta^2 \quad +$ unknown $\times\,\delta^3$

$\Longrightarrow$ **Estimate next term *“most conservatively”* as** $|$**unknown** $c_3| \lesssim R := \max\{|c_0|;|c_1|;|c_2|\}$.



No infinite sampling pool; data fixed; more data changes confidence.

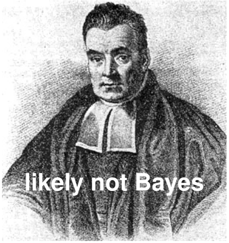
Call upon the Reverend Bayes for probabilistic interpretation!

e.g. **BUQEYE collaboration** **[Furnstahl/Phillips/. . . 1506.01343+1511.01952+. . .]**

New information increases level of confidence.

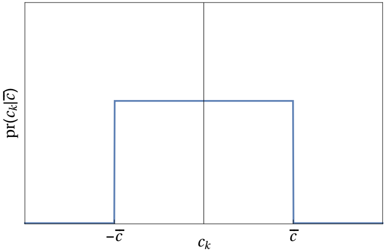
$\Longrightarrow$ **Smaller corrections, more reliable uncertainties.**

Clearly state your premises/assumptions – including *naturalness*.

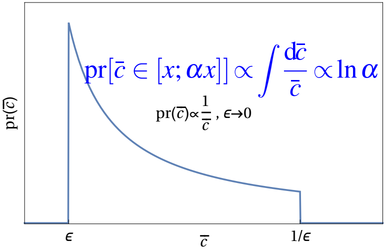


Priors: leading-omitted term dominates ($\delta\ll 1$); putative distributions of *all* c_k’s and of largest value $\bar{c}$ in series.

Uniform ~~“least informed/-ative”~~: All values c_k equally likely, given upper bound $\bar{c}$ of series.



“Any upper bound” (*Benford’s Law*): ln-uniform prior sets no bias on scale of $\bar{c}$



$\mathrm{pr}[\bar{c}\in[x;\alpha x]] \propto \int\frac{\mathrm{d}\bar{c}}{\bar{c}} \propto \ln\alpha$ indep. of x

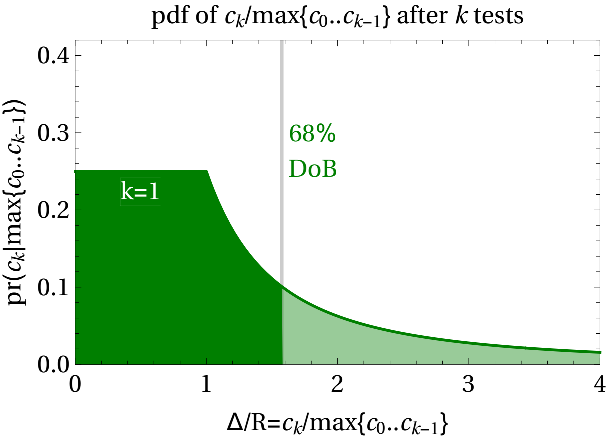
equi-distribution on ln scale

Quantifying Beliefs in $\mathcal{O} = \delta^n(c_0 + c_1\,\delta^1 + c_2\,\delta^2 + \dots) = 11.2 - 9.1\,\delta^1 - 0.6\,\delta^2 \pm 0.6_{\text{th}}$

Input: **Expansion parameter $\delta \simeq 0.4$, number of orders $k = 1$ (LO)**
and probable "largest number" $R = \delta^{k=1} \times \max\{|c_0 = 11.2| \qquad\qquad \} = 4.5$.
Result: Posterior $\equiv$ *Degree of Belief (DoB)* that next term $c_k\delta^k$ differs from order-k central value by Δ.

[BUQEYE 1506.01343 eq. (22)]

$$\mathrm{pr}(\Delta|\max.\,R,\text{order }k) \propto \int_0^\infty \mathrm{d}\bar{c}\;\mathrm{pr}(\bar{c})\;\mathrm{pr}(c_k = \frac{\Delta}{\delta^k}|\bar{c}) \prod_n^{k-1} \mathrm{pr}(c_n|\bar{c}) \to \frac{k}{k+1}\,\frac{1}{2R} \begin{cases} 1 & |\Delta| \le R \\ \left(\frac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R \end{cases}$$


pdf of $c_k/\max\{c_0..c_{k-1}\}$ after k tests

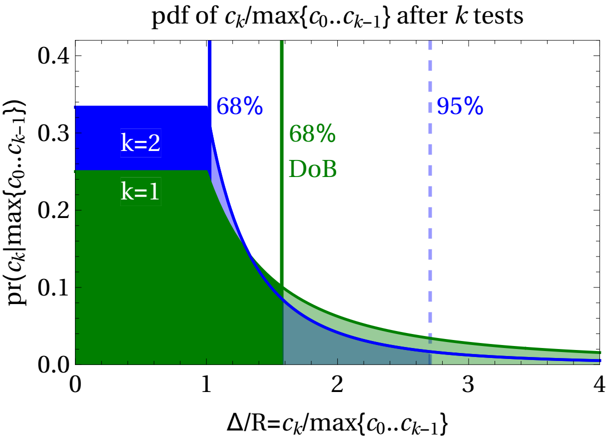
order	DOB in $\pm R$	σ: 68%	$\Delta(95\%)$
LO	$\frac{1}{2} = 50\%$	$1.6\,R$	$11R = 7\sigma$
Gauß	68.27%	$1.0\,R$	2.0σ

Quantifying Beliefs in $\mathcal{O} = \delta^n(c_0 + c_1\,\delta^1 + c_2\,\delta^2 + \dots) = 11.2 - 9.1\,\delta^1 - 0.6\,\delta^2 \pm 0.6_{\text{th}}$

Input: **Expansion parameter** $\delta \simeq 0.4$, **number of orders** $k = 2$ **(NLO)**
and probable "largest number" $R = \delta^{k=2} \times \max\{|c_0 = 11.2|;|c_1 = -9.1| \qquad\qquad \} = 1.7$.
Result: Posterior $\equiv$ ***Degree of Belief (DoB)*** **that next term** $c_k\delta^k$ **differs from order-**k **central value by** Δ.

[Buqeye 1506.01343 eq. (22)]

$$\mathrm{pr}(\Delta|\text{max. }R,\text{order }k) \propto \int\limits_0^\infty \mathrm{d}\bar{c}\;\mathrm{pr}(\bar{c})\;\mathrm{pr}\Big(c_k = \frac{\Delta}{\delta^k}\Big|\bar{c}\Big)\prod_n^{k-1}\mathrm{pr}(c_n|\bar{c}) \to \frac{k}{k+1}\,\frac{1}{2R}\begin{cases} 1 & |\Delta| \le R \\ \left(\dfrac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R \end{cases}$$



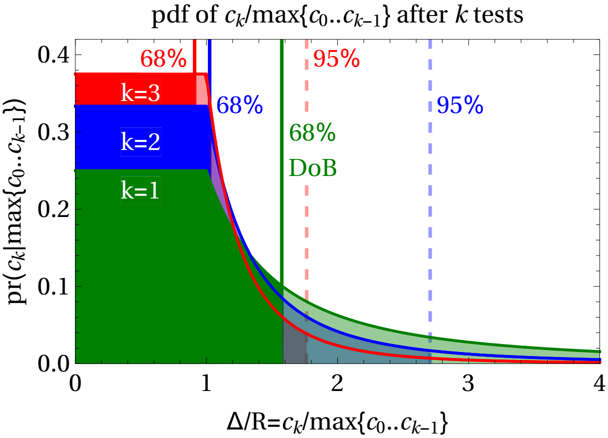
order	DOB in $\pm R$	σ: 68%	$\Delta(95\%)$
LO	$\frac{1}{2} = 50\%$	$1.6\,R$	$11R = 7\sigma$
NLO	$\frac{2}{3} = 66.7\%$	$1.0\,R$	$2.7R = 2.6\sigma$
Gauß	68.27%	$1.0\,R$	2.0σ

Quantifying Beliefs in $\mathcal{O} = \delta^n(c_0 + c_1\,\delta^1 + c_2\,\delta^2 + \dots) = 11.2 - 9.1\,\delta^1 - 0.6\,\delta^2 \pm 0.6_{\text{th}}$

Input: Expansion parameter $\delta \simeq 0.4$, number of orders $k = 3$ (N^{2}LO)
and probable "largest number" $R = \delta^{k=3} \times \max\{|c_0 = 11.2|;|c_1 = -9.1|;|c_2 = -0.6|\} = 0.7$.
Result: Posterior ≡ *Degree of Belief (DoB)* that next term $c_k\delta^k$ differs from order-k central value by Δ.

[BUQEYE 1506.01343 eq. (22)]

$$\mathrm{pr}(\Delta|\mathrm{max.}\ R, \mathrm{order}\ k) \propto \int\limits_0^\infty \mathrm{d}\bar{c}\ \mathrm{pr}(\bar{c})\ \mathrm{pr}(c_k = \frac{\Delta}{\delta^k}|\bar{c}) \prod_n^{k-1} \mathrm{pr}(c_n|\bar{c}) \to \frac{k}{k+1}\,\frac{1}{2R}\begin{cases} 1 & |\Delta| \leq R \\ \left(\frac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R \end{cases}$$



order	DOB in $\pm R$	σ: 68%	$\Delta(95\%)$
LO	$\frac{1}{2} = 50\%$	$1.6\,R$	$11R = 7\sigma$
NLO	$\frac{2}{3} = 66.7\%$	$1.0\,R$	$2.7R = 2.6\sigma$
N^{2}LO	$\frac{3}{4} = 75\%$	$0.9\,R$	$1.8R = 1.9\sigma$
N^{k-1}LO k terms	$\frac{k}{k+1}$	$0.68\frac{k+1}{k}R(k \geq 2)$	
Gauß	68.27%	$1.0\,R$	2.0σ

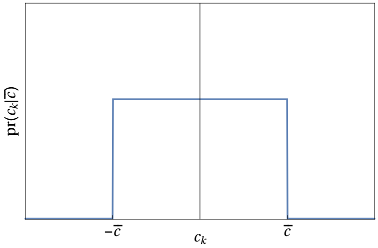
For "high enough" order, largest number R limits
$\gtrsim 68\%$ **degree-of-belief interval.**

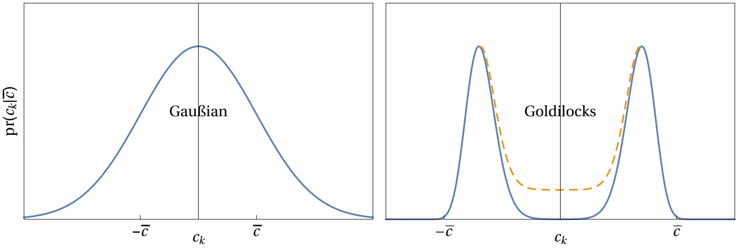
$\Longrightarrow$ ***Interpretation of all theory uncertainties, with these priors; "$A \pm \sigma$": 68% DoB interval $[A - \sigma; A + \sigma]$.***

Prior Choice: What is “Natural Size”? (SCOTUS: I Know It When I see It.)

Observable $\mathcal{O} = c_0 + c_1\delta^1 + c_2\delta^2 +$ unknown $\times\,\delta^3$: *assumed* $\delta \approx 0.4$ *& “naturally-sized coefficients”* c_i.

[Buqeye 1511.03618]: Bayesian technology to extract value of δ from (many) observables, with degree of belief.



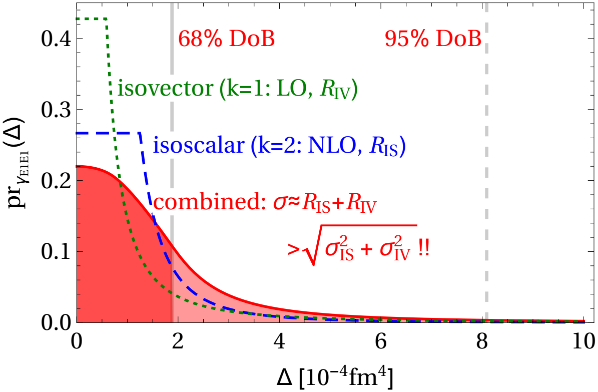


Uniform ~~“Least informative/-ed”~~:
characterised by 1 number: $\bar{c}$.

“More informed choices”: more complicated structures, more thought,
more parameters: $\bar{c}$, typ. size, spread,...

[Buqeye:] When $k \geq 2$ orders known, DoBs with
different assumptions about $\bar{c}$, c_n vary by $\lesssim \pm 20\%$ for some “reasonable priors”.

Final Bayes Comments



Posterior pdf *not Gauß'ian:*

Plateau & power-law tail.

$\Longrightarrow$ **Do not add in quadrature for convolution**

(more like linear).

Bayes provides well-defined *procedure*!

Bayes in EFTs also used to estimate: [BUQEYE Furnstahl/Phillips/... 1506.01343, 1511.03618,...]

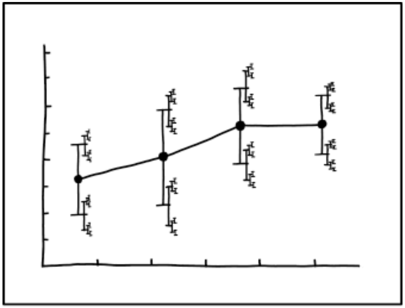
- **k-dependent $\delta(k)$ estimate from (many) observables ($\delta\approx 0.4$✓);**
- **breakdown scale $\overline{\Lambda}_{\text{EFT}}$;**
- **momentum-dependent data-weighting for LEC fitting/extraction;**
- **build LEC hierarchy into fit;**
- **"model quality"$\equiv$ correctness of EFT assumptions,...**

$\Longrightarrow$ **Quantitative theoretical uncertainties make EFT falsifiable:**

Economical, reproducible procedure: argue about "error on error".

"The aim is to estimate the uncertainty, not to state the exact amount[...]"

[PRA Editorial 2011]



I DON'T KNOW HOW TO PROPAGATE ERROR CORRECTLY, SO I JUST PUT ERROR BARS ON ALL MY ERROR BARS.

Error Estimates: Cooking Recipe with Water

Recipe for cooking with wine or oil/interest in more/details:
Previous slides & consult literature.

Observable $\mathcal{O} = \underbrace{(c_0 + c_1\,\delta^1 + c_2\,\delta^2 + \dots)}_{\text{known/calculated}} + \underbrace{c_{k+1}\,\delta^{k+1}}_{\text{unknown}}$

High orders like N^{5}LO are extremely rare.

$\mathcal{O} = 7.5 = 11.2_{\text{LO}} - 3.6_{\text{NLO}} - 0.1_{\text{N}^2\text{LO}}$

(1) Calculate quantity at every order in your expansion up to and including N^{k}LO ($k = 0$ is LO). N^{2}LO $\Rightarrow k = 2$

(2) Make a reasonable guess about the expansion parameter $\delta = \frac{\text{low momentum } p_{\text{typ.}}}{\text{breakdown } \Lambda_{\text{EFT}}}$. $\sqrt{\frac{m_\pi}{\Lambda_\chi}} \approx \frac{M_\Delta - M_{\text{N}}}{\Lambda_\chi} \approx 0.4$

(3) Identify the coefficients c_i, $i = 0, \dots, k$ (up to highest known order N^{k}LO). $\{11.2; -9.1; -0.6\}$

(4) Identify the largest-in-magnitude coefficient $\max\{|c_i|\}$. 11.2

(5) Find the "probable largest number at N^{k+1}LO": $R := \delta^{k+1} \times \max\{|c_i|\} \overset{\text{probably}}{>} |\delta^{k+1}\, c_{k+1}|$. $11.2 \times \delta^3 \approx 0.7$

(6) You can rescale R to an $\alpha\%$ (e.g. 68%, 95%) interval R_α; see table on slide 51 (link). $0.7 \times 0.9_{68\%} \approx 0.6_{68\%}$

(7) R and R_α is a reasonable estimate of your theory error: $\mathcal{O} \pm R_{(\alpha)}$. 7.5 ± 0.6

(8) Reproducibly describe in publication what you did. e.g. arXiv:1511.01952

At LO, R is somewhat less than a 68% DoB interval, and the tail is very "fat".

At NLO (N^{2}LO), R contains a bit more (less) than 70% (see comment in box), and the tail is a bit less "fat".

At N$^{k\geq 3}$LO, R contains increasingly more than 70%, the tail gets increasingly less "fat", but exceptional DoBs like 99.5% are still farther out than $3R$ or $3R_\alpha$. **Rule of Thumb: Tails fatter than you think, never Gaußian.**

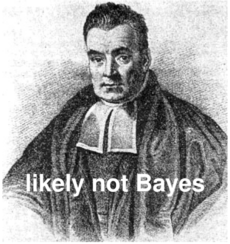
Remember: You estimate the error. Errors have errors. I do not trust any estimate to better than 20% of R_α.

If you worry about whether R (R_α) contains 65% or 70%, you have mis-understood the exercise.

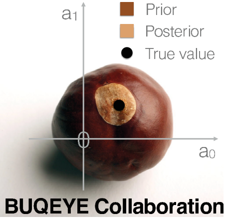
0.433 ± 0.325 **makes no sense: precision vs. accuracy.**

Bayesian Posterior Shrinkage by Intelligent Design

BUQEYE: Melendez/Furnstahl/Pratola/
DRP/hgrie/JAMcG/...
EPJA **57** (2021) 81 2004.11307



Apply Bayesian Experimental Design:
Be explicit about assumptions/prejudices.
Maximise benefits – minimise cost
(time, money, workforce, data not taken).
Jupyter notebook: `buqeye.github.io`

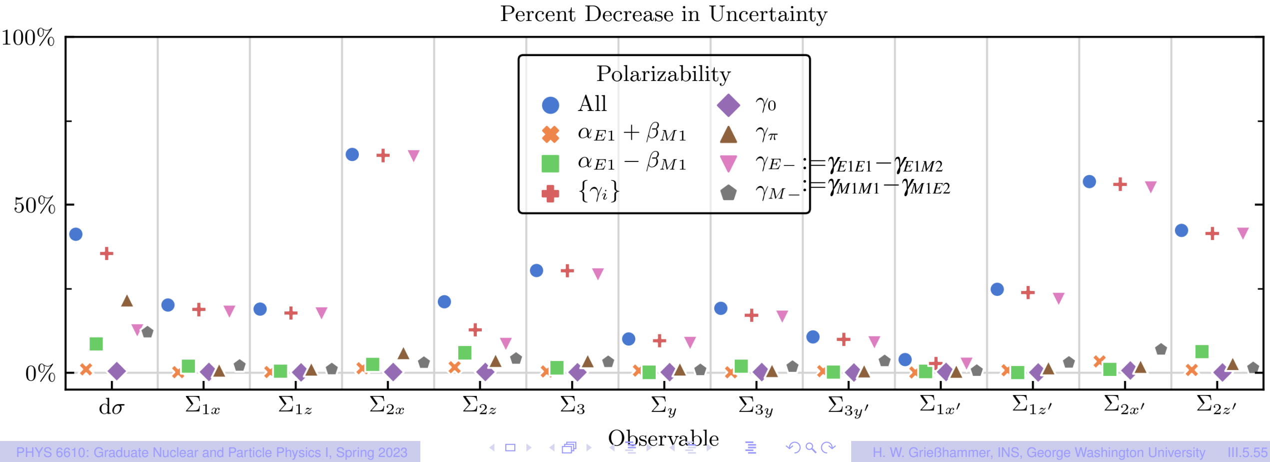


Bayesian
Uncertainty
Quantification:
Errors in
Your
EFT

Given: (1) Present polarisability errors (exp info); (2) χEFT accuracy decreases as $\omega \nearrow$; (3) exp constraints.

Assumption "Doable": Get cross section to $\pm 4\%$ or asymmetry to ± 0.06 (absolute) at 1 energy and 5 angles.

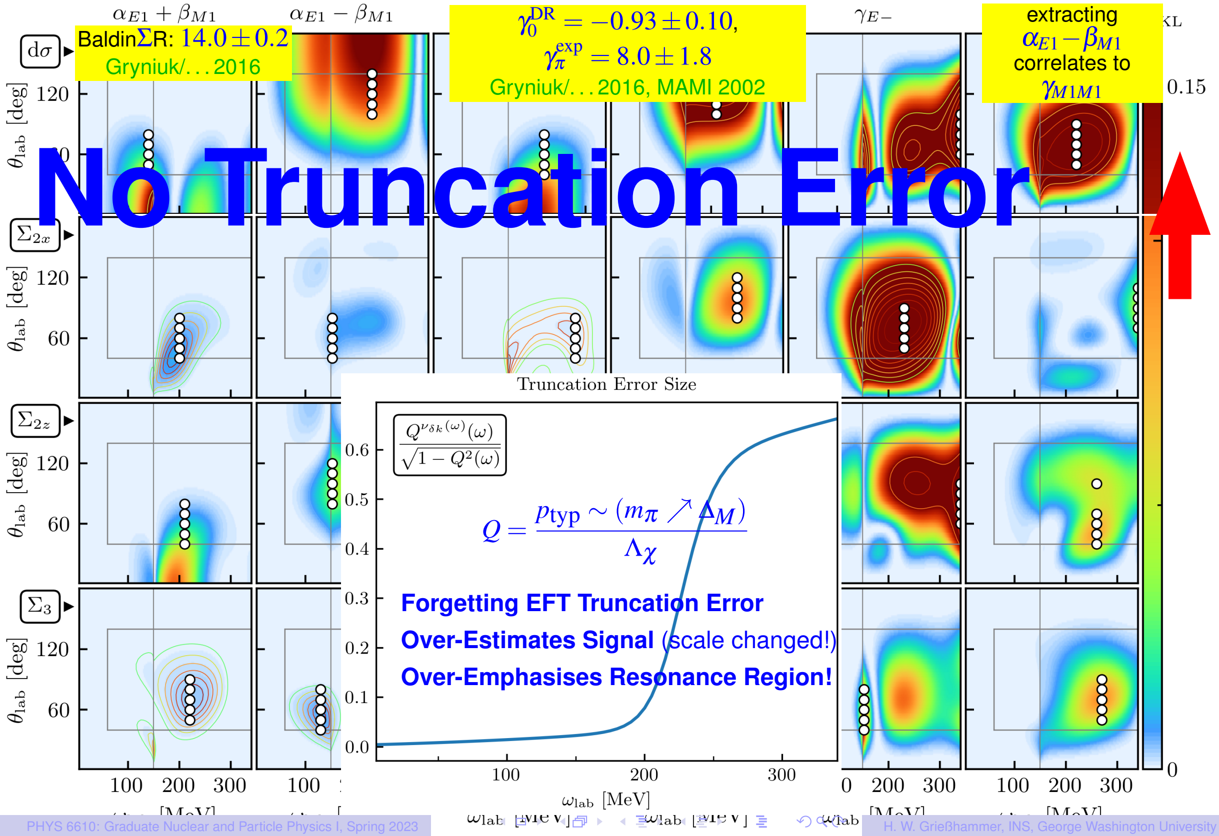
Gaußian Process
$\Longrightarrow$ ***Likely* impact** on errors $\Delta(\alpha_{E1}, \beta_{M1}, \gamma_i)$: Utility(new data) $= \left\langle \frac{\text{error's hypervolume after new data}}{\text{error's hypervolume before new data}} \right\rangle_{\text{avg}}$



Bayesian Posterior Shrinkage by Intelligent Design

BUQEYE: Melendez/Furnstahl/Pratola/
DRP/hgrie/JAMcG/…
EPJA **57** (2021) 81 2004.11307

Which 5 angles on proton have biggest impact on a particular polarisability?



BaldinΣR: 14.0 ± 0.2
Gryniuk/…2016

$\gamma_0^{\rm DR} = -0.93 \pm 0.10$,
$\gamma_\pi^{\rm exp} = 8.0 \pm 1.8$
Gryniuk/…2016, MAMI 2002

extracting
$\alpha_{E1}-\beta_{M1}$
correlates to
γ_{M1M1}

No Truncation Error

$$Q = \frac{p_{\rm typ} \sim (m_\pi \nearrow \Delta_M)}{\Lambda_\chi}$$

Forgetting EFT Truncation Error
Over-Estimates Signal (scale changed!)
Over-Emphasises Resonance Region!

Bayesian Posterior Shrinkage by Intelligent Design

BUQEYE: Melendez/Furnstahl/Pratola/
DRP/hgrie/JAMcG/...
EPJA **57** (2021) 81 2004.11307

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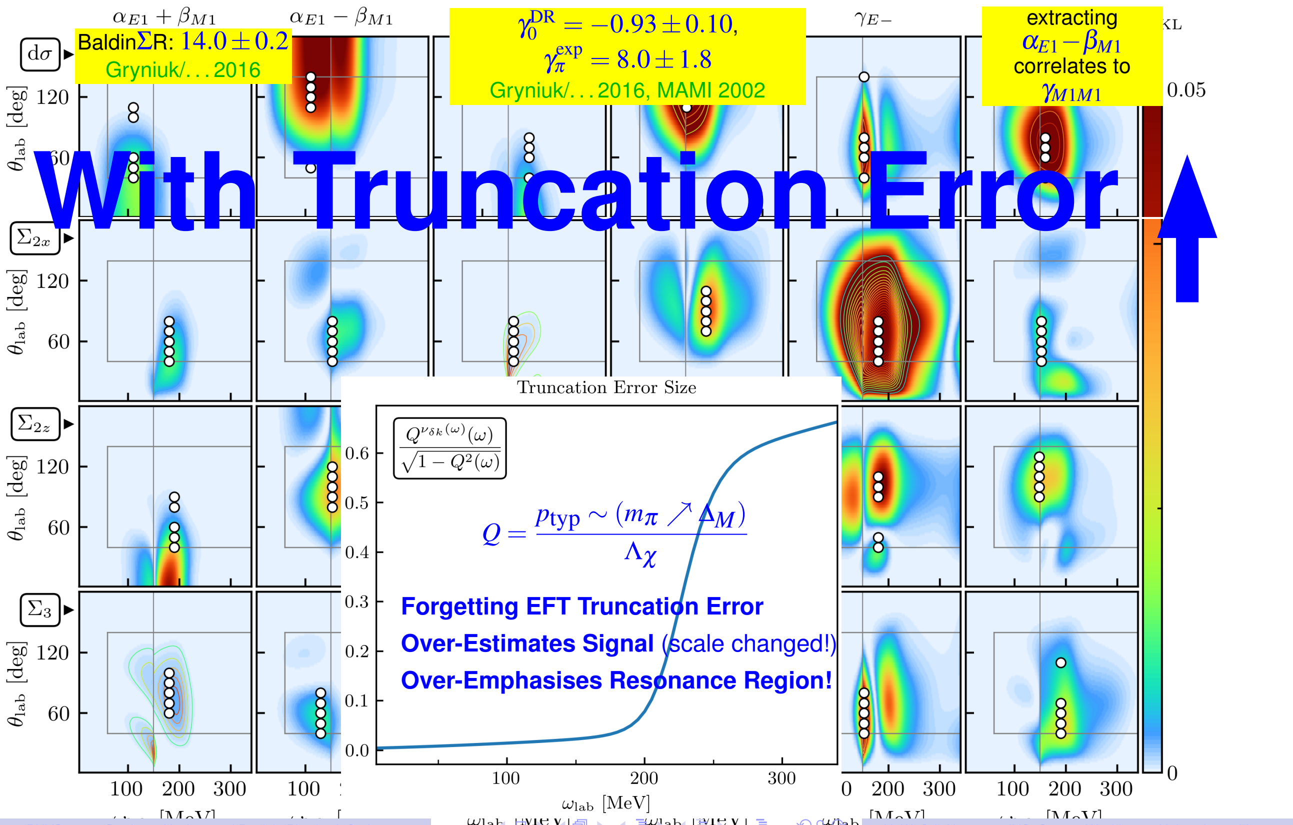

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Gryniuk/...2016, MAMI 2002

extracting
$\alpha_{E1}-\beta_{M1}$
correlates to
γ_{M1M1}

With Truncation Error

$$Q = \frac{p_{\text{typ}} \sim (m_\pi \nearrow \Delta_M)}{\Lambda_\chi}$$

Forgetting EFT Truncation Error
Over-Estimates Signal (scale changed!)
Over-Emphasises Resonance Region!

(g) Statistical Interpretation of the Max-Criterion: A Simple Example

I take this table of πN scattering parameters in χEFT with effective $\Delta(1232)$ degrees of freedom from a talk by Jacobo Ruiz de Elvira. Here, I am not interested in the Physics, but use it as series $c_i = c_{i0} + c_{i1}\epsilon^1 + c_{i2}\epsilon^2$ in a small expansion parameter.

parameter $[\mathrm{GeV}^{-1}]$	LO total	NLO total	N^{2}LO total	expansion $= c_{i0} + c_{i1}\epsilon^1 + c_{i2}\epsilon^2$	perturbative expansion $\epsilon \approx 0.4$ (guess)
c_1	-0.69	-1.24	-1.11	$= -0.69 + 0.55 - 0.13$	$= -0.69 + 1.38\epsilon^1 - 0.81\epsilon^2$
c_2	$+0.81$	$+1.13$	$+1.28$	$= +0.81 - 0.32 - 0.15$	$= +0.81 - 0.80\epsilon^1 - 0.94\epsilon^2$
c_3	-0.45	-2.75	-2.04	$= -0.45 + 2.30 - 0.71$	$= -0.45 + 5.75\epsilon^1 - 4.44\epsilon^2$
c_4	$+0.64$	$+1.58$	$+2.07$	$= +0.64 - 0.94 - 0.49$	$= +0.64 - 2.35\epsilon^1 - 3.06\epsilon^2$

Now pick the largest absolute coefficient to estimate typical size of next-order correction $c_{i(n+1)} = c_{i3}$ in our case:

Max-Criterion: $c_{i(n+1)} \lesssim \max\limits_{n\in\{0;1;2\}} \{|c_{in}|\} =: R$ **is labelled as red in the table.**

This criterion has been applied since "Time Immemorial"
See example on the next slide which predates EKM by 4 years.

Multiply that number with ϵ^3 to finally get a corridor of uncertainty/typical size of the ϵ^3 contribution.

For c_1: $\max\limits_{n\in\{0;1;2\}} \{|-0.69|;|1.38|;|-0.81|\} = 1.38 \implies$ error $\pm 1.38 \times (\epsilon = 0.4)^3 \approx 0.09 \implies c_1 = -0.69 \pm 0.09$.

Similar: $c_2 = 1.28 \pm 0.06$, $c_3 = -2.04 \pm 0.37$, $c_4 = 2.07 \pm 0.20$ (round significant figures conservatively).

But what's the statistical interpretation? $\implies$ Next slide!

Notes: **(1)** Provide a theoretical error *estimate* that is *reproducible*. You can then discuss with others who have different opinions. No estimate, no discussion possible. – **(2)** Sometimes, one discards the LO→NLO correction if it's anomalously large. That is a "prior information" you need to disclose as "bias" of your estimate. – **(3)** Coefficients c_{in} appear "more natural" for c_1 and c_2 than for c_4 – c_4 not that well-converging? – **(4)** The uncertainty estimate is agnostic about the Physics details. Somebody just handed me a table. – **(5)** If you are not happy with the input "$\epsilon \approx 0.4$", pick another number. BUQEYE 1511.03618 developed the Bayesian technology to extract degrees of belief on what value of the expansion parameter the series suggests. – **(6)** The c_i are not observables, but they are renormalised couplings which – according to Renormalisation – should follow a perturbative expansion.

(g) Statistical Interpretation of the Max-Criterion: A Simple Example

The Bayesian interpretation of the max-criterion on the next slide will provide probability distribution (pdf)/degree-of-belief functions using a “reasonable” set of assumptions (“priors”) which give nice, analytic expressions. That’s one choice of assumptions, but other reasonable assumptions provide very similar pdf’s see BUQEYE: 1506.01343, 1511.03618,. . . .

But before that, let’s do something intuitive which gives the same statistical likeliness interpretation of the max-criterion as the Bayesian one. The Bayesian analysis formalises the example and provides actual pdf’s.

Estimating a Largest Number: Given a finite set of (finite, positive) numbers in an urn. You get to draw one number at a time.

Your mission, should you choose to accept it: Guess the largest number in the urn from a limited number of drawings.

For c_1, we first draw $c_{10} = 0.69$. I would say it’s “natural” to guess that there is a 1-in-$2 = 50\%$ chance that the next number is lower. But there is also a pretty good chance that if it is higher, then its distribution up there is not Gauß’ian but with a stronger tail.

Next, we draw $c_{11} = 1.38$ which is larger. So I revise my largest-number projection to $R = 1.38$, but I also get more confident that this may be pretty high (if not he highest already). After all, I already found one number which is lower, namely $c_{10} = 0.69$. With 2 pieces of information (0.69 and 1.38), it’s “natural” that the 3rd drawing has a 2-in-3 or $2/3$ chance to be lower.

Next, we draw $c_{12} = 0.81 < R$. Looking at my set of 3 numbers, I am even more confident that $R = c_{11} = 1.38$ is the largest number, with 3-in-4 or 75% confidence. For c_1, evil forces interfere and we have no more drawings to draw information from.

But if we could reach into the urn k times and look at the collected k results, every time revising our max-estimate, it’s “natural” to assign a $100\% \times k/(k+1)$ confidence that I have actually gotten the largest number R.

The Bayesian procedure on the next slide provides the same result. Read the BUQEYE papers for details and formulae!

In our example, we had $k = 3$ terms (drawings) for c_1. So the confidence that $R = 1.38$ is indeed the highest number is $3/4 = 75\%$, which is larger than $p(1\sigma) \approx 68\%$. For a 1σ corridor, I reasonably assume that the numbers are equi-distributed between 0 and the maximum R. Then, the 68%-error corridor is set by $\pm 68\% \times (k+1)/k \times R$ amongst the known numbers.

Now, I multiply that number with 3 powers of the expansion parameter $\epsilon \approx 0.4$ (estimate N^{3}LO terms!) (but see **Note (5)** on the previous slide): $\pm 1.38 \times (68\%/75\%) \times 0.4^3 = \pm 0.08$ is a good uncertainty estimate for a traditional 68% confidence region. I also get a feeling that the probabilities outside the interval $[0;R]$ may not be Gauß’ian-distributed. Bayes will confirm that.

Physical Models vs. Physical Theories – A Sliding Scale

Model: **Parametrise** data, Capture *some* aspects with lots of data – no “fail” but “tuning”. Cargo Cult mode.

The Trouble With Nuclear Physics

In fact the trouble in the recent past has been a surfeit of different *models* [of the nucleus], each of them successful in explaining the behavior of nuclei *in some situations*, and each in *apparent contradiction with other successful models* or with our ideas about nuclear forces.

[Rudolph E. Peierls: “The Atomic Nucleus”, *Scientific American* **200 (1959)**, no. 1, p. 75; emph. added]

Theory: **Predictive**, comprehensive, prescriptive, may fail. Explain-All-To-Some-Degree mode.

Gelman’s Totalitarian Principle/Swiss Basic Law/ Weinberg’s “Folk Theorem”: Throw In the Kitchen Sink

As long as you let it be the most general possible Lagrangian consistent with the symmetries of the theory, you’re simply writing down the most general theory you could possibly write down.

[Original: Weinberg: Physica 96A (1979) 327 – here 1997 version]

Quality Check: EXISTENCE: Are there Theory-uncertainties/errors?

REPRODUCIBILITY: Clear discussion how they are assessed?

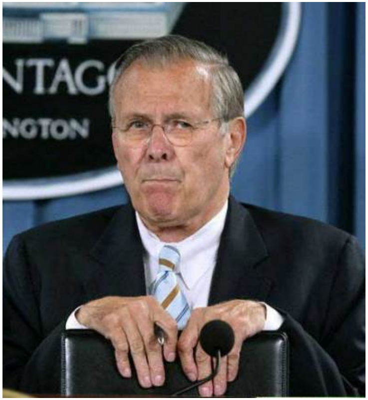
Why I Do Effective Field Theories

Scientific Approach

As we know,

there are known knowns.

There are things we know we know.



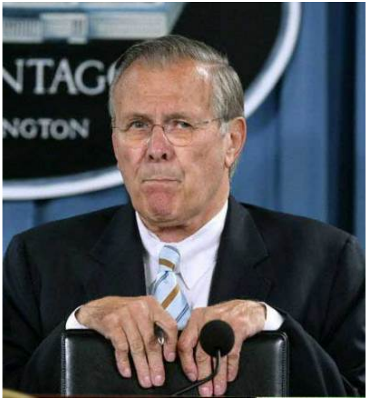
Why I Do Effective Field Theories

Scientific Approach

As we know,
there are known knowns.
There are things we know we know.

We also know
there are known unknowns.

That is to say
we know there are some things
we do not know.



Why I Do Effective Field Theories

Scientific Approach

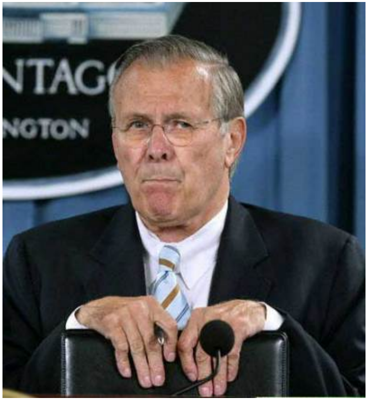
As we know,
there are known knowns.
There are things we know we know.

We also know
there are known unknowns.

That is to say
we know there are some things
we do not know.

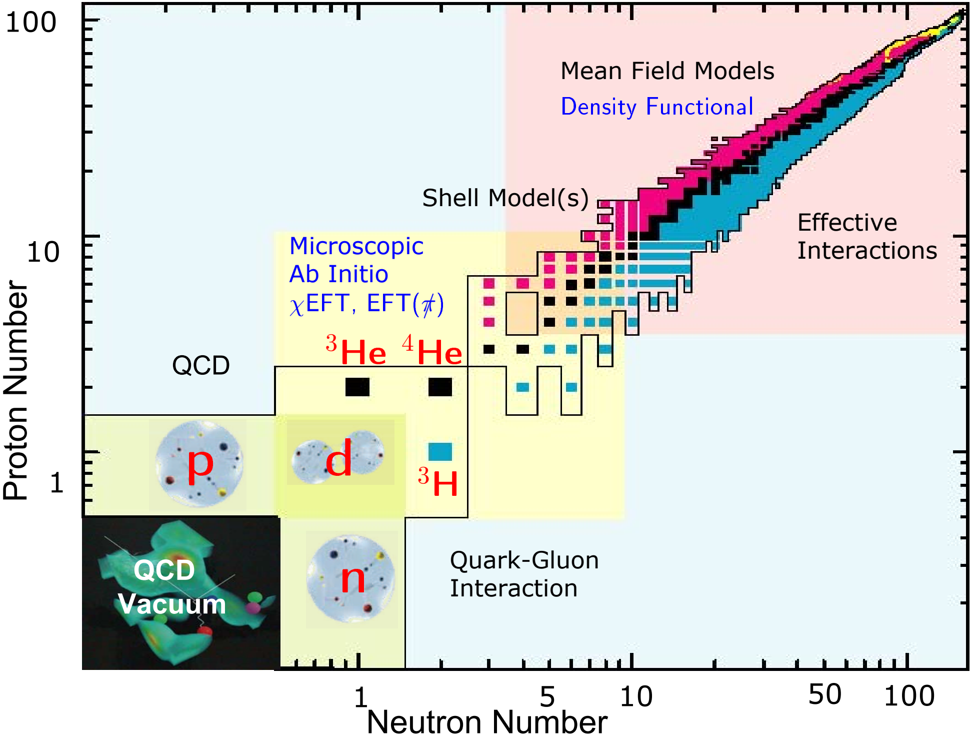
But there are also unknown unknowns,
the ones we don't know
we don't know.

Donald Rumsfeld, 12 Feb 2002

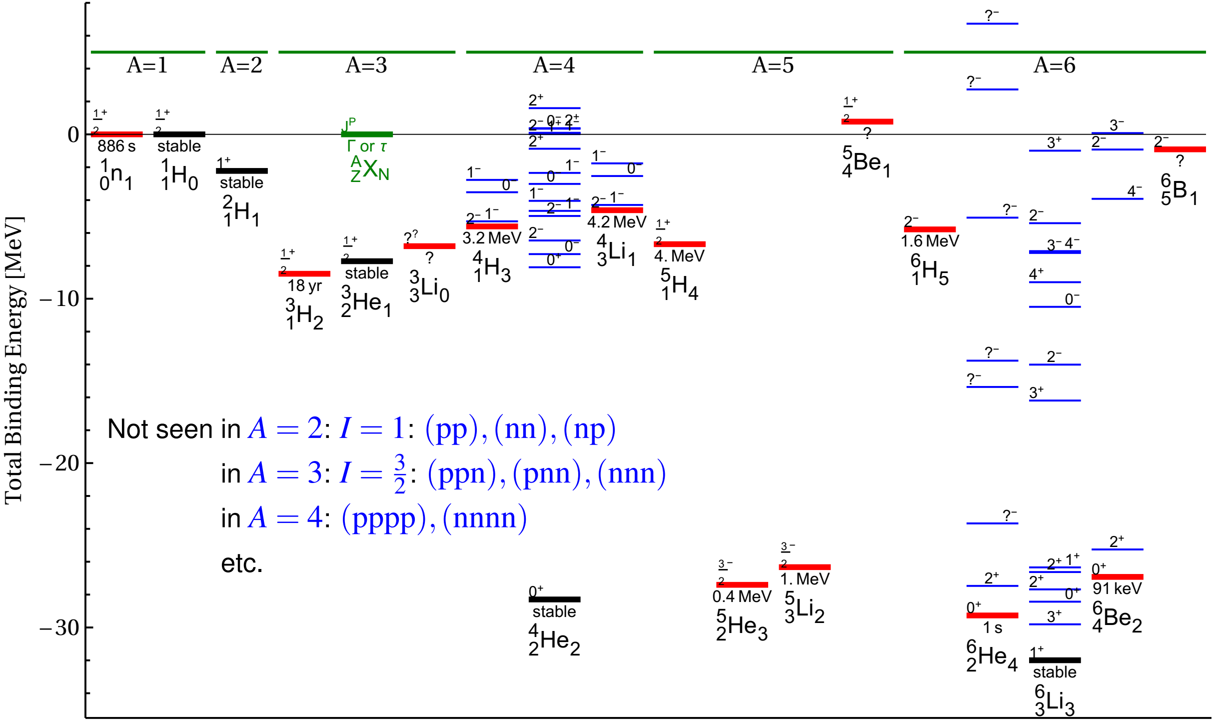


(h) χEFT In All Its Glory: Few-Nucleon Systems

NN **System is gateway to understanding microscopic structure of nuclear structure from QCD.**



Few-Nucleon Spectra “Should” follow from QCD



Not seen in $A = 2$: $I = 1$: $(pp), (nn), (np)$

in $A = 3$: $I = \frac{3}{2}$: $(ppn), (pnn), (nnn)$

in $A = 4$: $(pppp), (nnnn)$

etc.

Whence the Patterns? How much is special to QCD?

Quantum Numbers of the NN System

Couple 2 nucleons with spin $S_\mathrm{N}=\frac{1}{2}$, isospin $I_\mathrm{N}=\frac{1}{2}$:

Spin $\vec{S}=\frac{\vec{\sigma}_1}{2}+\frac{\vec{\sigma}_2}{2}$			Isospin $I^a=\frac{\tau_1^a}{2}+\frac{\tau_2^a}{2}$			
$S=0$	$m_s=0$	$\frac{1}{\sqrt{2}}[\lvert\uparrow\downarrow\rangle-\lvert\downarrow\uparrow\rangle]$	$I=0$	$I_3=0$	$\frac{1}{\sqrt{2}}[\lvert pn\rangle-\lvert np\rangle]$	**anti-symmetric**
$S=1$	$m_s=+1$	$\lvert\uparrow\uparrow\rangle$	$I=1$	$I_3=+1$	$\lvert pp\rangle$	**symmetric**
	$m_s=0$	$\frac{1}{\sqrt{2}}[\lvert\uparrow\downarrow\rangle+\lvert\downarrow\uparrow\rangle]$		$I_3=0$	$\frac{1}{\sqrt{2}}[\lvert pn\rangle+\lvert np\rangle]$	
	$m_s=-1$	$\lvert\downarrow\downarrow\rangle$		$I_3=-1$	$\lvert nn\rangle$	

$$\Psi^{NN}_{\text{total}}=\lvert\textbf{spin}\rangle\otimes\lvert\textbf{isospin}\rangle\otimes\lvert\textbf{orb. ang. mom. } Y_{lm}(\theta,\phi)\rangle\otimes\lvert\textbf{radial }(r)\rangle$$

Pauli Principle: Total wave function anti-symmetric under exchange of identical fermions:

$$(-)^1\overset{!}{=}(-)^{S+1}\,(-)^{I+1}\,(-)^L\implies S+I+L\text{ must be odd!}$$

Angular momentum coupling: Eigenvalues to $\vec{J}^2=(\vec{L}+\vec{S})^2$ are $J=0,1,2,\dots$

Lowest Partial Waves in Spectroscopic Notation $^{2S+1}L_J$:

$I=1$ in pp, np, nn: $^1\mathrm{S}_0$; $^3\mathrm{P}_{0,1,2}$; $^1\mathrm{D}_2$;... $I=0$ *only in* np!: $^3\mathrm{S}_1$; $^1\mathrm{P}_1$; $^3\mathrm{D}_{1,2,3}$; ...

Waves with same J^P mix, most importantly: $^3\mathrm{S}_1$-$^3\mathrm{D}_1$ (see in a minute...)

Generic Structure of the NN Potential at Low Energies [Ber 3; EW 3]

Pions π^a & non-relativistic nucleons: $N = \begin{pmatrix} p \\ n \end{pmatrix}_{\text{isospin}} \otimes \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}_{\text{spin}}$

Most general form which depends only on relative distance r of nucleons (“local”)

and is isospin, rotation, parity symmetric:

$$V_{NN}(\vec{r}, \vec{\sigma}_i, \tau_i^a, \vec{L}) = \overbrace{\delta_{I0}\, V^{(I=0)}}^{\text{iso-scalar}} + \overbrace{4\delta_{I1}\, V^{(I=1)}}^{\text{iso-vector}}$$

with $V^{(I)} = \underbrace{V_C^{(I)}}_{\text{central}} + \underbrace{\vec{\sigma}_1 \cdot \vec{\sigma}_2\, V_S^{(I)}}_{\text{spin-spin}} + \underbrace{\vec{L} \cdot \vec{S}\, V_{LS}^{(I)}}_{\text{spin-orbit}} + \underbrace{S_{12}(\vec{e}_r)\, V_T^{(I)}}_{\text{tensor}}$

Tensor operator $S_{12}(\vec{e}_r) = 3(\vec{\sigma}_1 \cdot \vec{e}_r)(\vec{\sigma}_2 \cdot \vec{e}_r) - \vec{\sigma}_1 \cdot \vec{\sigma}_2 = 6(\vec{S} \cdot \vec{e}_r)^2 - 4\delta_{S1}$:

– Analogous to elmag. dipole-dipole $V_{\text{dd}} = -\dfrac{3(\vec{\mu}_1 \cdot \vec{e}_r)(\vec{\mu}_2 \cdot \vec{e}_r) - \vec{\mu}_1 \cdot \vec{\mu}_2}{r^3}$.

– Mixes partial waves with same J and parity, most importantly: ${}^3\mathrm{S}_1$-${}^3\mathrm{D}_1$.

Solve Schrödinger $\left[E - V + \dfrac{\vec{\partial}^2}{M_N}\right] \Psi_{NN} = 0$ **or Lippmann-Schwinger** $T = V + TGV$

in Partial-Wave basis: Decouple into 1-dimensional problems for $S = 0$; (2×2) for $S = 1$.

Partial Wave Analysis and (More) Phenomenological Potentials

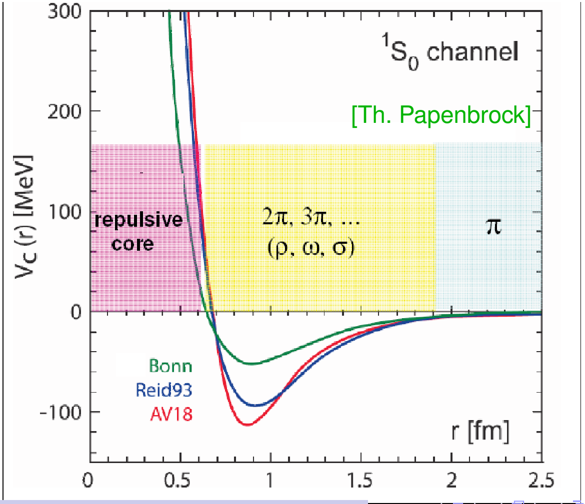
Nijmegen Partial Wave Analysis 1993-present: $> 6000\ pp$ and np scattering data for $p \lesssim 300\ \mathrm{MeV}$

Sketch of Phenomenological approaches: long-range: OPE – short-range: reasonable guesses…

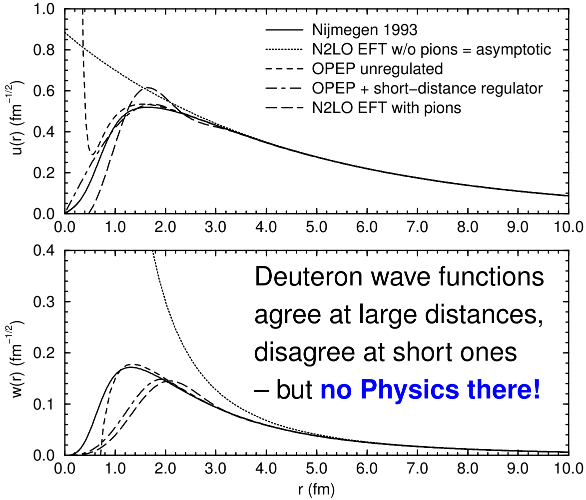
One Boson Exchange Potentials (Bonn BC, Paris,…): $V_{\text{core}} = \sum\limits_{\omega,\rho,\sigma,\dots} g_i^2 \times (\text{spin-structure})\ \dfrac{\mathrm{e}^{-m_i r}}{r}$

Short-Distance Core (Nijmegen 93, AV18, Reid,…): e.g. $V_{\text{core}}(r) \sim \sum\limits_{\text{LSJ}} \dfrac{A[{}^{2S+1}\mathrm{L}_J]}{1+\exp-(r-r_0)/a}$

Suitably flexible, ~ 40 parameters, same fit quality: Systematic? Resolved for $p < 300\ \mathrm{MeV}$?



[Th. Papenbrock]



Deuteron wave functions agree at large distances, disagree at short ones – but **no Physics there!**

Non-Relativistic Reduction of an EFT for $Q = \frac{p_{\text{typ.}}}{M} \ll 1$

Find kinetic energy $T = p_0 - M \ll M$ of free nonrelativistic boson (including spin is “trivial”):

$$\mathcal{L} = \Phi^\dagger \left[(T+M)^2 - \vec{p}^2 - M^2 \right] \Phi = \left(\sqrt{2M}\Phi^\dagger \right) \Bigg[\underbrace{T - \frac{\vec{p}^2}{2M}}_{\substack{\text{inv.}\\ \text{propagator}}} - \underbrace{\frac{\vec{p}^4}{8M^3}}_{\substack{\text{relative}\\ \mathcal{O}(Q^2)}} + \dots \Bigg] \underbrace{\left(\sqrt{2M}\Phi \right)}_{\substack{=:\,\phi\\ \text{non-rel. field}}}$$

$\Longrightarrow$ Treat higher orders in $\beta = \frac{|\vec{p}|}{T+M} \approx \frac{|\vec{p}|}{M}$ as perturbation:

$-\mathrm{i}\frac{\vec{p}^4}{8M^3}$

$\Longrightarrow$ Propagator pole at $T = \frac{\vec{p}^2}{2M} - \mathrm{i}\epsilon > 0$ $\Longrightarrow$ Anti-N effects $\approx 2M_{\mathrm{N}} \gg p_{\text{typ}}$ in LECs.✓

$\Longrightarrow T \sim \frac{\vec{p}^2}{2M} \ll |\vec{p}| \ll M$ as expected – and Pauli spinors $N = \begin{pmatrix} p \\ n \end{pmatrix}_{\text{isospin}} \otimes \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}_{\text{spin}}$.

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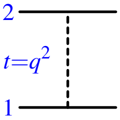
$\Longrightarrow T \sim \frac{\vec{p}^2}{2M} \ll |\vec{p}| \ll M$ as expected – and Pauli spinors $N = \begin{pmatrix} p \\ n \end{pmatrix}_{\text{isospin}} \otimes \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}_{\text{spin}}$.

$\Longrightarrow$ Nonrel. point-N with anom. mag. moment κ: $\mathcal{L}_{\text{Pauli}} = N^\dagger \left[T - \frac{(\vec{p} - eQ\vec{A})^2}{2M} - \frac{(Q+\kappa)}{2M}\sigma \cdot \vec{B}\right] N$

Pion-exchange in t-channel between nonrelativistic nucleons becomes instantaneous: $\frac{(q_0 = T_1 - T_2)^2}{(\vec{q} = \vec{p}_1 - \vec{p}_2)^2} \ll 1$



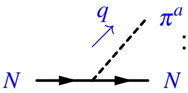
$$\frac{\mathrm{i}}{q_0^2 - \vec{q}^2 - m_\pi^2} \to \frac{-\mathrm{i}}{\vec{q}^2 + m_\pi^2} + \dots$$

$\Longrightarrow$ Use non-relativistic QM for few-N bound states from potentials: Schrödinger eq.,...

χEFT at Leading Order (LO): One Pion Exchange

Pion lightest meson $\Longrightarrow$ dominates V_{NN} at large distance   Yukawa 1935: $\sim \frac{e^{-m_\pi r}}{r}$

πN symmetries: **chiral, isospin, parity, rotation**, plus simplest form: fewest derivatives



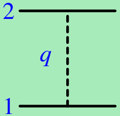
$: -\frac{g_A}{2f_\pi} \underbrace{\tau^a \vec{\sigma}}_{N \text{ isospin \& spin}} \cdot \vec{q}$

No interaction for $\vec{q} \to 0$: π decouples by chiral symmetry.

$\Longrightarrow$ **One Pion Exchange Potential (OPE)**

all parameters fixed by πN!

$$V_{OPE} = -\frac{g_A^2}{4f_\pi^2} \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^2 + m_\pi^2} \tau_1^a \tau_{2a}$$

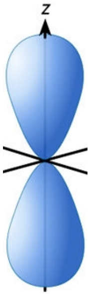


$\vec{\sigma} \cdot \vec{q}$ **Spin-dependent**: strongest $\substack{\text{attraction} \\ \text{repulsion}}$ for $\vec{q}$ $\substack{\text{along} \\ \text{opposite}}$ N spin.

$\Longrightarrow$ πN is a P-wave interaction, like magnetic dipole in external field $\vec{\sigma} \cdot \vec{B}$.

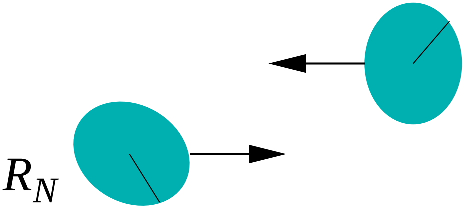
$\Longrightarrow$ NN interacts like dipole-dipole: **tensor force, angle-dependent**.

$\tau_1^a \tau_{2a} = 2I(I+1) - 3$: **Isospin-dependent "iso-tensor" interaction.**



$\Longrightarrow$ **Study partial-wave decomposition in isospin, spin and angular momentum!**

NN in the $^1S_0$- and $^3S_1-{}^3D_1$ Waves: Unnatural Scales are Natural



Zero-momentum cross section $\sigma(k=0)=4\pi\,a^2$.

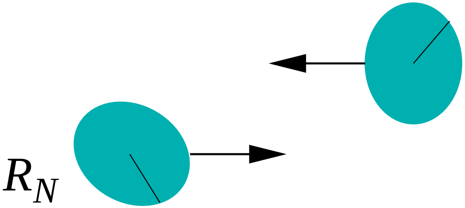
If $a>0 \Longrightarrow$ bound state with energy $B\approx\dfrac{1}{2\mu a^2}$

Scatt. length a: naïve hard-sphere geometry: $a=2R_N$

$\Longrightarrow$ "Natural size": $a\sim R_N\sim\dfrac{1}{m_\pi}\approx 1.5\,\mathrm{fm}$ Yukawa range.

πN System: $a^+=\dfrac{0.008}{m_\pi}$, $a^-=\dfrac{0.078}{m_\pi}$ anomalously small – understood: **chiral symmetry**.

NN in the ${}^1\mathrm{S}_0$- and ${}^3\mathrm{S}_1-{}^3\mathrm{D}_1$ Waves: Unnatural Scales are Natural



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NN System:
$$\begin{cases} a({}^1\mathrm{S}_0)=[-23.71\pm0.03]\mathrm{fm}\approx\dfrac{-16.9}{m_\pi} & \textbf{nearly bound}\\ a({}^3\mathrm{S}_1)=[+5.432\pm0.005]\mathrm{fm}\approx\dfrac{3.9}{m_\pi} & \textbf{bound}\end{cases} \gg \text{range}\sim\frac{1}{m_\pi}=1.4\mathrm{fm}$$

The Deuteron $I(J^{PC})=0(1^{+-})\;[{}^{2S+1}L_J=({}^3\mathrm{S}_1\text{-}{}^3\mathrm{D}_1)]$ is the *only* NN bound state:

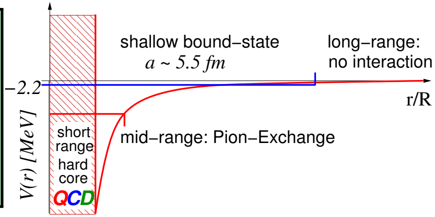
binding energy $B_d=2.2244573\,\mathrm{MeV}\ \ll$ natural size $\dfrac{m_\pi^2}{M_N}\approx 20\,\mathrm{MeV}$ if Yukawa alone (dim. an.)

Unnaturally shallow bound state & large scattering lengths:

χEFT long-range attractive Yukawa, but phenomenology:

compensate by short-distance: *repulsive core*.

Necessary fine-tuning not yet fully understood in QCD!



There is no principle built into the laws of Nature that says that theoretical physicists have to be happy.

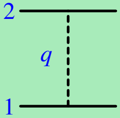
S. Weinberg (3.5.1933 – 23.7.2021)
on NOVA: THE FABRIC OF THE COSMOS
– Episode 3: Quantum Leap (2011)

Deuteron and $^3\mathrm{S}_1$-$^3\mathrm{D}_1$: The Partial-Wave Projected LO OPE

Project into partial waves

& Fourier transform:

$$V_{OPE} = -\frac{g_A^2}{4f_\pi^2}\,\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{\vec{q}^{\,2}+m_\pi^2}\;\tau_1^a\tau_{2a}$$



Central Potential is Yukawa: $V_C(r) = -\frac{g_A^2\,m_\pi^2}{16\pi f_\pi^2}\,\frac{\mathrm{e}^{-m_\pi r}}{r} < 0 \quad \xrightarrow{\text{chiral limit}} 0$

Tensor Potential: $V_T(r) = -\frac{g_A^2\,m_\pi^2}{16\pi f_\pi^2}\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\frac{\mathrm{e}^{-m_\pi r}}{r} < 0 \quad \xrightarrow{\text{chiral limit}} -\frac{3g_A^2}{16\pi f_\pi^2}\,\frac{1}{r^3}$

Strength: $\frac{g_A^2\,m_\pi^2}{16\pi f_\pi^2} \overset{\text{Goldberger-Treiman}}{=} \frac{g_{\pi NN}^2}{4\pi}\,\frac{m_\pi^2}{4M_N^2} = \alpha_{NN}\,\frac{m_\pi^2}{4M_N^2}$ – “nuclear” $\alpha_{NN} \approx 13.9 \Longrightarrow$ **Nonperturbative!**

$$V_{OPE}[S=0] = V_C(r)\times\begin{cases} -3 & \text{: repulsive for } I=0\text{, i.e. } L \text{ odd} \\ +1 & \text{: attractive for } I=1\text{, i.e. } L \text{ even } ({}^1\mathrm{S}_0!) \end{cases}$$

$$V_{OPE}[S=1] = \frac{1}{3}\left[V_C(r)+S_{12}(\vec{\mathrm{e}}_r)\;V_T(r)\right]\times\begin{cases} +3 & \text{: attractive for } I=0\text{, i.e. } L \text{ even (deuteron!)} \\ -1 & \text{: repulsive for } I=1\text{, i.e. } L \text{ odd} \end{cases}$$

Pion tensor force couples S and D waves in deuteron:

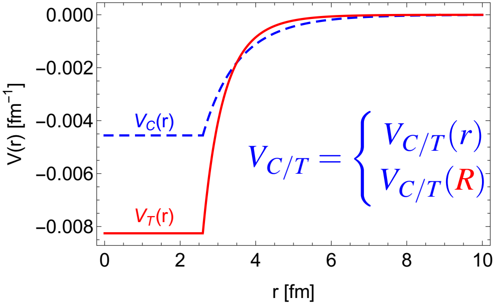
$$\frac{1}{M}\,\frac{\partial^2}{\partial r^2}\begin{pmatrix} u(r) \\ w(r) \end{pmatrix} = \begin{pmatrix} -E+V_C(r) & \sqrt{8}\;V_T(r) \\ \sqrt{8}\;V_T(r) & -E+\underbrace{\frac{6}{Mr^2}}_{\text{centrifugal}}+[V_C(r)-2V_T(r)] \end{pmatrix}\begin{pmatrix} u(r) \\ w(r) \end{pmatrix}$$

The Problem: Wave Functions Collapse at Short Range

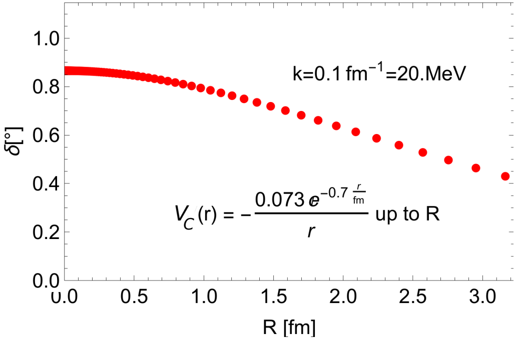
For $(m_\pi r) \to 0$ (short distance/chiral limit): $-\frac{3g_A^2}{16\pi f_\pi^2}\,\frac{1}{r^3}\begin{pmatrix} 0 & \sqrt{8} \\ \sqrt{8} & -2 \end{pmatrix}$ with EVals $\begin{pmatrix} 4 \\ -2 \end{pmatrix}\frac{3g_A^2}{16\pi f_\pi^2}\,\frac{1}{r^3}$.

A little project: Sensitivity of phase-shift

on short-distance with shooting method. [HH: QM-I/II]



$$V_{C/T} = \begin{cases} V_{C/T}(r) & \text{unchanged} \\ V_{C/T}(R) & \text{“cut off" at } R \end{cases}$$



Use “realistic” parameters for V_C ($^1\mathrm{S}_0$) & V_T ($^3\mathrm{S}-\mathrm{D}_1$).

$\Longrightarrow$ V_C stronger than V_T for $r \gtrsim 3\text{fm}$.

—- Only at short distances does V_T win.

Take $k_{\text{cm}} = 20\text{MeV} \ll \Lambda_\chi \lesssim \frac{2\pi}{R}$: **EFT Folk Theorem**

Expect no sensitivity on short-distance, i.e. on R or form.

✓ for V_C (Coulombic)

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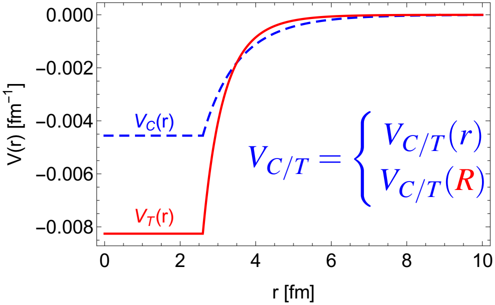

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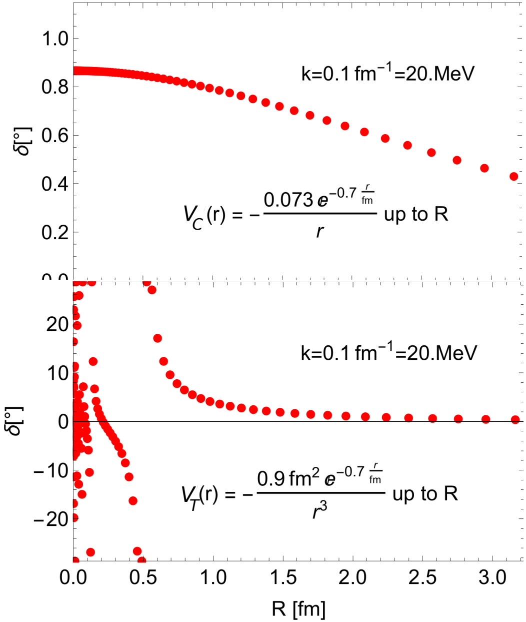


Thomas Effect: Attr. $\frac{1}{r^3}$ not self-adjoint!

$\Longrightarrow$ Wave function collapses to $r = 0$!

The Solution to Collapsing Wave Functions: EFT

The EFT Tenet **Weinberg1979**

Short-distance physics does not have to be right for a good calculation, because a low-energy process cannot probe details of the high-energy structure.

χEFT: long-range/low-energy correct.

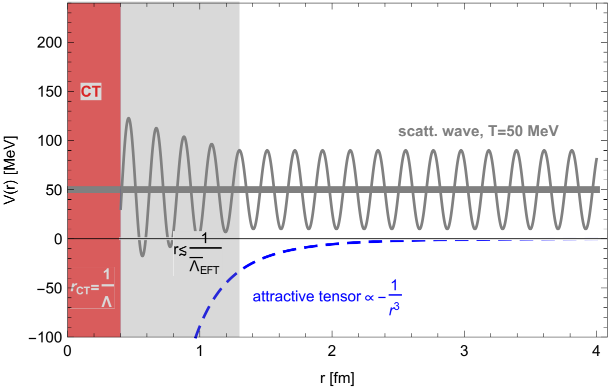
$\Longrightarrow$ Add short-range repulsive core

to stabilise system against collapse!

Simplest: Point-interaction  : $-\mathrm{i}\,C$

without structure/derivative/form factor

renders cutoff-independence at *all(!)* k.



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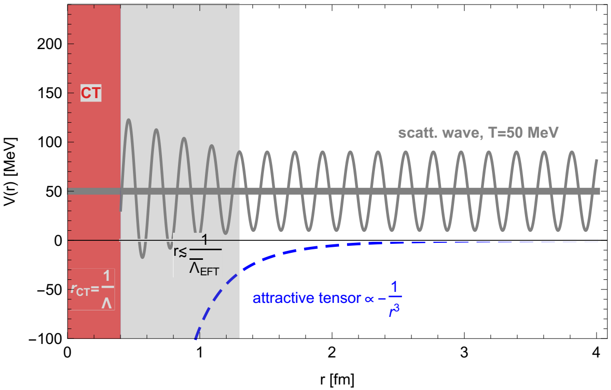


RGE: Adjust CT strength $C(R = \frac{1}{\Lambda})$ with $R = \frac{1}{\Lambda \gtrsim \overline{\Lambda}_\chi}$ so that observables cutoff-independent.

Initial condition set by one datum: scatt. length, B_d,. . . ; $\mathcal{O}(k)$ predicted, only residual Λ-dep.

In line with unnaturally shallow bound state & large scattering lengths in ${}^3\mathrm{S}_1$ and ${}^2\mathrm{S}_0$:

OPE should be attractive, but not too much: compensate by *repulsive core*.

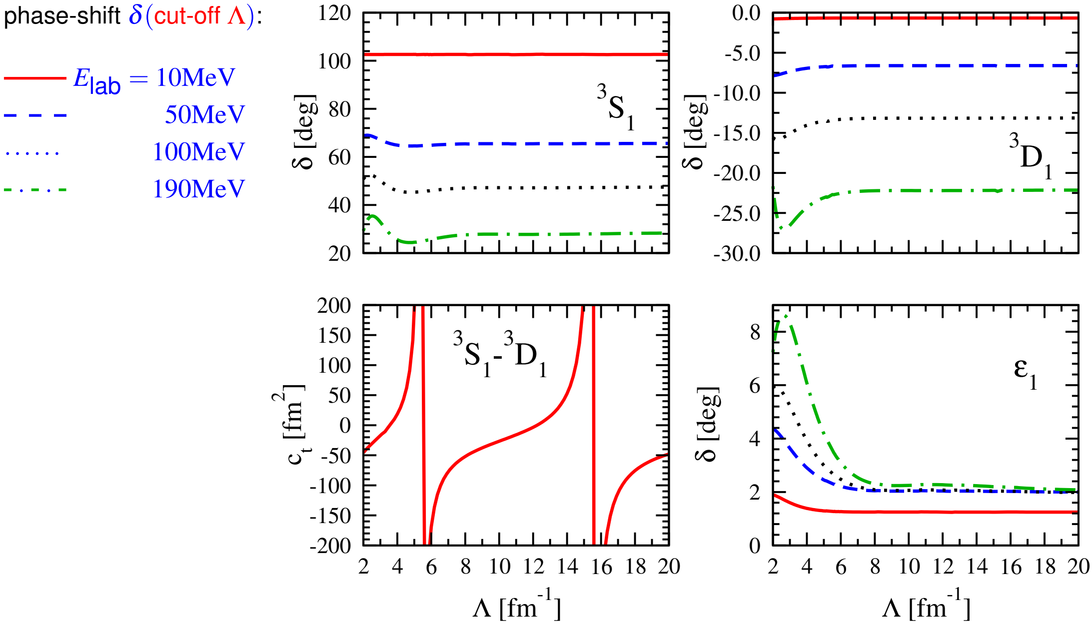
Counter Terms by Λ Independence

Beane/. . . 2002, Nogga/Timmermans/van Kolck 2005, Birse 2005-07;
NLO: Song/Lazauskas/van Kolck 1612.09090

Check: Observables dependent on cut-off $\Lambda = 1/R$ at LO with $c(\Lambda)$ fixed by B_d?

Other channels: Need 4 more, *new*, momentum-dependent LECs for low attractive triplets: ${}^3\mathrm{P}_{0,2}$, ${}^3\mathrm{D}_{2,3}$.

phase-shift $\delta(\text{cut-off } \Lambda)$:



Similar for all repulsive waves & attractive singlets $(\pm\frac{1}{r})$.

NN χEFT Power Counting Comparison

prepared for Orsay Workshop by Grießhammer 7.3.2013
based on and approved by the authors in private communications

Derived with explicit & implicit assumptions; contentious issue.

All but WPP: RGE as construction principle, but different approximations at short-range lead to variant interpretations.

Proposed order Q^n at which counter-term enters *differs*. $\Longrightarrow$ Predict *different* accuracy, # of parameters.

<table>
<tr><th>order</th><th>Weinberg (modified)
[PLB251 (1990) 288 etc.]</th><th>Birse
[PRC74 (2006) 014003 etc.]</th><th>Pavon Valderrama et al.
[PRC74 (2006) 054001 etc.]</th><th>Long/Yang
[PRC86(2012) 024001 etc.]</th></tr>
<tr><td>Q^{-1}</td><td colspan="4">LO of $^1S_0$, $^3S_1$, OPE</td></tr>
<tr><td></td><td></td><td>plus $^3D_1$, $^3SD_1$</td><td>plus $^3P_{0,2}$, $^3D_2$</td><td>plus $^3P_{0,2}$</td></tr>
<tr><td>$Q^{-\frac{1}{2}}$</td><td>none</td><td>LO of $^3P_{0,1,2}$, $^3PF_2$, $^3F_2$, $^3D_2$</td><td>LO of $^3SD_1$, $^3D_1$, $^3PF_2$, $^3F_2$</td><td>none</td></tr>
<tr><td>Q^0</td><td>none</td><td colspan="3">NLO of $^1S_0$</td></tr>
<tr><td>$Q^{\frac{1}{2}}$</td><td>none</td><td>NLO of $^3S_1$, $^3D_1$, $^3SD_1$</td><td>none</td><td>none</td></tr>
<tr><td>Q^1</td><td>LO of $^3SD_1$, $^1P_1$, $^3P_{0,1,2}$; NLO of $^1S_0$, $^3S_1$</td><td>none</td><td>none</td><td>LO of $^3SD_1$, $^1P_1$, $^3P_1$, $^3PF_2$; NLO of $^3S_1$, $^3P_0$, $^3P_2$; N^{2}LO of $^1S_0$</td></tr>
<tr><td># at Q^{-1}</td><td>2</td><td>4</td><td>5</td><td>4</td></tr>
<tr><td># at Q^0</td><td>$+0$</td><td>$+7$</td><td>$+5$</td><td>$+1$</td></tr>
<tr><td># at Q^1</td><td>$+7$</td><td>$+3$</td><td>$+0$</td><td>$+8$</td></tr>
<tr><td>total at Q^1</td><td>9</td><td>14</td><td>10</td><td>13</td></tr>
</table>

With same χ^2/d.o.f., proposal with least parameters *wins*: minimum information bias.

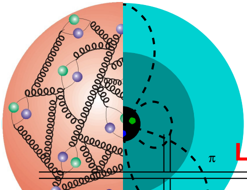
Few-Nucleon Interactions in χEFT

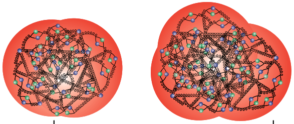
Weinberg, Ordóñez/Ray/van Kolck, Friar/Coon, Kaiser/Brockmann/Weise, Epelbaum/Glöckle/Meißner, Entem/Machleidt, Kaiser, Higa/Robilotta, Epelbaum, . . .

$\frac{\text{typ. momentum}}{\text{breakdown scale}} \ll 1$

Long-Range: correct symmetries and IR degrees of freedom: **Chiral Dynamics**

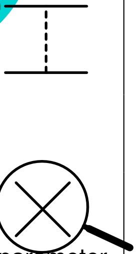
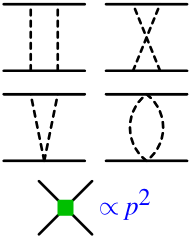
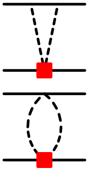
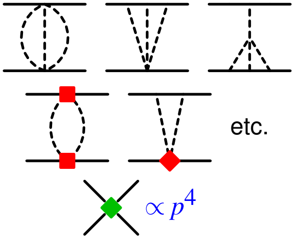
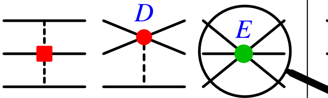
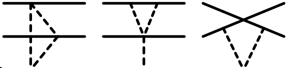
Short-Range: symmetries constrain contact-ints to simplify UV: **Minimal parameter-set**





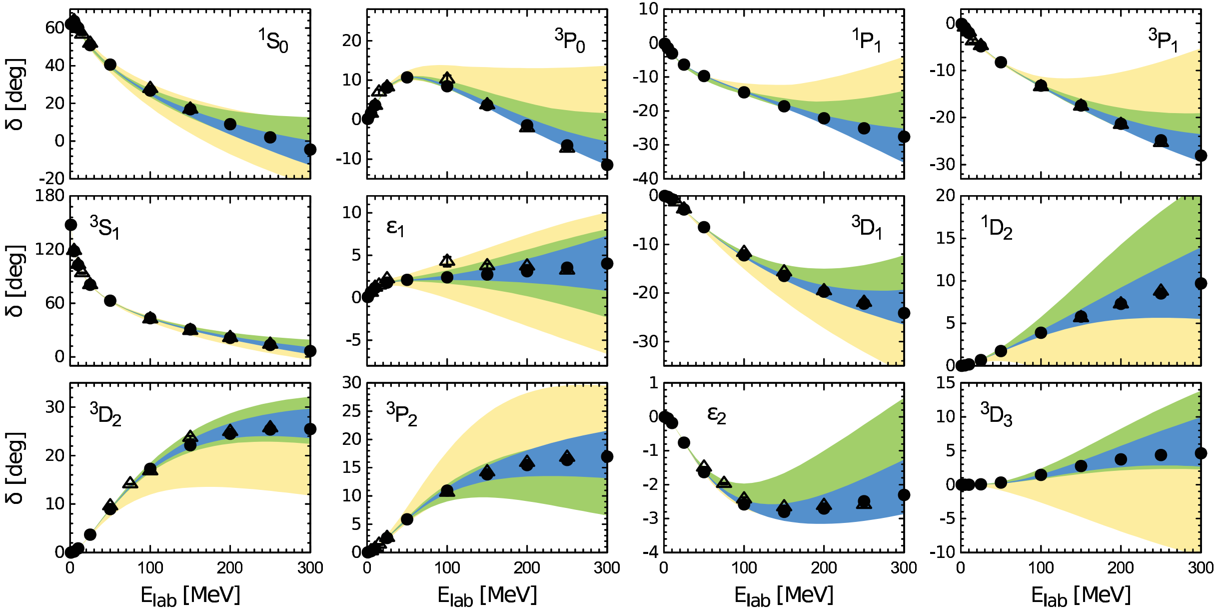
Add LEC only to ensure independence of short-distance.

Hierarchy: 2NF-effects $\gg$ 3NF-effects $\gg$ 4NF-effects

	LO	NLO	N^{2}LO	N^{3}LO
$2N$ ints	 2 parameter	 $\propto p^2$ $+7$ parameter	 — $+0$ parameter	 etc. $\propto p^4$ $+15 = 24$ param.
$\frac{\chi^2}{\text{d.o.f}}$ in np		36.2	10.1	1.06 (AV 18: 1.04)
$3N$ ints	—	—	 D E 2 parameter	 etc. parameter-free, in progress
$4N$ ints	—	—	—	 etc. parameter-free

(i) Selected (Biased) Accomplishments

np Scattering Phase Shifts: Bands Estimate Higher-Order Effects



Fewer free parameters than traditional. [Epelbaum/…1412.0142]

Converges order-by-order
– and even to Nature.

	LO	NLO	N^{2}LO	N^{3}LO	AV 18
# of parameters	2	$+7$	$+0$	$+15=24$	~ 40
χ^2/d.o.f in *np*		36.2	10.1	1.06	1.04

The Deuteron $I(J^{PC}) = 0(1^{+-})[{}^3\mathrm{S}_1\text{-}{}^3\mathrm{D}_1]$

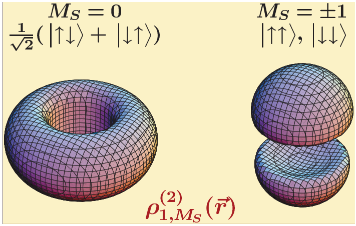
$$\Psi_d(r)\frac{1}{\sqrt{4\pi r}}\Big[\underbrace{u(r)}_{S\text{ wave}}+\underbrace{S_{12}(\vec{e}_r)\frac{w(r)}{\sqrt{8}}}_{D\text{ wave: tensor}}\Big]\chi_{1M}\longleftarrow\text{spin-triplet wf}\qquad\text{with}\int\limits_0^\infty\mathrm{d}r\,[u(r)^2+w(r)^2]=1$$

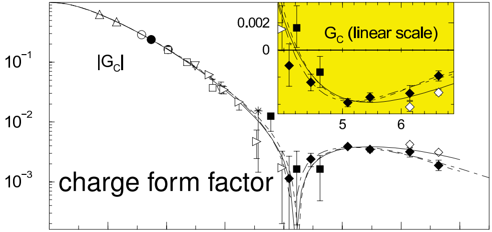
Asymptotic wave function decays with **“binding momentum”** $\gamma = \sqrt{MB_d} = 45.70\ldots\ \mathrm{MeV}$:

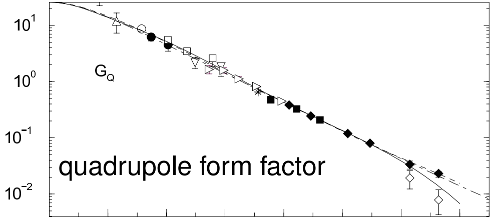
$$\lim_{r\to\infty}\begin{pmatrix}u(r)\\w(r)\end{pmatrix}\propto\begin{pmatrix}1\\ \eta\end{pmatrix}\mathrm{e}^{-\gamma r}$$

with **asymptotic D-to-S wave ratio** $\eta_{\text{exp}} = 0.02544$

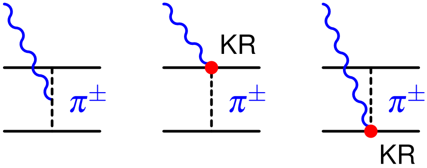
D wave $\Longrightarrow$ deformation $\Longrightarrow$ electric quadrupole moment $Q_d = [0.2859 \pm 0.0003]\ \mathrm{fm}^3$ [II.1.b]





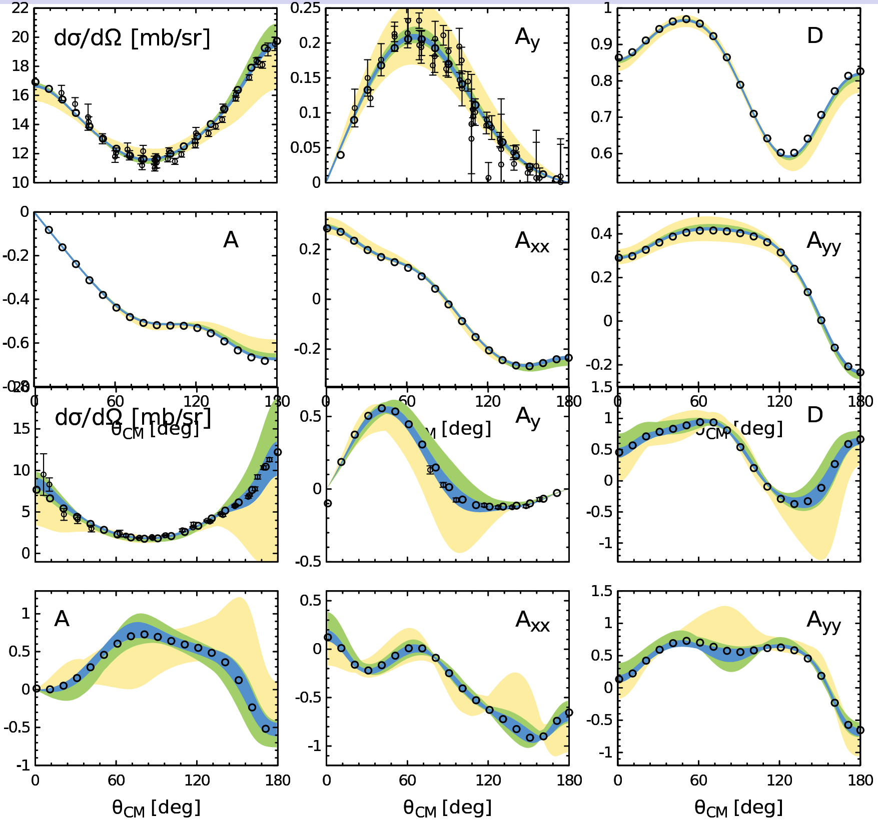


Tests Kroll-Rudermann in coupling to $\pi^\pm$ exchange:



np Scattering Observables at $E_{\rm cm} = 50$ & $200 \rm MeV$

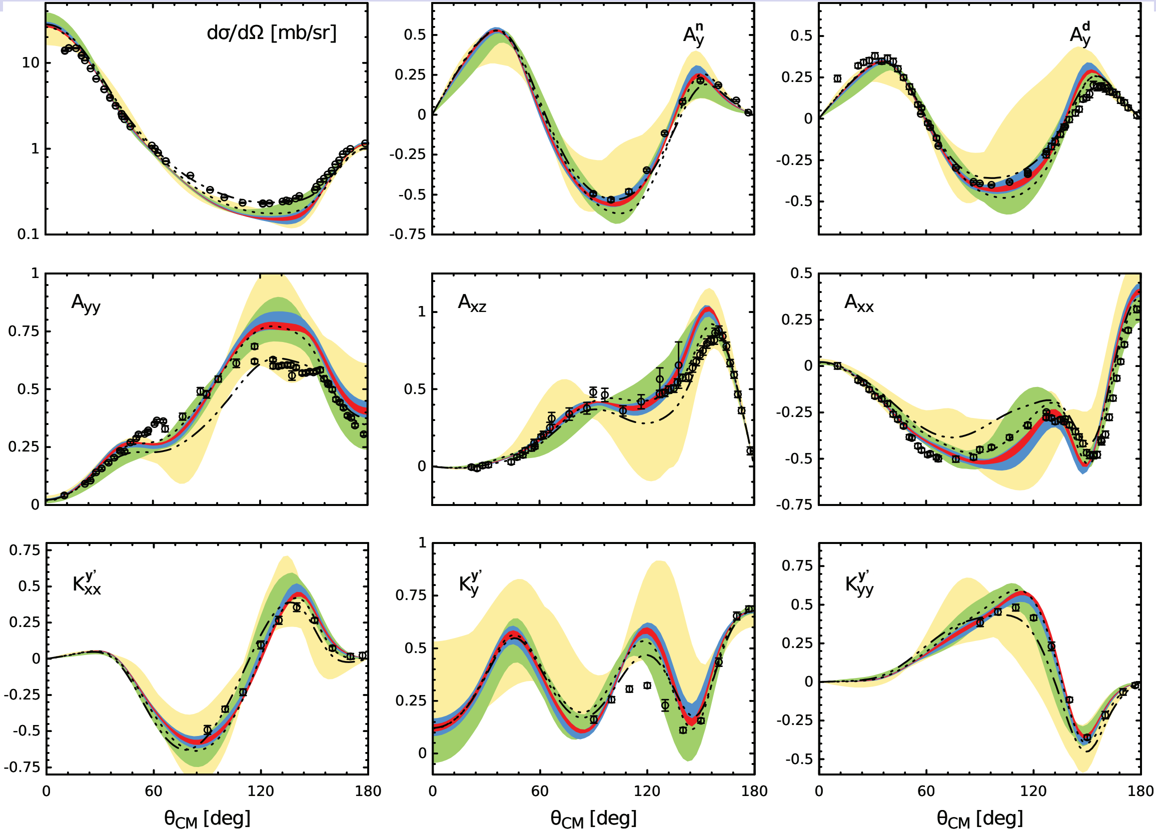
Epelbaum/…1412.0142



Bands estimate theoretical uncertainties by higher-order effects: LO → **NLO** → **N^{2}LO**

3N: Polarised Deuteron-Proton Scattering

Epelbaum/. . . [arXiv:1802.08584]



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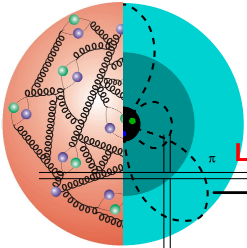
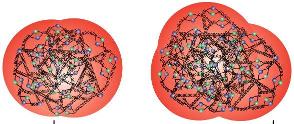
Long-Range: correct symmetries and IR degrees of freedom: **Chiral Dynamics**

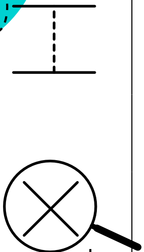
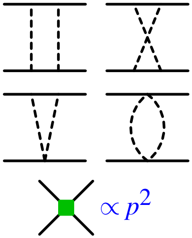
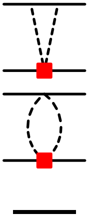
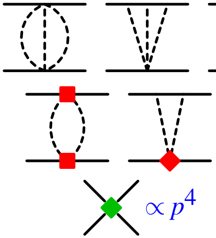
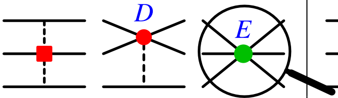
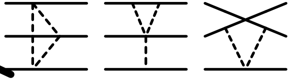
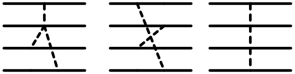
Short-Range: symmetries constrain contact-ints to simplify UV: **Minimal parameter-set**





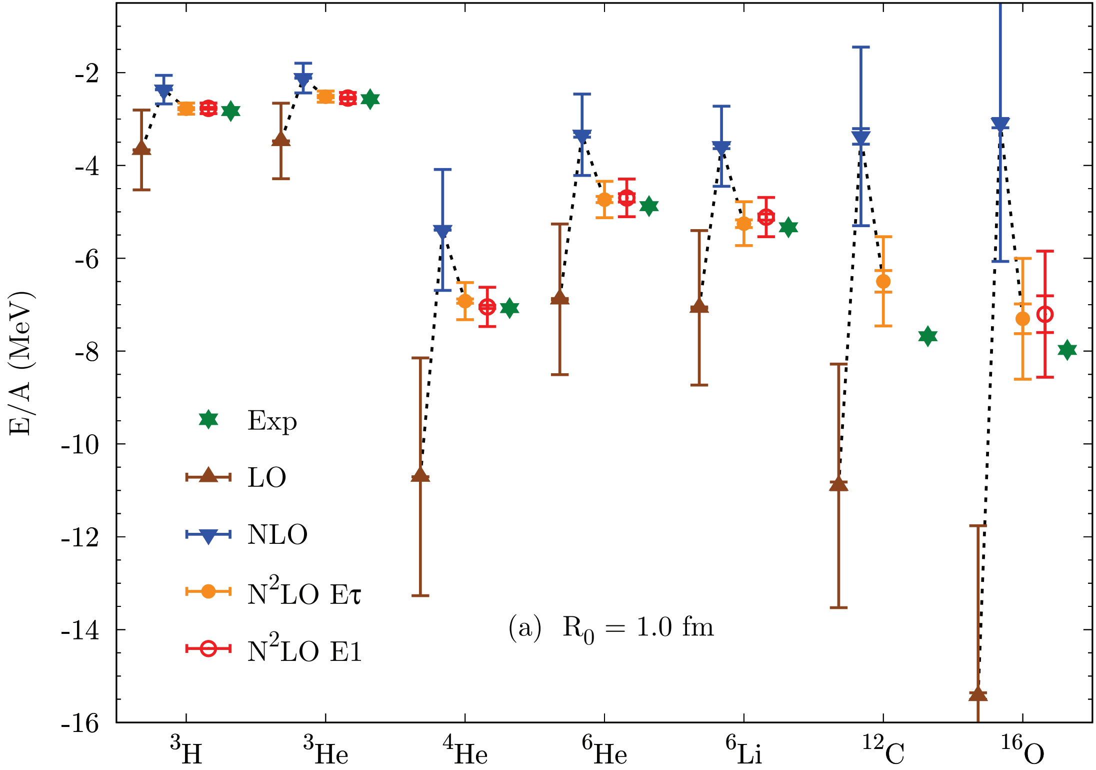
Add LEC only to ensure independence of short-distance.

Hierarchy: 2NF-effects ≫ 3NF-effects ≫ 4NF-effects

	LO	**NLO**	**N^{2}LO**	**N^{3}LO**
$2N$ ints	 2 parameter	 $\propto p^2$ $+7$ parameter	 $+0$ parameter	 $\propto p^4$ etc. $+15 = 24$ param.
$\frac{\chi^2}{\text{d.o.f}}$ in np		36.2	10.1	1.06 (AV 18: 1.04)
$3N$ ints	—	—	 D E 2 parameter	 etc. parameter-free, in progress
$4N$ ints	—	—	—	 etc. parameter-free

Ground States of Light Nuclei

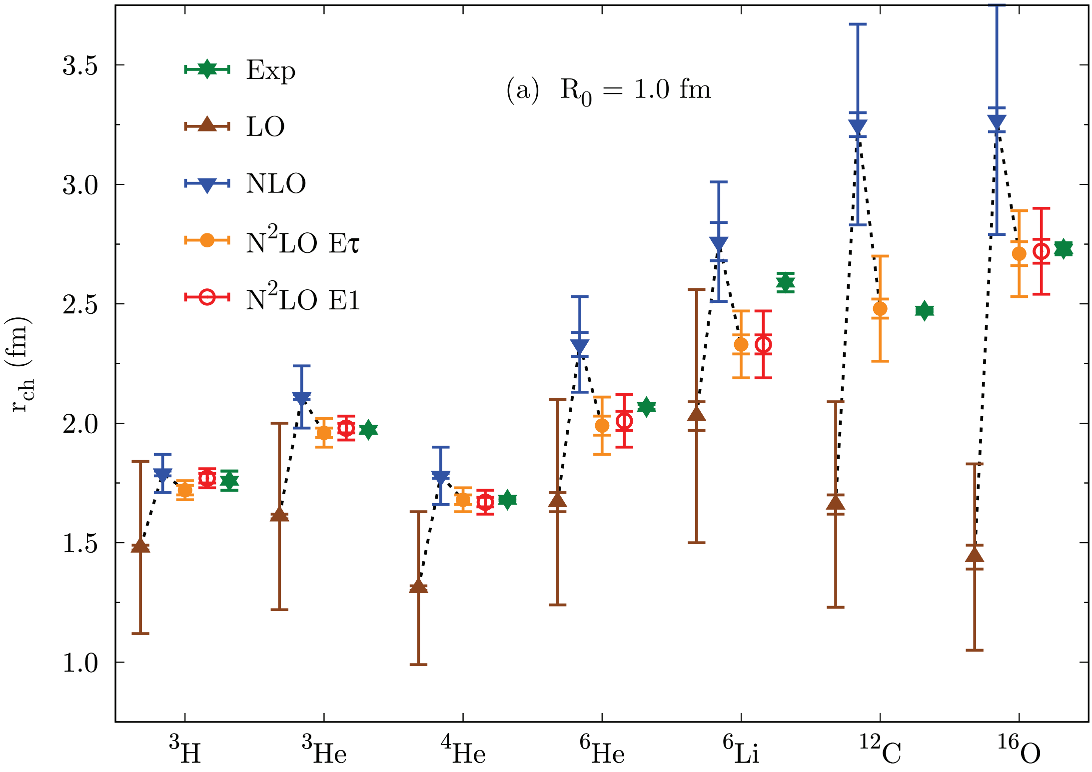
Lonardoni/. . .
PRC 97 (2021) 044318



Notice order-by-order shrinking theory uncertainties (Bayesian assessment).

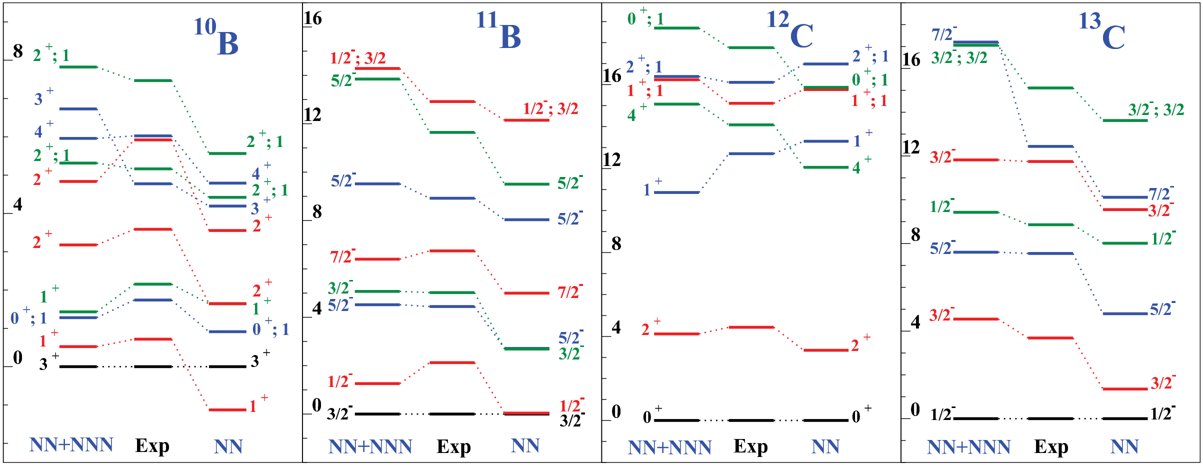
Charge Radii of Light Nuclei

Lonardoni/. . .
PRC 97 (2021) 044318



Notice order-by-order shrinking theory uncertainties (Bayesian assessment).

Starting on Spectra of Less-Light Nuclei (with 3NI)



[Navratil/. . . : Phys. Rev. Lett. **99** (2007) 042501]

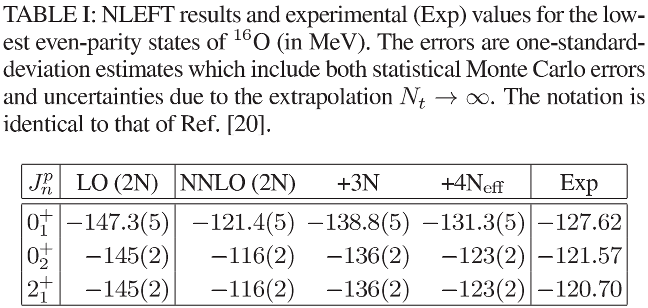


TABLE I: NLEFT results and experimental (Exp) values for the lowest even-parity states of ^{16}O (in MeV). The errors are one-standard-deviation estimates which include both statistical Monte Carlo errors and uncertainties due to the extrapolation $N_t \to \infty$. The notation is identical to that of Ref. [20].

J_n^p	LO (2N)	NNLO (2N)	+3N	+4N$_{\rm eff}$	Exp
0_1^+	$-147.3(5)$	$-121.4(5)$	$-138.8(5)$	$-131.3(5)$	-127.62
0_2^+	$-145(2)$	$-116(2)$	$-136(2)$	$-123(2)$	-121.57
2_1^+	$-145(2)$	$-116(2)$	$-136(2)$	$-123(2)$	-120.70

[Epelbaum/. . . : Phys. Rev. Lett. **112** (2014) 102501]

Heavier Nuclei: Ca Isotope Binding

Forssen/Ekström/. . . [arXiv:2006.16774]

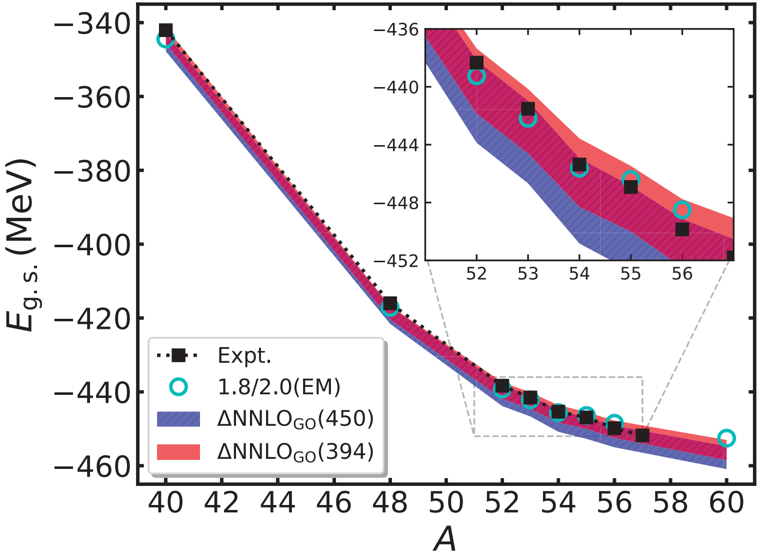
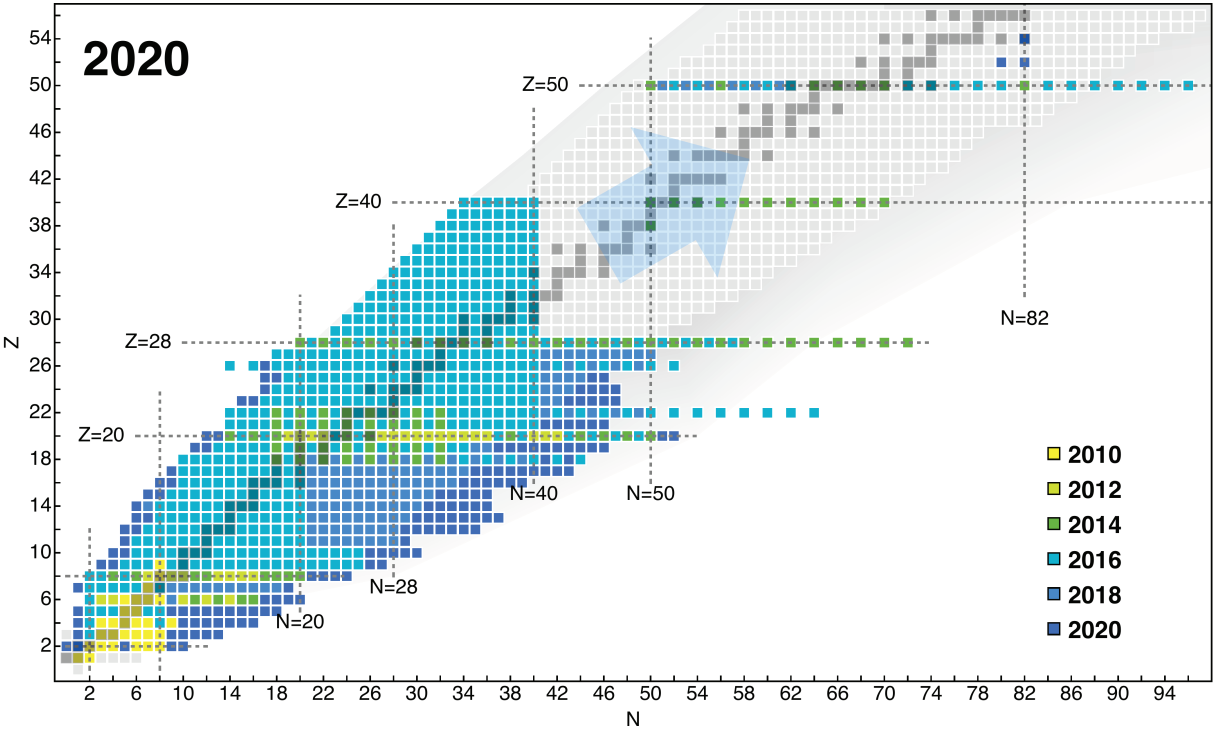


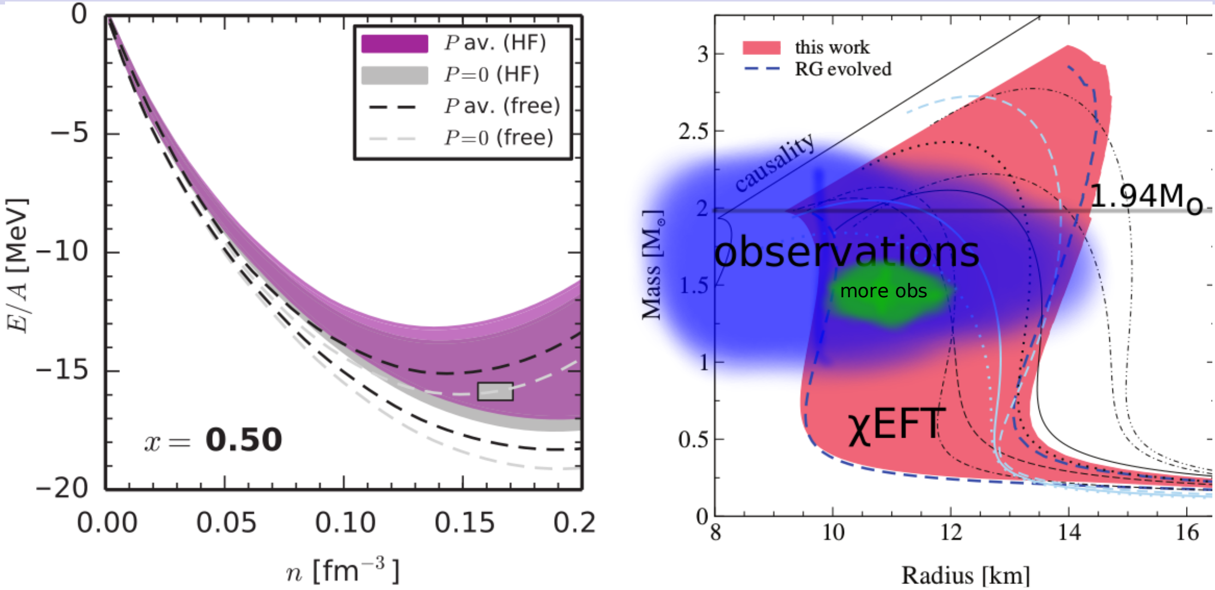
FIG. 3. (Color online) The ground-state energies of calcium isotopes obtained with ΔNNLO$_{\rm GO}$ and 1.8/2.0(EM) interaction compared with experiment (data of $^{55-57}$Ca are taken from Ref. [70]).

Heavy Nuclei: How Far With “Microscopic” Interactions?

Hergert [2008.05061]



Chiral EoS: Neutron Star Mass-Radius Relation



[Drischler/. . . [arXiv:1510.06728 [nucl-th]]; Krüger/. . . [arXiv:1304.2212 [nucl-th]]]

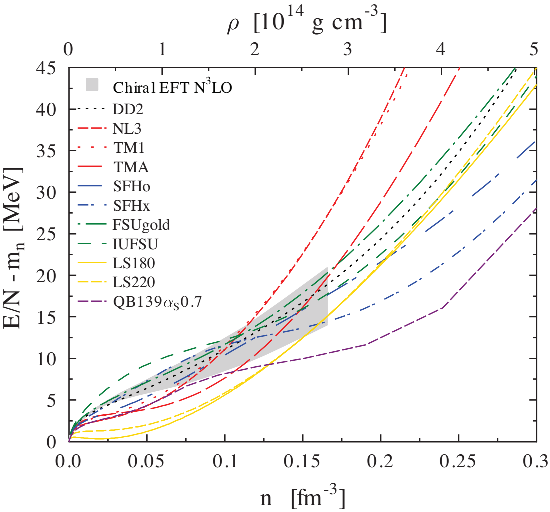
Astro data added by hand (hgrie) – certainly more. . .

Corridors provide honest uncertainty assessment: know what to improve and how.

Right now, can explain bulk observations *without* exotic matter inside neutron star.

What about flares/glitches/. . . ?

Chiral EoS for Neutron Matter and Neutron Stars



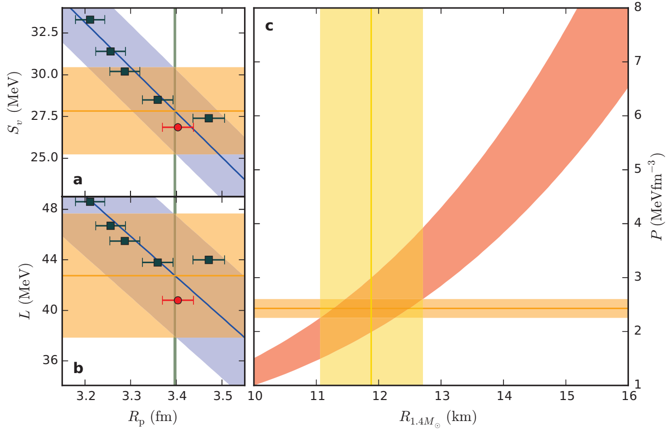


Figure 4 | Properties of the nuclear equation of state and neutron star radii based on chiral interactions. a, The symmetry energy S_v and **b**, the slope L of the symmetry energy at predicted saturation densities versus the point-proton radius in ^{48}Ca. **c**, Pressure-radius relationship for a neutron star of mass M=1.4$M_\odot$ (red band) from the phenomenological expression of refs. 30,31. The predicted pressure (horizontal orange band) constrains the neutron star radius (vertical yellow band).

Fig. 2 Energy per baryon in pure neutron matter for different supernova EoS, compared to results of χEFT (grey band [228]), from Ref. [229].

[Blaschke/…[arXiv:1803.01836 [nucl-th]]]

[Hagen/…[arXiv:1509.07169 [nucl-th]]]

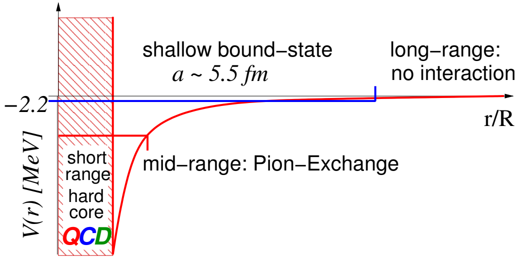
Error bars in χEFT vs. no error bars in models. – More work needed!

χEFT and Lattice QCD: Exploring Alternative Worlds

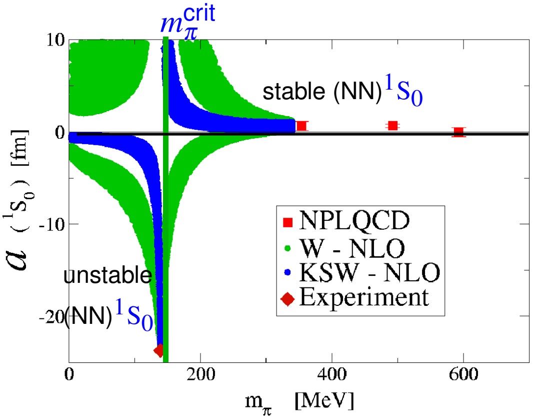
Vary QCD parameters using χEFT: $m_q \propto m_\pi^2, \dots$

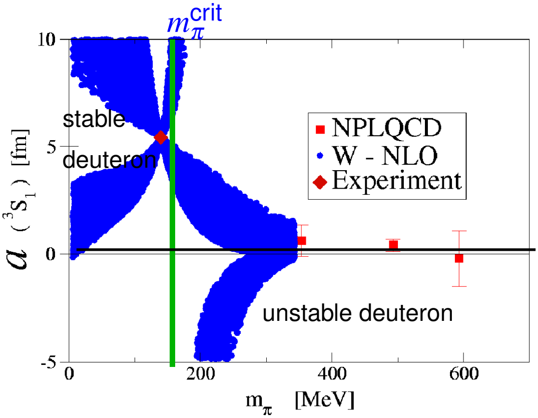
$\Longrightarrow a_{\text{NN}}$ **diverge at** $m_\pi^{\text{crit}} \approx 197\text{MeV}$**??**

$\Longrightarrow$ **QCD Critical Point: zero NN binding energy.**



m_π-dependence of NN-scatt. lengths: χEFT & lattice



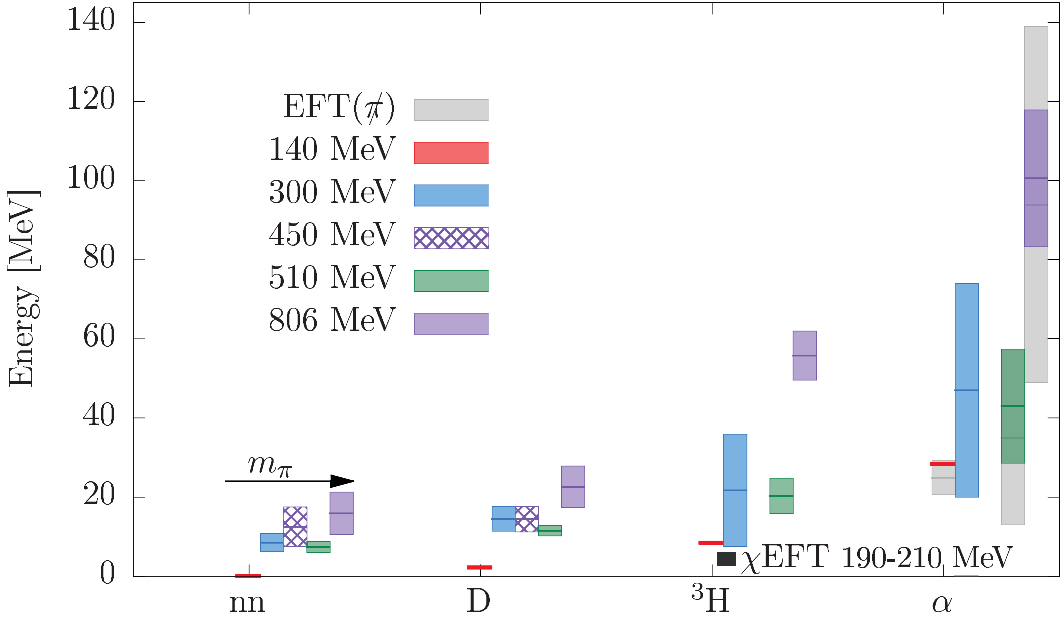


- $^1\text{S}_0$ with bound state for $m_\pi > 160$ MeV? $\Longrightarrow nn, pp$ bound!
- What is the deuteron binding energy for $m_\pi \neq 140$ MeV?
- Explain fine-tuning of NN-scattering lengths, origin of few-N interactions.
- Fix parameters hard to determine experimentally: **weak int.'s** test SM; πNN- & YN-couplings. . .

Alternative Worlds: Lightest Nuclei at Higher Pion Masses

NPLQCD
HALQCD

Merger of EFT and lattice has started exploring how few-nucleon systems emerge from QCD.



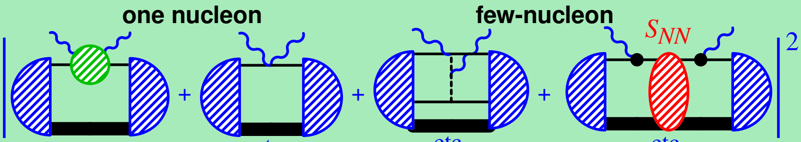
[J. Kirscher [arXiv:1509.07697 [nucl-th]] (got his PhD in GW's EFT group)]

Surprisingly little change in few-nucleon systems – but nn becomes bound when m_π increased!

(j) Neutron Polarisabilities & Nuclear Binding

How to Get to the Neutron?

deuteron: hg/.../+Phillips/+McGovern 2004-
MECs: Beane/... 1999-2005
^{3}**He:** Shukla/... 2009 +Strandberg/Margaryan/hg/... 1804.01206



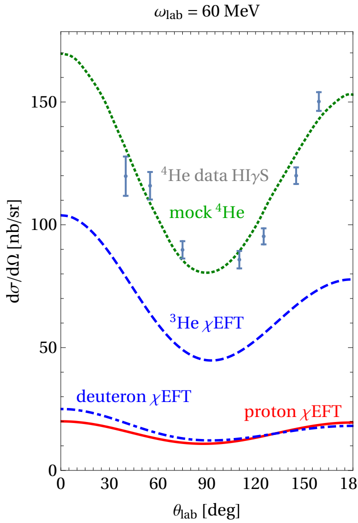
N **structure** $\alpha_{E1}, \beta_{M1}, \gamma_i$

etc.
N Born: charge & mag. moment

etc.
$\pi^\pm$ exchange
largest part

etc.
coherent rescattering relevant for $\omega \to 0$

Parameter-free, except N polarisabilities.



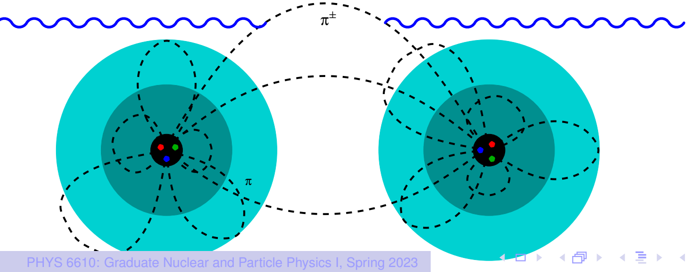
Experiment: More charge & MECs $\Longrightarrow$ more counts $\Longrightarrow$ *heavier nuclei*

Theory: Reliable only if nuclear binding & levels accurate $\Longrightarrow$ *lighter nuclei*

Find *sweet-spot* between competing forces: deuteron, ^{3}He, ^{4}He.

Deuteron, ^{4}He: sensitive to $\alpha_{E1}^p + \alpha_{E1}^n, \quad \beta_{M1}^p + \beta_{M1}^n \Longrightarrow$ **neutron pols**

^{3}**He**: sensitive to $2\alpha_{E1}^p + \alpha_{E1}^n, 2\beta_{M1}^p + \beta_{M1}^n \Longrightarrow$ **neutron pols**



Model-independently subtract binding effects.

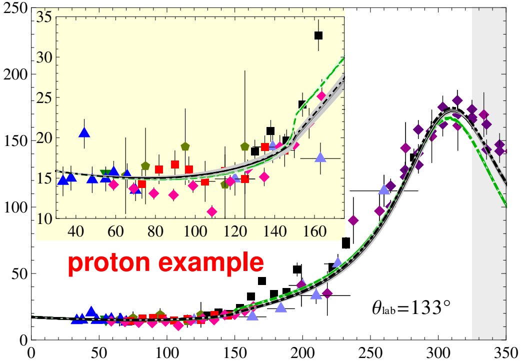
$\Longrightarrow$ χEFT: reliably quantify uncertainties.

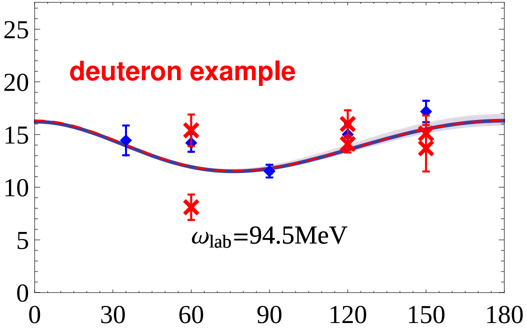
Chirally consistent 1N & few-N: potentials, wave functions, currents, π-exchange.

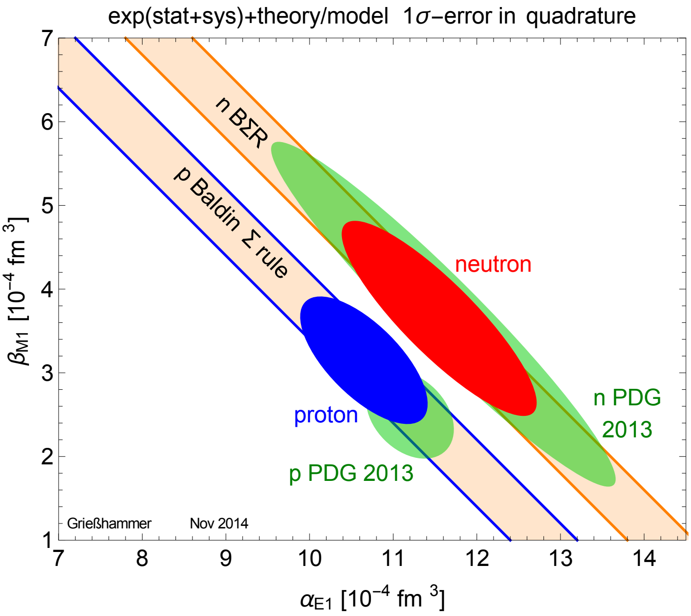
Test charged-pion component of NN force.

Proton & Neutron Values, Data & Theory Errors

McG/Ph/hg 2014
+Feldman/MAXlab 2014/15





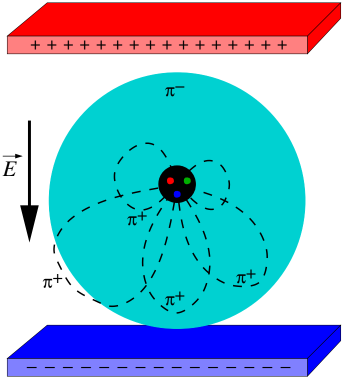


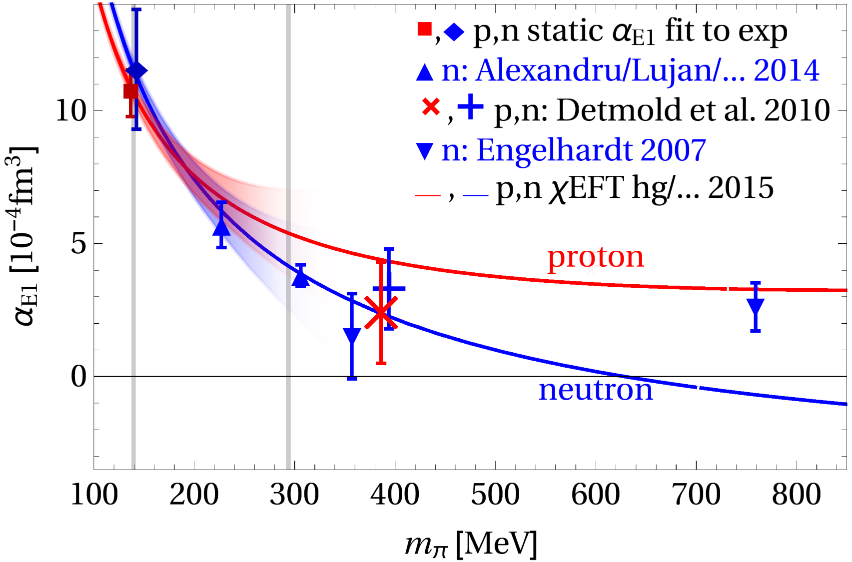
$\Longrightarrow$ **Neutron ≈ proton polarisabilities; exp. error dominates.**

Downie, Feldman,. . . spokespersons of Compton efforts at HIγS, MAMI,. . .

Hadron Polarisabilities: GW Leads Connecting Data & QCD GW focus

Needs to be phrased as energy-difference: $\Delta E = -2\pi\alpha_{E1}^{(N)}\vec{E}^2$.





Neither Approach Uses The Other To Fit!

[lattice: Lujan/Alexandru/Freeman/Lee [arXiv:1411.0047 [hep-lat]];
chiral extrapolation: hgrie/McGovern/Phillips [arXiv:1511.01952 [nucl-th]];
Downie/Feldman take data at HIγS, MAMI,. . .]

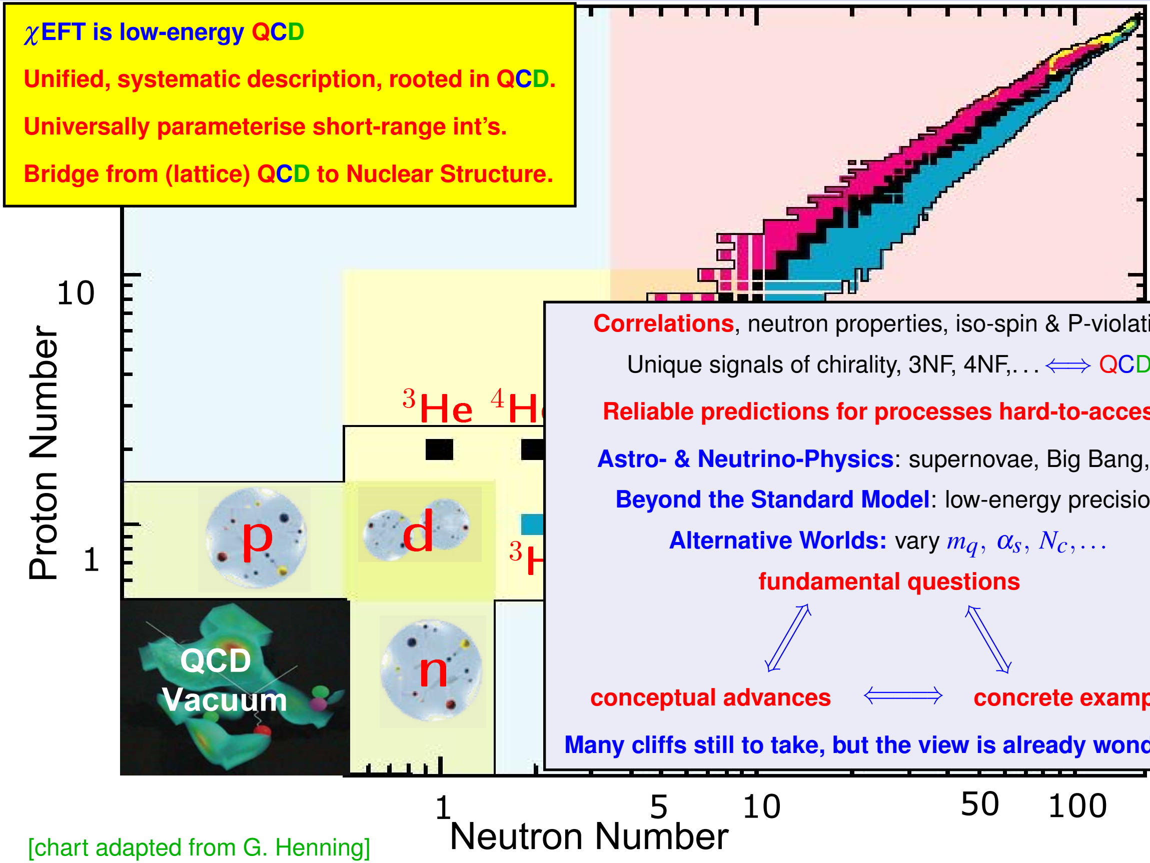
(k) Error-Bars for Nuclear Physics!

χEFT is low-energy QCD

Unified, systematic description, rooted in QCD.

Universally parameterise short-range int's.

Bridge from (lattice) QCD to Nuclear Structure.



Correlations, neutron properties, iso-spin & P-violation,

Unique signals of chirality, 3NF, 4NF,. . . $\Longleftrightarrow$ QCD

Reliable predictions for processes hard-to-access:

Astro- & Neutrino-Physics: supernovae, Big Bang,. . .

Beyond the Standard Model: low-energy precision

Alternative Worlds: vary $m_q,\ \alpha_s,\ N_c, \ldots$

fundamental questions

$\Updownarrow$ $\Updownarrow$

conceptual advances $\Longleftrightarrow$ **concrete examples**

Many cliffs still to take, but the view is already wonderful!

[chart adapted from G. Henning]

Next: 5. Weak Interactions

Familiarise yourself with: [phenomenology: PRSZR 10, 11, 12, 18.6; Per 7.1-6 – theory: Ryd 8.3-5; CL 11, 12; Per 7, 8, 5.4; most up-to-date: PDG 10, 12, 14 and reviews inside listings]